Universal Source Coding with Codebook Transmission

Kenneth Zeger, Anurag Bist, and Tamás Linder

Abstract—**A universal source coding system with vector quantizer codebook transmissions is studied using high resolution quantization theory. Conditions are derived for the optimal tradeoff between quantizer resolution and the information rate used to transmit codebooks. A formula that tightly bounds the mean squared error of the universal coding system as a function of the time between codebook transmissions is experimentally verified and found to be tight, and a new and simpler derivation is given. Other research in the literature has proposed vector quantizing the transmitted codebooks; one conclusion we prove here is that under some reasonable conditions uniform scalar quantization of the transmitted codebooks performs as well as vector quantizing them. Experimental results are given that support the analytic derivations.**

Keywords—**Universal lossy source coding, codebook transmission, vector quantization.**

I. INTRODUCTION

Vector Quantization (VQ) plays a critical role as an important building block of many lossy data compression systems and is generally designed based on the long term statistical behavior of a source. In many situations, however, (e.g. image coding) sources are encountered where real-time adaptation is desirable. An approach to providing this need is for a quantizer to be both *adaptive* and *universal* in nature. An adaptive quantizer is one in which changing source statistics induce changes in the quantization procedure or parameters, and a universal quantizer is one which is a priori able to successfully encode a large class of distinct sources. These two notions are very closely related and are encountered in lossless source coding, such as with Ziv-Lempel coding and Gallagher's adaptive Huffman coding. However, for lossy source coding, there is a significant gap in this area.

Universal schemes were shown to exist for both fixed rate and variable rate lossy coding. Ziv [1] has shown that for metric space valued alphabets satisfying some regularity conditions and for metric distortion measures, there exist fixed rate universal algorithms for the class of all stationary sources that asymptotically do as well for each source as an optimum source code designed for that source. Neuhoff et al. [2] developed a unified theory for fixed rate universal source coding, allowing different source and reproduction alphabets and more general distortion measures, and Matsuyama and Gray [3] have applied the idea of universal coding to tree encoding of speech. Chou [4] has designed weighted universal codes for image coding by using a finite collection of predesigned VQ codebooks. A short binary index is occasionally transmitted to specify to the decoder which codebook is being used. This scheme is limited in the sense that the number of different codebooks that can be used is rather small due to memory constraints.

Nasrabadi and Feng [9] used the notion of a "super-codebook", a large ordered codebook available at both the encoder and decoder, from which the first N codevectors serve as an operational vector quantizer in coding an image. The operational codebook can be adapted by trickling vectors in the super-codebook to the top and allowing others to fall down to lower positions. Their reordering method is heuristic in nature, and depends on the statistics of the previously used codevector indices. One potential difficulty with this scheme is that being finite state in nature, it cannot easily recover from channel errors. An earlier method for adaptive quantization was proposed by Gersho and Yano [10] that adaptively replenishes codevectors as the source statistics change, attempting to keep the partial distortions constant at each stage. In [11] [12] subsets of a universal codebook are used as reduced size codebooks, which are then used to encode the source training set. Also, in [11], adaptive quantization is performed by transmitting (as overhead) new codebooks to a receiver. For variable length universal codes see e.g. [5]-[7].

In this paper, we analyze a universal quantization technique based upon the occasional transmission of new codebooks. Strictly speaking, the notion of universality we consider is that of weak minimax universality (see [2]). This means that the analyzed scheme asymptotically achieves the rate distortion performance limit for stationary real sources. The number of input vectors between successive codebook transmissions will be called the *block size*. The main idea is that periodically, as the source statistics change, a new *target* VQ codebook, $\mathcal{C}_t$, of size N, is designed in real-time for the current source statistics and is then transmitted to the receiver. The real-time VQ design issue will not be addressed here, though we note that some fast design algorithms do currently exist. These include a fast "on-line" clustering technique [13], [14], a method using Kohonen's self-organizing feature maps [15], competitive learning [16], and gradient descent algorithms [17], [18]. Transmission of some approximation of the target codebook to the receiver requires that side-information be sent. The proposed technique is a fixed rate transmission scheme and potential channel errors will not propagate indefinitely.

If large amounts of side information are used to describe each new target codebook then these transmitted codebooks

Paper approved by Arun N. Netravali, the Editor for Image Processing of the IEEE Communications Society. Manuscript received July 23, 1992. This research was supported in part by the National Science Foundation under Grants No. NCR-90-09766 and NCR-92-96231 and by Hewlett-Packard Co.

Kenneth Zeger is with the Coordinated Science Laboratory and the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801 USA.

Anurag Bist is with the Department of Electrical Engineering, University of Hawaii, Honolulu, HI 96822 USA.

Tamás Linder is with the Faculty of Electrical Engineering, Technical University of Budapest, Hungary.

IEEE Log Number 9400915.

can be conveyed quite accurately and the performance of the actual quantizer that the receiver uses will be very close to that of the intended quantizer. However, accurately transmitting the target codebook "takes away" bits from the overall rate that could instead be used to transmit VQ indices from higher resolution codebooks. Thus, for a fixed transmission rate, there is a performance tradeoff in how many bits are sent as side information and how many as codevector indices.

Using both the MSE upper bound and the high resolution quantization development in [19], Chou and Effros [20] derived a formula for the optimal tradeoff. The formula shows that for large block sizes, the optimal number of bits used to transmit each codevector component grows as the logarithm of the block size. In Appendix A, we present an alternate and simpler derivation of the formula given in [20]. Also, we give, as a function of the block size, upper and lower asymptotic bounds on the optimal amount of side-information to transmit per codebook component.

We use high resolution quantization theory to analyze the performance of the universal coding system. Specifically, the optimal tradeoff between overhead bits used for transmitting new codebooks and the encoding bits sent as codevector indices is determined for various universal codebook designs, and is compared to experimental results. Two methods of codebook transmission are considered: (i) uniform scalar quantizing the codevector components, and (ii) vector quantizing the codevectors themselves. In the second scheme, which we call the *vector universal scheme* (see [21, pg. 620]), the main idea is that periodically, as the source statistics change, a new *target* VQ codebook, $\mathcal{C}_t$, of size N is designed for the source and then matched in a nearest neighbor manner to the N closest vectors in a large *universal* codebook, $\mathcal{C}_u$, of size M. The N matched vectors from $\mathcal{C}_u$ constitute the *operational* codebook, $\mathcal{C}_o$, which is used for coding by both the encoder and receiver as an approximation to $\mathcal{C}_t$. The operational codebook can be conveyed to the receiver by transmitting side information specifying some N-vector subset of $\mathcal{C}_u$. In this manner, the vector quantizer is itself being vector quantized for the purposes of transmitting its codebook. This technique improves upon those described in [4] and [9]. A theoretical and experimental comparison is given for the two schemes considered and we conclude that they perform approximately the same over a wide range of block sizes α.

Simulations results show that for a fixed stationary source and transmission rate, as the block size increases so does the performance. In practice, however, source statistics often vary quite quickly such as in an image source. Thus, there is a tradeoff in the choice of block size. If a smaller block size is chosen then the model is allowed to be more adaptive, whereas choosing a large block size improves the SNR for a fixed stationary source.

The paper is organized as follows. Section II describes the universal source coding scheme using quantization and transmission of the codebooks, and gives a tractable upper bound on the overall MSE of the system. Both scalar and the vector universal schemes are discussed. In Section III the tradeoff between quantization resolution and codebook transmission resolution is analyzed and experimental results are given. Section IV provides a theoretical justification for the use of the given distortion upper bound in optimizing the system's performance.

II. UNIVERSAL SOURCE CODING WITH CODEBOOK TRANSMISSION

Let X be a k-dimensional random vector with density f, and suppose α independent samples of X are encoded by a k-dimensional vector quantizer Q having a codebook of size N. Further suppose that a partial description of Q is transmitted to the receiver consisting on average of b bits per scalar component for each of Q's codevectors. Let r be the overall average rate of this quantization system measured in bits per input vector component. Denote Q's codebook by $\mathcal{C}_t$ (the "target codebook"), denote the codebook derived from the received description of $\mathcal{C}_t$ by $\mathcal{C}_o$ (the "operational codebook"), and denote by $\mathcal{C}_q$ the codebook consisting of all possible 2^{kb} codevectors that could be transmitted to describe vectors in $\mathcal{C}_o$. Note that $\mathcal{C}_o \subset \mathcal{C}_q$. For every α input vectors, a new target codebook $\mathcal{C}_t$ is designed and transmitted and a new operational codebook $\mathcal{C}_o$, approximating $\mathcal{C}_t$, is received and then used for quantization. Equating two expressions for the total number of bits transmitted between codebook updates gives

$$\alpha r k = \alpha \log_2 N + kbN. \tag{1}$$

The term kbN is the total number of bits used to quantize and transmit the N codevectors of $\mathcal{C}_t$ and $\alpha \log_2 N$ is the number of bits transmitted as codevector indices for encoding the α source vectors (Fig. 1).

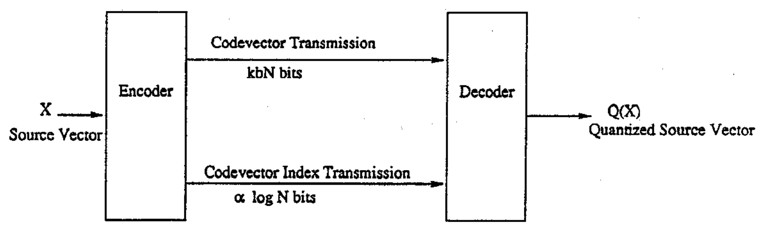

Fig. 1. Universal quantization scheme using codebook transmission.

For a fixed number α of input vectors to quantize with $\mathcal{C}_o$, and a fixed transmission rate r, the overall system's mean squared error (MSE) is a function that varies with the choices of b and N, subject to the constraint in (1). As b is increased, the codebook $\mathcal{C}_t$ is transmitted more accurately so that $\mathcal{C}_o$ becomes a closer approximation of $\mathcal{C}_t$. This helps reduce the overall MSE by ensuring that input vectors are quantized by codevectors that are accurate representations of their intended codevectors, thus reducing the additional component of distortion formed by an inaccurate description of $\mathcal{C}_t$. However, as b increases, the constraint equation (1) demands that the codebook size N must decrease, thus reducing the quantizer's resolution and increasing the system's MSE. Using high resolution quantization theory, we analyze this tradeoff to find the optimal pair (b_{opt}, N_{opt}) minimizing the overall MSE.

For each i denote the i^{th} codevector and partition cell of $\mathcal{C}_t$ respectively by y_i and R_i, and the corresponding quantized approximation codevector in $\mathcal{C}_o$ and its cell by $\hat{y}_i$ and $\hat{R}_i$. For any codebook $\mathcal{C}$, let $D(\mathcal{C})$ denote its MSE. Throughout this paper, the source will be either specified or obvious from the context. Assume $\mathcal{C}_t$ is optimally designed for the source so that each codevector y_i is the centroid of its cell R_i and each cell R_i is a nearest neighbor region, and further assume that the cells $\hat{R}_i$ of $\mathcal{C}_o$ are nearest neighbor regions (though the codevectors of $\mathcal{C}_o$ are *not* in general centroids of these cells). Then we have:

$$\begin{aligned} D(\mathcal{C}_o) &= \frac{1}{k}\sum_{i=1}^{N}\hat{P}_i E(\|X-\hat{y}_i\|^2 | X\in\hat{R}_i) && (2)\\ &\le \frac{1}{k}\sum_{i=1}^{N} P_i E(\|X-\hat{y}_i\|^2 | X\in R_i) && (3)\\ &= \frac{1}{k}\sum_{i=1}^{N} P_i E(\|X-y_i\|^2 | X\in R_i)\\ &\quad +\frac{1}{k}\sum_{i=1}^{N} P_i\|y_i-\hat{y}_i\|^2 && (4)\\ &= D(\mathcal{C}_t)+D(\mathcal{C}_q), && (5)\end{aligned}$$

where $P_i = \Pr[X\in R_i]$, $\hat{P}_i = \Pr[X \in \hat{R}_i]$, and

$$D(\mathcal{C}_q) = \frac{1}{k}\sum_{i=1}^{N} P_i\|y_i-\hat{y}_i\|^2 \qquad (6)$$

is the quantization distortion when the codevectors y_i are treated as a source and are quantized with the $\hat{y}_i$'s. The inequality in (3) follows from the fact that the encoding regions $\hat{R}_i$ are optimal for the codebook $\mathcal{C}_o$. Equation (4) is obtained using the fact that the codevectors of $\mathcal{C}_t$ are the conditional means of their cells.

While the distortion term in (2) is difficult to analyze, the upper bound (4) provides a useful decomposition that is more amenable to analysis, and is asymptotically accurate since $\alpha \to \infty$ since in this case $D(\mathcal{C}_q) \to 0$, as shown in Section III. Furthermore, experiments show that the "$\le$" in (3) can effectively be replaced by "$\approx$". Also, in Section IV a theoretical justification of the tightness of this bound is given using an additive noise model for quantizing a scalar target codebook. Thus, the overall quantizer distortion can be reasonably approximated as the distortion of the target quantizer plus the distortion incurred in transmitting the target codebook.

Zador's formula [8] gives the asymptotic r^{th}-power distortion of an N-point, k-dimensional vector quantizer having a density f as

$$D_{r,k,N} = b_{r,k}\|f\|_{k/(k+r)}N^{-r/k}, \qquad (7)$$

where $b_{r,k}$ is a constant independent of f and N, and $\|f\|_p = \left(\int_{\mathcal{R}^k}|f|^p\right)^{1/p}$. The distortion $D(\mathcal{C}_t)$ can asymptotically be computed from (7). The quantity $D(\mathcal{C}_q)$ is the distortion resulting from transmitting an imperfect description of the codebook $\mathcal{C}_t$. This is the error incurred by quantizing $\mathcal{C}_t$, using on average b bits per codebook component, where $\mathcal{C}_t$ is viewed as a source, whose probability density is the same as the point density of the target quantizer.

Two possibilities for quantizing $\mathcal{C}_t$ are considered: (i) uniform scalar quantizing each vector component of $\mathcal{C}_t$, and (ii) vector quantizing $\mathcal{C}_t$ with a "universal" codebook $\mathcal{C}_u$.

A. Uniform Scalar Quantization of the Codebook

Suppose each scalar component of every codevector in the codebook $\mathcal{C}_t$ is uniformly quantized (Fig. 2) and the support of the density f is $[u, v]$. The MSE of a uniform scalar quantization of $\mathcal{C}_t$ with 2^b output levels spread over a support region $[u, v]$ is asymptotically

$$D(\mathcal{C}_q) = \frac{(v-u)^2}{12}2^{-2b}. \qquad (8)$$

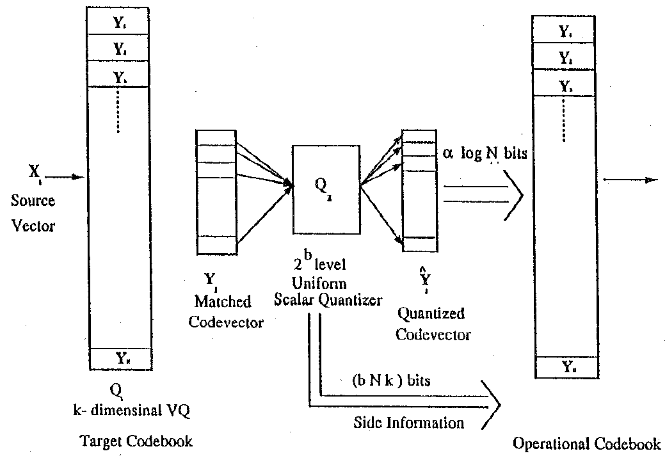

Fig. 2. Scalar universal scheme for transmission of VQ codebooks. Each component of $\mathcal{C}_t$ is uniform scalar quantized and transmitted. The received description forms $\mathcal{C}_o$. Every α input vectors, bNk bits are transmitted to describe $\mathcal{C}_o$ and $\alpha \log_2 N$ bits are sent as codevector indexes.

The distortion upper bound in (5) can be expressed explicitly as a function of b and N (large) as

$$D(N, b) = C_1 N^{-2/k} + C_2 2^{-2b}, \qquad (9)$$

where the values of N and b satisfy the constraint (1) for a fixed transmission rate r, $C_1 = b_{2,k}\|f\|_{k/(k+2)}$ from (7), and $C_2 = (v-u)^2/12$ from (8).

One motivation for scalar uniform quantization of the codebook, apart from the fact that this is the simplest available method, stems from a minimax viewpoint. Suppose the statistics of X are unknown except that it takes its values in the bounded interval $[u, v]$. Then among all N-level quantizers for X, the uniform quantizer over $[u, v]$ is optimal in the sense that its squared error is less than $(v-u)^2(2N)^{-2}$ for all input values, while this obviously does not hold for any other N-level quantizer. Moreover, the MSE is also upper bounded by $(v-u)^2(2N)^{-2}$ for the uniform quantizer over $[u, v]$. For any other quantizer we can always find a source, with a density of support $[u, v]$ whose MSE is larger than this upper bound. A more general statement concerning the minimax optimality of lattice quantizers for sources of a certain bounded support is proved in Appendix B.

B. Vector Quantization of the Codebook

This section examines the case where the N vector target codebook $\mathcal{C}_t$ is encoded for transmission by a k-dimensional "universal" vector quantizer with codebook $\mathcal{C}_u$ containing $M \gg$

N codevectors. This generalizes the uniform quantization scheme presented earlier. C_u is assumed available to both the encoder and decoder.

A description is transmitted to the receiver by vector quantizing the codevectors of C_t to form an operational codebook C_o (Fig. 3). As in the scalar case, the target codebook is designed in real-time based on the current (but unknown a priori) source statistics. For each codevector in C_t, a nearest codevector in C_u is chosen and added to C_o. If a codevector in C_u is chosen twice it is thrown out and instead the next nearest unused codevector in C_u is added to C_o. Similar systems are described in [21] and [11]. It will be seen that asymptotically as the block length α grows, both VQ and uniform scalar quantization of the codebook C_t perform about equally well.

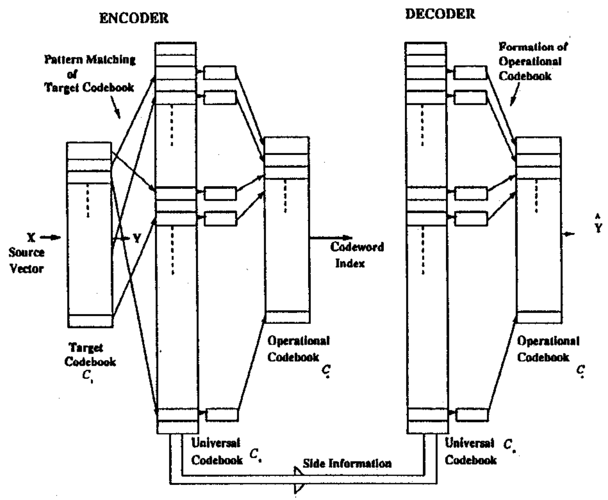

Fig. 3. Vector universal scheme for transmission of VQ codebooks. Each codevector of C_t is itself vector quantized by being matched to a nearest codevector in C_u.

The target codebook C_t is treated as a *vector source* that is itself quantized by C_u. The universal quantizer, however, is in general mismatched to the statistics of the codebook source C_t since the input source X is assumed unknown ahead of time. On the other hand, the target codebook C_t is assumed to be matched to the statistics of the source X, since C_t is designed "on the fly" (e.g. using the generalized Lloyd algorithm) based on recently observed training set vectors.

As M increases the operational codebook can more closely approximate the target codebook, but more of the available bits must be dedicated to transmitting C_o. We determine the optimum tradeoff between M and N, for a fixed overall rate r (bits/sample), analogous to the previous section. C_o is treated as a "source" to C_u and its probability density is assumed equal to the k-dimensional point density function given by asymptotic theory as

$$\lambda_o(x) = \frac{f(x)^{\frac{k}{k+2}}}{\int_{\mathcal{R}^k} f(z)^{\frac{k}{k+2}} dz}. \tag{10}$$

The asymptotic MSE of the upper bound in (4) is

$$D(N, b) = C_1 N^{-2/k} + C_3 M^{-2/k}, \tag{11}$$

where the values of N and b (implicitly a function of M) satisfy the constraint (1) for a fixed transmission rate r. The constants in (11) are

$$\begin{aligned} C_1 &= b_{2,k} \cdot \|f\|_{k/(k+2)} \\ C_3 &= b_{2,k} \int_{\mathcal{R}^k} \frac{\lambda_o(x)}{[\lambda_u(x)]^{\frac{2}{k}}} dx, \end{aligned} \tag{12}$$

where $\lambda_u(x)$ is the point density function of the universal codebook C_u which is an M vector codebook used on a source with probability density $\lambda_o(x)$.

The amount of side information used to describe an operational codebook C_o depends on what type of codebook description is sent. For fixed rate transmission, the minimum total number of bits of side-information needed is the same as the number of bits required to specify an arbitrary subset of size N from a larger set of size M. Thus, the average rate, b, in bits per codevector component is lower bounded by

$$b_1 = \frac{1}{kN} \log_2 \binom{M}{N}. \tag{13}$$

To describe C_o by transmitting $b = b_1$ bits requires a sophisticated encoding scheme for determining which b bit word corresponds to which N vector subset of the M vector codebook C_u. In principle, though, there do exist methods to accomplish this task [22]. A related technique used for source coding is described in [23]. To reduce the complexity of describing the appropriate N vector subset of C_u, we consider two simplified techniques.

In the first, N binary words are transmitted, each containing $\log_2 M$ bits that describe the "addresses" of the N codevector subset of C_u. In this case, b takes on the value

$$b_2 = \frac{1}{k} \log_2 M. \tag{14}$$

The second technique uses M bits to describe whether each codevector of C_u is either in C_o or not in C_o. Here, b takes on the value

$$b_3 = \frac{M}{kN}. \tag{15}$$

Simple combinatorial inequalities imply that

$$b_1 \le \min\{b_2, b_3\}. \tag{16}$$

Furthermore, it can be seen that $b_2 \le b_3$ whenever $N \le M/\log_2 M$. Since $N \le 2^{rk}$ (i.e. bounded), this condition is met for large enough block lengths α. For this reason, we omit the case where $b = b_3$ in the present discussion. The case where $b = b_1$ is here denoted the *optimal* case, and $b = b_2$ the *suboptimal* case since the latter uses more than the minimum number of bits to describe C_o as a subset of C_u. If the value of $b = b_1$ is substituted into the constraint equation (1), then for fixed α, r, and k, and a given N, one method of computing the value of M is by iterative solution.

Using the inequalities [24, eq. (7.14)]

$$\frac{1}{M+1}2^{Mh(N/M)} \leq \binom{M}{N} \leq 2^{Mh(N/M)} \quad (17)$$

where $h(x) = -x\log_2 x - (1-x)\log_2(1-x)$ is the binary entropy function, gives

$$0 \leq \frac{M}{kN}h(N/M) - b_1 \leq \frac{\log_2(M+1)}{kN}. \quad (18)$$

For large N and a bounded, but large universal codebook size M, this implies

$$b_1 \approx \frac{M}{kN}h(N/M), \quad (19)$$

keeping in mind that $N \leq M$.

For a fixed operational codebook size N, and for a large ratio of codebook sizes M/N, b can be approximated from (19) as

$$b_1 \approx \frac{1}{k}\left(\log_2 e + \log_2 \frac{M}{N}\right). \quad (20)$$

Let us examine how the suboptimal description of the vector operational codebook effects the performance of the universal scheme. Fix the values of N, M, and α (this way the distortion is also fixed) and let us determine the difference $\Delta r = r_2 - r_1$ in the transmission rate when r_1 is the transmission rate of the scheme using the optimal description $b = b_1$, and r_2 is the transmission rate of the scheme with the suboptimal description $b = b_2$. By (1) and (20)

$$\alpha r_1 k = \alpha \log_2 N + N\left(\log_2 e + \log_2 \frac{M}{N}\right),$$

and

$$\alpha r_2 k = \alpha \log_2 N + N \log_2 M,$$

giving

$$\Delta r = r_2 - r_1 = \frac{N\log_2(N/e)}{\alpha k}. \quad (21)$$

Equation (21) indicates that $\Delta r \approx 0$ when $N \ll \alpha$, i.e., the suboptimal description performs as well as the optimal one for a large block size α. In this case, due to complexity considerations, one might prefer the suboptimal scheme. On the other hand, for even moderately smaller values of α, (21) predicts a significant decrease in the transmission rate when the optimal scheme is used. For example, if $\alpha = 10{,}000$, $N = 2^{10}$, and $k = 1$, then $\Delta r = .88$, corresponding to a difference in SNR of about 5 dB (as seen later in (27)).

It was noted that the suboptimal scheme with $b = b_3$ is generally inferior to the optimal scheme with $b = b_1$. However, if a variable transmission rate is allowed then in fact they perform equally well asymptotically. In the fixed rate suboptimal scheme a single bit is transmitted for each codevector in C_u to signify whether or not it belongs to C_o. Instead suppose the collection of these M transmitted bits is treated as the output of a memoryless binary source with the probability of a one equal to N/M, since exactly N of the M transmitted bits are one and the rest are zero. If this bit stream is entropy coded (e.g. with an arithmetic coder) then the average rate per codevector component of C_o will be $b_3h(N/M)$, which is the same as the asymptotic rate for b in (19).

III. Resolution Tradeoff Between Codebook Transmission and Source Quantization

This section examines the tradeoff between N and b for a given rate r as the blocksize α varies. As it will turn out, an explicit optimum can be found with the assumption that α is large. Equations are given based on the scalar universal scheme, though the analysis can easily be extended to the vector universal scheme.

By building on our asymptotic development and MSE upper bound decomposition here (and in [19]), and assuming the suboptimal case $b = b_2$, Chou and Effros [20] recently have used a Lagrangian constrained optimization approach to show that the b and N which minimize (9) while satisfying (1) are related implicitly by

$$\begin{aligned} b = \; & \frac{1}{2}\log_2 \alpha + \frac{1}{2}\left(\frac{2}{k} - 1\right)\log_2 N + \frac{1}{2}\log_2 \frac{C_2}{C_1} \\ & + \frac{1}{2}\log_2\left(1 + \frac{Nkb\ln 2}{\alpha}\right). \end{aligned} \quad (22)$$

They also argue that the last term in (22) becomes negligible as the block size increases so that for large α the optimal b can be explicitly written as

$$b_{opt} = \frac{1}{2}\log_2 \alpha + \frac{1}{2}\left(\frac{2}{k} - 1\right)\log_2 N_{opt} + \frac{1}{2}\log_2 \frac{C_2}{C_1}. \quad (23)$$

In Appendix A we use an alternative approach to that in [20] to prove (22) and (23) with a simpler derivation.

By observing that $0 \leq N \leq 2^{rk}$, we can obtain asymptotic upper and lower bounds on b_{opt} from (23):

For $k = 1$:

$$\frac{1}{2}\log_2 \alpha + \frac{1}{2}\log_2 \frac{C_2}{C_1} \leq b_{opt} \leq \frac{1}{2}\log_2 \alpha + \frac{r}{2} + \frac{1}{2}\log_2 \frac{C_2}{C_1}. \quad (24)$$

For $k = 2$:

$$b_{opt} = \frac{1}{2}\log_2 \alpha + \frac{1}{2}\log_2 \frac{C_2}{C_1}. \quad (25)$$

For $k \geq 3$:

$$\begin{aligned} \frac{1}{2}\log_2 \alpha - r\left(\frac{k}{2} - 1\right) \; + \; & \frac{1}{2}\log_2 \frac{C_2}{C_1} \leq b_{opt} \\ \leq \; & \frac{1}{2}\log_2 \alpha + \frac{1}{2}\log_2 \frac{C_2}{C_1}. \end{aligned} \quad (26)$$

It is shown in Appendix A that $N \to 2^{rk}$ as $\alpha \to \infty$, so that for large input block sizes b_{opt} approaches the lower bound if $k \geq 3$ and the upper bound if $k = 1$.

This behavior is depicted in Fig. 4 for $k = 1$ where b is plotted versus $\log_2 \alpha$. The upper and lower bounds are given by the two solid parallel lines. It should be noted that the "Experimental" optimal curve goes slightly above the upper bound for some α. This is due to the fact that the Experimental curve numerically (using training sets) finds the pair (b, N) that minimizes $D(C_o)$

whereas the "Theoretical" curve analytically determines the pair (b, N) that minimizes the upper bound $D(N, b)$. Also b_{opt} is numerically sensitive to small changes in the distortion since $\partial D/\partial b$ is zero at b_{opt}.

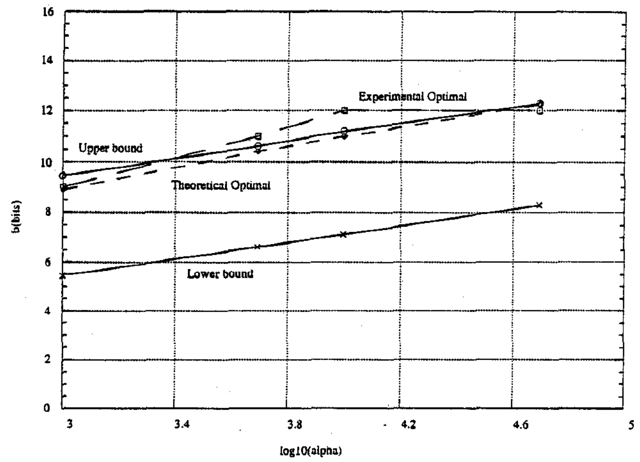

Fig. 4. Variation of b_{opt} with α. Source is Laplacian truncated to $[-4.0, 4.0]$, $k = 1$ and $r = 8$ bits/sample. The upper and lower bounds are straight lines obtained from Eq. (24) by setting $\log_2 N = rk$ and $\log_2 N = 0$ respectively. The Experimental curve gives the b which experimentally minimizes $D(\mathcal{C}_o)$. The Theoretical curve gives the b which minimizes $D(N, b)$ subject to $\alpha rk = \alpha \log_2 N + kbN$.

The optimal distortion for large α is computed in [20] by substituting (23) into (9) to get

$$D_{opt}(\alpha) = C_1 {N_{opt}}^{-2/k}\left(1 + \frac{N_{opt}}{\alpha}\right). \tag{27}$$

Since $N_{opt} \to 2^{rk}$ as $\alpha \to \infty$, considering signal-to-noise ratios we have for large α,

$$\mathrm{SNR}_{opt}(\alpha) = \mathrm{SNR}_{LM} - 10\log_{10}\left(1 + \frac{2^{rk}}{\alpha}\right), \tag{28}$$

where SNR_{LM} is the signal-to-noise ratio of the optimal (Lloyd-Max) k-dimensional quantizer with per sample rate r. As α grows, the minimum MSE of the universal quantization scheme approaches that of an optimal quantizer for an a priori known source (i.e. no codebook transmission). The cost of transmitting the codebook becomes negligible as the number of input vectors encoded with it increases. Equation (28) gives a precise way to determine how closely the codebook transmission scheme performs to the Lloyd-Max performance.

The MSE of a uniform quantizer can be used as a simple baseline comparison to the MSE of any universal quantization system. The MSE advantage of our codebook transmission scheme over plain uniform quantization can be seen from (27) to be the same as the usual Lloyd-Max advantage but multiplied by a factor of $1 + (2^{rk}/\alpha)$. It is thus advantageous to use this codebook transmission scheme so long as the codebooks can be transmitted infrequently enough to guarantee that α is large enough to satisfy

$$b_{2,k}\|f\|_{k/(k+2)}\left(1 + \frac{2^{rk}}{\alpha}\right) < \frac{(v-u)^2}{12}. \tag{29}$$

The development in Appendix A can be repeated for the vector universal scheme to obtain the same expressions for b_{opt}. If $b = b_2$ then the optimal universal codebook size M is given by $M_{opt} = 2^{kb_{opt}}$. If $b = b_1$, then M_{opt} can be found iteratively from (13).

A. Experimental Results

In this section experimental results are presented for Gaussian and Laplacian sources. Some of the sources have been truncated to have bounded support regions of various widths. This is a realistic model for many sources and also facilitates the design of the uniform quantizers for codebook transmission. Otherwise one would have to determine the best support region for the uniform quantizers which would distract from the main points presented here. In computing various asymptotic distortions for the experiments, the constant $b_{2,k}$ in Zador's formula is taken as $1/12$ for $k = 1$ and $.08333$ for $k = 4$. The latter case assumes Gersho's Conjecture and uses the Cube Upper Bound [25].

In Fig. 5 the optimal SNR is plotted for truncated Laplacian data as a function of α. For each given α the Experimental Universal curve is obtained by determining the N that minimizes $D(\mathcal{C}_o)$ by iteratively trying pairs (b, N) with $0 \le N \le 2^{rk}$. The Theoretical Curve is computed according to (28). For comparison, the performance of a uniform quantizer is shown in the figure and acts as a lower bound, and the performance of a Lloyd-Max quantizer designed with a priori knowledge of the source statistics is shown as an upper bound. For small α the overhead of transmitting codebooks is costly in terms of bits per source vector so that universal source coding with codebook transmission has little advantage over uniform quantization. On the other extreme, for large α the overhead of transmitting codebooks is negligible and the performance of the universal system approaches that of Lloyd-Max with a known source.

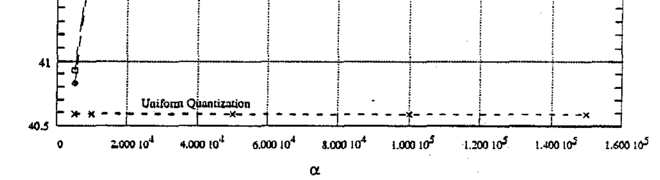

Fig. 5. Variation of optimal SNR with the block size α. The source is from i.i.d. zero mean, unit variance, Laplacian data truncated to $[-4, 4]$, and $k = 1$, $r = 8$ bits/sample.

Fig. 6 shows the comparative SNRs of the universal source coding scheme for a truncated Laplacian source and a scalar quantizer. The curves show that the universal scheme performs as well as the Lloyd-Max case for practically all rates when

$\alpha = 50,000$.

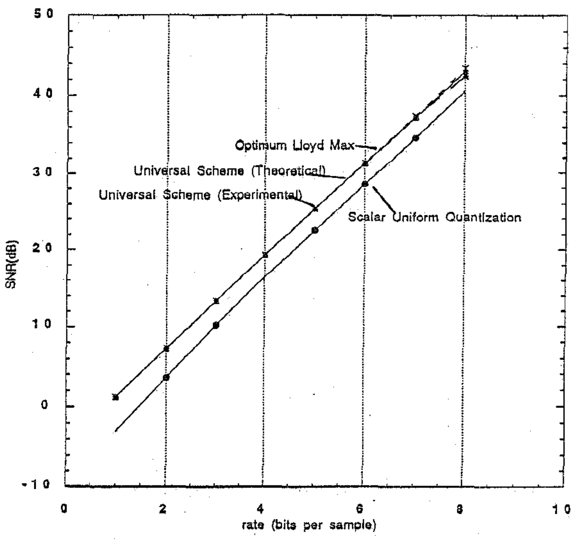

Fig. 6. Comparison of different quantization schemes. Source is from i.i.d. zero mean, unit variance, Laplacian data truncated to $[-4, 4]$ and is scalar quantized, with $\alpha = 50,000$.

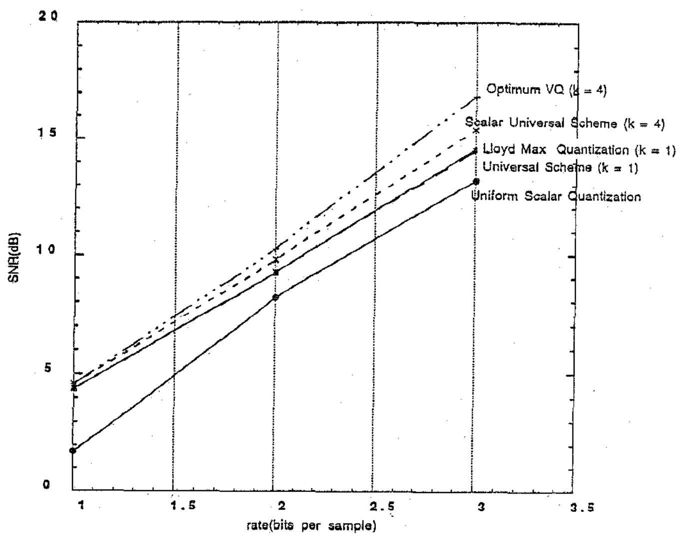

Fig. 7. Experimental performance curves for different schemes. The source is from i.i.d. zero mean, unit variance, Gaussian data truncated to $[-2, 2]$ and $\alpha = 50,000$. The five curves shown from top to bottom are: (1) VQ ($k = 4$) matched to the source, (2) Scalar universal scheme for VQ ($k = 4$), (3) Lloyd-Max scalar quantization matched to the source, (4) Scalar universal scheme for a scalar quantizer, and (5) Uniform scalar quantization.

Fig. 7 compares the performances for uniform scalar quantizing the codebook. From bottom to top, the curves shown are uniform scalar quantization of the source, uniform scalar quantization of the scalar codebook (universal), Lloyd-Max scalar quantization (known source), uniform scalar quantization of the VQ ($k = 4$) codebook (universal), and non-universal VQ ($k = 4$) designed with the generalized Lloyd algorithm (known source). Each of the universal schemes outperforms the uniform scalar quantizer and each performs nearly as well as the quantizers designed for a known source.

Fig. 8 examines the bit tradeoff for the scalar universal scheme, for an i.i.d. Gaussian source with rate $r = 2$.

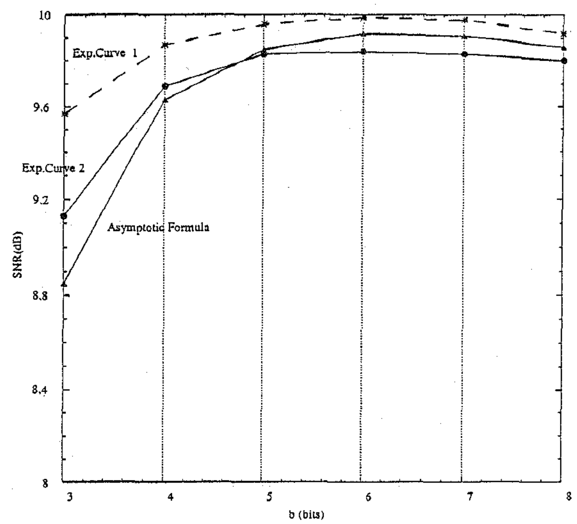

Fig. 8. Scalar universal scheme for Gaussian source, with $k = 4$, $r = 2$ bits/sample, and $\alpha = 50,000$. Experimental Curve 1 plots $D(\mathcal{C}_o)$ and Experimental Curve 2 plots $D(\mathcal{C}_t) + D(\mathcal{C}_q)$. The Asymptotic Formula curve is D_{opt}.

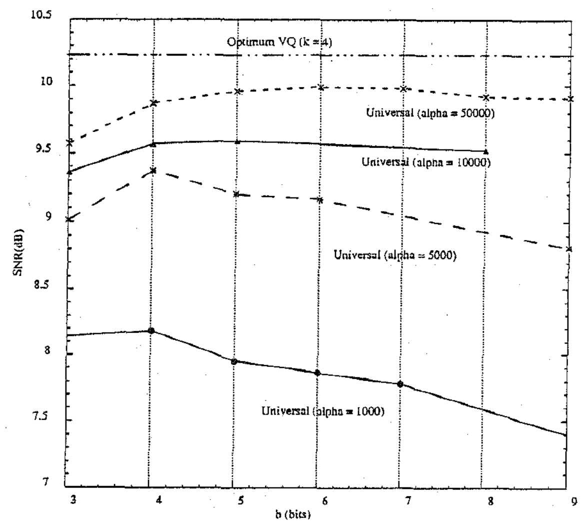

Fig. 9. Comparison of vector universal and scalar universal schemes. The source is from i.i.d. zero mean, unit variance, Gaussian data truncated to $[-2, 2]$, $k = 4$, $r = 2$. The vector universal $\mathcal{C}_u$ used is designed for a Laplacian.

The vector dimension of $\mathcal{C}_t$ is $k = 4$, $\alpha = 50,000$, and b is varied. It is seen that the overall SNR first increases and then begins to decrease. The place where the SNR peaks gives the optimum choice of b. Two experimental curves are given in this figure. Experimental Curve 1 gives the SNR from equation (2) and Experimental Curve 2 gives the SNR from MSE upper bound in equation (4). All three curves are very close to each other in dBs and thus the Asymptotic Formula curve provides an accurate technique for computing the optimal choice of b.

Fig. 9 shows the dependence of the overall SNR on b for various different fixed values of α. As α increases the curves approach the curve for VQ with known source statistics. The peak of each curve occurs at b_{opt}.

Fig. 10 compares the scalar universal scheme and the vector universal scheme as the blocksize varies. The scalar and vector universal schemes are seen to perform nearly equally well and the theoretical predictions are also accurate.

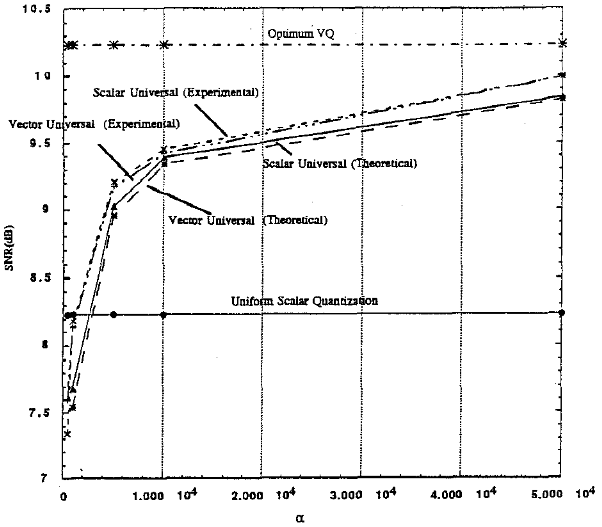

Fig. 10. Comparison of vector universal and scalar universal schemes. The source is from i.i.d. zero mean, unit variance, Gaussian data truncated to $[-2,2]$, $kk=4$, $r=2$. The vector universal $\mathcal{C}_u$ is designed for a Laplacian.

IV. Additive Noise Model for Codebook Quantization

In this section a particular quantization model is introduced in order to provide a theoretical justification of the tightness of the MSE upper bound in Section III.

The optimization described in Section III and in [20] minimizes an upper bound on the system's MSE. In order for the minimization to be useful it is important that the upper bound be accurate. Experimental results suggest that the upper bound (4) on the distortion of the universal scheme is tight. Unfortunately it seems very difficult to prove rigorously that the "$\leq$" in (3) can effectively be replaced by "$\approx$". One can realize the difficulties when considering the question of what exactly happens to the multidimensional quantization regions when the codevectors are slightly displaced. For this reason we consider a simple model for scalar uniform quantization of the scalar target codebook and develop an alternative expression for $D(N,b)$ using this model. It turns out that the result will not differ significantly from (9), the formula for the distortion derived from the upper bound.

Our approach is to model the codebook quantization as a process where the quantization levels of the scalar quantizer are perturbed by some noise independent of the quantized signal. The MSE of the resulting quantizer is then used as the distortion of the universal scheme.

We next formally define an *additive noise model* of quantization. Some of the properties of this model were studied in [26]. Let the target N-level quantizer have codebook $\mathcal{C}_t=\{y_1,y_2,\ldots,y_N\}$ (all quantizers use nearest neighbor regions) and define an (random) operational quantizer by its codebook $\mathcal{C}_o=\{y_1+Z_1,y_2+Z_2,\ldots,y_N+Z_N\}$, where the random variables Z_i represent the "noise" introduced by uniform quantization of the codebook $\mathcal{C}_o$. Assume that the Z_i's are i.i.d. and independent of the quantized random variable X and that their ranges are small enough compared to the quantization intervals so that $|Z_i|<\frac{1}{2}\min\{y_i-y_{i-1},y_{i+1}-y_i\}$. Let σ^2 denote the variance of each Z_i.

We do not assert that the above model accurately describes the process of codebook quantization. The condition on the independence of the Z_i's, for example, can hardly be justified. But lacking a more exact model, we use this one to heuristically derive an analytic expression on the effect of codebook quantization.

The following proposition asserts that the average distortion in this model, $E_z[D(\mathcal{C}_o)]$, can be expressed in terms of $D(\mathcal{C}_t)$ and σ^2, the common variance of the Z_i's.

Proposition 1. If the source X has a continuous density f, then for a large number of quantization levels N,

$$E_z[D(\mathcal{C}_o)]\approx D(\mathcal{C}_t)+\frac{1}{2}\sigma^2, \tag{30}$$

where the expectation is with respect to the Z_i's, and the approximate identity means that

$$N^2\left(E_z[D(\mathcal{C}_o)]-\frac{1}{2}\sigma^2\right)\to N^2D(\mathcal{C}_t)\quad\text{as}\quad N\to\infty.$$

Proof. For each i, define $\hat{y}_i=y_i+Z_i$. Then,

$$\begin{aligned}D(\mathcal{C}_o) &= \int_{-\infty}^{\hat{y}_1}(\hat{y}_1-x)^2f(x)\,dx\\ &\quad+\int_{\hat{y}_N}^{\infty}(\hat{y}_N-x)^2f(x)\,dx\\ &\quad+\sum_{i=1}^{N-1}\left(\int_{\hat{y}_i}^{\frac{1}{2}(\hat{y}_i+\hat{y}_{i+1})}(x-\hat{y}_i)^2f(x)\,dx\right.\\ &\quad\left.+\int_{\frac{1}{2}(\hat{y}_i+\hat{y}_{i+1})}^{\hat{y}_{i+1}}(x-\hat{y}_{i+1})^2f(x)\,dx\right).\end{aligned} \tag{31}$$

If $\Delta_i=y_{i+1}-y_i$ is small, then since f is continuous, we can use the approximation $f(x)\approx p_i/\Delta_i$, where $p_i=\int_{y_i}^{y_{i+1}}f(x)\,dx$. Using this approximation and the definitions of Δ_i and $\hat{y}_i$, we get that both terms in the summation in (31) are approximately

$$\frac{p_i}{\Delta_i}\int_0^{\frac{1}{2}(\Delta_i-Z_i+Z_{i+1})}x^2\,dx=\frac{p_i}{24\Delta_i}(\Delta_i-Z_i+Z_{i+1})^3. \tag{32}$$

Some algebra and the fact that the Z_i's are mutually independent and have common first, second, and third moments show that

$$\begin{aligned}E_z[(\Delta_i-Z_i+Z_{i+1})^3] &= E_z[\Delta_i^3-3\Delta_i^2(Z_i-Z_{i+1})\\ &\quad+3\Delta_i(Z_i-Z_{i+1})^2-(Z_i-Z_{i+1})^3]\\ &= \Delta_i^3+6\Delta_i\sigma^2.\end{aligned} \tag{33}$$

It is easy to show that the first two integrals in (31) are negligible for large N, therefore asymptotically

$$E_z[D(\mathcal{C}_o)] = \frac{1}{12}\sum_{i=1}^{N}\frac{p_i}{\Delta_i}(\Delta_i^3 + 6\Delta_i\sigma^2) = \frac{1}{12}\sum_{i=1}^{N}\Delta_i^2 p_i + \frac{1}{2}\sigma^2. \quad (34)$$

Since the first term in (34) is known from the asymptotic theory to be approximately the distortion of the fine quantizer $\mathcal{C}_t$, the proof is complete. □

Note that although a sketch of proof is actually given, the proof can be made exact, assuming that the conditions on the Z_i's hold.

Let us now examine the relation between (30) and (9). Since the variance σ^2 corresponds to the variance of the quantization noise of uniform quantization of the target codebook, we can put $\sigma^2 = \frac{(u-v)^2}{12}2^{-2b}$. Thus we obtain the alternative expression (in place of (9)) for the overall distortion

$$D(N,b) = D(Q_N) + \frac{1}{2}\sigma^2 = C_1 N^{-2} + \hat{C}_2 2^{-2b}, \quad (35)$$

where $\hat{C}_2 = \frac{1}{2}C_2$. In this way, the entire minimization procedure of Section III applies with the substitution of C_2 by $\hat{C}_2$. In what follows we show that the changes in the optimizing parameters are negligible.

Let us consider expression (23) for the optimizing parameters when $k = 1$. We have

$$b_{opt} = \frac{1}{2}\log_2\alpha + \frac{1}{2}\log_2 N_{opt} + \frac{1}{2}\log_2\frac{C_2}{C_1}, \quad (36)$$

and denoting by $\hat{b}_{opt}$ and $\hat{N}_{opt}$ the parameters optimizing (35) (for the given blocklength α) we obtain

$$\hat{b}_{opt} = \frac{1}{2}\log_2\alpha + \frac{1}{2}\log_2\hat{N}_{opt} + \frac{1}{2}\log_2\frac{C_2}{C_1} - \frac{1}{2}. \quad (37)$$

It follows from the rate constraint (1) that either $\hat{b}_{opt} \le b_{opt}$ and $\hat{N}_{opt} \ge N_{opt}$, or else $\hat{b}_{opt} > b_{opt}$ and $\hat{N}_{opt} \le N_{opt}$. We will show that the latter case cannot happen. First, introduce the quantities

$$D_1 = C_1 N_{opt}^{-2} + C_2 2^{-2b_{opt}} \qquad D_2 = C_1 N_{opt}^{-2} + \hat{C}_2 2^{-2b_{opt}}$$
$$D_3 = C_1 \hat{N}_{opt}^{-2} + \hat{C}_2 2^{-2\hat{b}_{opt}} \qquad D_4 = C_1 \hat{N}_{opt}^{-2} + C_2 2^{-2\hat{b}_{opt}}$$

and assume that $\hat{b}_{opt} > b_{opt}$ and $\hat{N}_{opt} < N_{opt}$. Then $D_2 > D_3$, since $\hat{b}_{opt}$ and $\hat{N}_{opt}$ minimize (35), and it follows that

$$\hat{C}_2\left(2^{-2b_{opt}} - 2^{-2\hat{b}_{opt}}\right) > C_1\left(\hat{N}_{opt}^{-2} - N_{opt}^{-2}\right) \quad (38)$$

where $2^{-2b_{opt}} - 2^{-2\hat{b}_{opt}} > 0$. But since $0 < \hat{C}_2 < C_2$ we have from (38)

$$C_2\left(2^{-2b_{opt}} - 2^{-2\hat{b}_{opt}}\right) > C_1\left(\hat{N}_{opt}^{-2} - N_{opt}^{-2}\right),$$

which is equivalent to $D_4 < D_1$, a contradiction since b_{opt} and N_{opt} minimize (9). Thus $b_{opt} < \hat{b}_{opt}$.

Subtracting (37) from (36) results in

$$0 < b_{opt} - \hat{b}_{opt} = \frac{1}{2}\log_2\frac{N_{opt}}{\hat{N}_{opt}} + \frac{1}{2} < \frac{1}{2},$$

which means that the optimal rate for the uniform quantizer is changed by less than 1/2 bit. The change $\hat{N}_{opt} - N_{opt}$ can be estimated by using the derivative (41) of N with respect to b:

$$\Delta N = \hat{N}_{opt} - N_{opt} \approx -\frac{N_{opt}^2}{\alpha\log_2 e + b_{opt}N_{opt}}(\hat{b}_{opt} - b_{opt}). \quad (39)$$

For example, if $\alpha = 10{,}000$, $N_{opt} = 2^8$, and $b_{opt} = 10$, then $\Delta N \approx 2$, a negligible increase.

Also, for large α since N_{opt} does not change very much, neither does the minimum MSE D_{opt} in (27). In summary, using the additive noise model above shows that the upper bound in (3) is in fact approximately equal to the distortion $D(\mathcal{C}_o)$ of the operational codebook, and that the optimal choice of b and N are closely preserved.

V. Conclusions

We have examined a universal quantization scheme that periodically transmits new VQ codebooks to update the decoder as the input statistics change. A mathematically tractable upper bound is presented that enables an asymptotic analysis of the tradeoff between quantization resolution and codebook transmission quality. Two models have been tested, one in which the initial target codebook is uniform scalar quantized, and the other in which the vector codebook is itself vector quantized. In both the cases there exists a tradeoff in the number of bits dedicated as side information and the number of bits used in specifying the codebook vectors once the transmitted codebook can be used. Asymptotic analysis gives a theoretical justification of this tradeoff and this is confirmed by experiments. The effect of block size on the performance is presented for some specific source distributions. For small block sizes the performance of neither schemes is very good. For a fixed memoryless source density the performance increases as the block size increases and the overhead of transmitting new codebooks becomes negligible.

Appendix A

A. Derivation of Equations (22) and (23)

Let us ignore for the moment that b and N are discrete and take the derivative of the distortion in (9) with respect to b:

$$\frac{\partial}{\partial b}D(N,b) = -\frac{2}{k}C_1 N^{-\frac{2}{k}-1}\frac{\partial N}{\partial b} - 2(\ln 2)C_2 e^{-2b\ln 2}, \quad (40)$$

where N is implicitly a function of b as given by (1). Though (1) can not be explicitly solved for N, it can be differentiated implicitly to give

$$\frac{\partial N}{\partial b} = -\frac{kN^2}{\alpha/\ln 2 + kbN}. \quad (41)$$

Now setting $\frac{\partial D}{\partial b} = 0$ in (40) and substituting (41) yields

$$2^{2b} = N^{2/k}\frac{C_2}{C_1}\left(\frac{\alpha}{N} + kb\ln 2\right). \quad (42)$$

After taking the logarithm of both sides and rearranging terms we get equation (22). To see that $\partial D(N,b)/\partial b = 0$ for the b which minimizes $D(N,b)$ subject to the constraint (1), it is enough to see that $\partial D(2^{rk},0)/\partial b < 0$ and $\partial D(1,\alpha r)/\partial b > 0$. From (40) it follows that these two inequalities are satisfied whenever α is large enough that

$$\alpha > \frac{1}{2r}\left(\log_2 \alpha + \log_2 \frac{C_2}{C_1} + \log_2(\ln 2)\right) \quad \text{and} \quad \alpha > \frac{C_1}{C_2}2^{2kr}.$$

The following simple argument shows that the last term in (22) becomes negligible as the block size increases. To see that

$$\lim_{\alpha\to\infty} \frac{1}{2}\log_2\left(1+\frac{Nkb\ln 2}{\alpha}\right) = 0, \tag{43}$$

use the inequality (9) to get

$$D(N,b) \geq C_1 2^{-2r}, \tag{44}$$

which follows from the fact that $N \leq 2^{kr}$ by (1). This implies that $\liminf_{\alpha\to\infty} D(N,b) \geq C_1 2^{-2r}$. If we make the dependence of N and b on α explicit by introducing the notations $N(\alpha)$ and $b(\alpha)$ it is easy to see that this lower bound is achieved in the limit if and only if $\lim_{\alpha\to\infty} N(\alpha) = 2^{kr}$ and $\lim_{\alpha\to\infty} b(\alpha) = \infty$. But by (1) this is possible if and only if $\lim_{\alpha\to\infty} b(\alpha)/\alpha = 0$ and $\lim_{\alpha\to\infty} b(\alpha) = \infty$. This in turn implies

$$\frac{kbN}{\alpha} \leq \frac{kb2^{kr}}{\alpha} \to 0 \quad \text{as} \quad \alpha \to \infty, \tag{45}$$

and (43) is proved.

We have found the best noninteger solution b_{opt}. Since the nearest integer to b_{opt} differs from b_{opt} by at most 1/2, it can be seen that the integer solution gives the same asymptotic distortion formula as the noninteger solution. This follows from the fact that $\partial N/\partial b$ tends to zero as α grows without bound. Thus, in (23) the quantity N_{opt} does not change much (in the ratio sense) from its value based on choosing the nearest integer to b_{opt}. □

APPENDIX B

A. Minimax Optimality of Best Lattice Quantizer

Consider the class of k-dimensional probability densities that vanish outside the bounded region S, and denote the volume of S by $V(S)$. We assert that the best resolution constrained k-dimensional lattice quantizer is the optimal minimax universal quantizer for this class of densities in the high resolution sense. By this we mean the following. Consider a sequence Q_n of k-dimensional n-level quantizers all matched to the support region S. Let Q_n^* be the sequence of quantizers whose codepoints are the intersection of the support region and an appropriately scaled given optimal lattice. (By the optimal lattice we mean the one with the smallest normalized second moment K_l). As it is usual in asymptotic quantization arguments we make the assumption that the codepoints of Q_n have a limiting point density λ, and their quantization cells have approximately the same normalized second moment K. Then for any density f in the class, the asymptotic distortion of Q_n is given by

$$\lim_{n\to\infty} n^{2/k} D(Q_n) = K \int_S \frac{f(x)}{[\lambda(x)]^{2/k}}\,dx. \tag{46}$$

Since the point density λ_u of the lattice quantizer is the uniform density over S, we have

$$\begin{aligned}\lim_{n\to\infty} n^{2/k} D(Q_n^*) &= K_l \int_S \frac{f(x)}{[V(S)]^{2/k}}\,dx \\ &= K_l [V(s)]^{2/k}.\end{aligned} \tag{47}$$

But by choosing from the class the uniform density $f = 1/V(S)$ over S, we get

$$\begin{aligned}\int_S \frac{f(x)}{[\lambda(x)]^{2/k}}\,dx &= \frac{1}{V(S)}\int_S \frac{1}{[\lambda(x)]^{2/k}}\,dx \\ &\geq \left(\frac{1}{V(S)}\int_S \lambda(x)\,dx\right)^{-2/k} \\ &= [V(S)]^{2/k},\end{aligned} \tag{48}$$

where Jensen's inequality has been applied to the convex function $\phi(z) = z^{-2/k}$. Since we assumed that the lattice has the smallest possible second moment, we have $K_l \leq K$, therefore (46), (47), and (48) show that $D(Q_n)$ is eventually larger than $D(Q_n^*)$.

ACKNOWLEDGEMENTS

The authors would like to thank Dr. Phil Chou of Xerox Palo Alto Research Center for helpful discussions during this work and in particular for pointing out the optimality of our suboptimal codebook transmission scheme if variable rate coding is allowed. We also thank Prof. Robert Gray of Stanford University for pointing out some references to prior related work.

REFERENCES

[1] J. Ziv, "Coding of Sources with unknown statistics - Part 2: Distortion relative to a fidelity criterion, "*IEEE Transactions on Information Theory*, vol. IT-18, pp. 389-394, May 1972.
[2] D. L. Neuhoff, R. M. Gray, and L. D. Davisson, "Fixed Rate Universal Block Source Coding with a Fidelity Criterion," *IEEE Transactions on Information Theory*, vol. IT-21, pp. 511-523, Sept. 1975.
[3] Y. Matsuyama and R. M. Gray, "Universal Tree Encoding of Speech," *IEEE Transaction on Information Theory*, vol. IT-27, pp. 31-40, Jan. 1981.
[4] P. A. Chou, "Code Clustering for Weighted Universal VQ and Other Applications," *IEEE International Symposium on Information Theory* (ISIT), Budapest, Hungary, June 1991.
[5] M. B. Pursley and L. D. Davisson, "Variable rate coding for nonergodic sources and classes of sources subject to a fidelity constraint," *IEEE Trans. Inform. Theory*, vol. IT-22, pp. 324-337, May 1976.
[6] "K. M. Mackenthum and M. B. Pursley, "Variable rate universal block source coding subject to a fidelity constraint," *IEEE Trans. Inform. Theory*, vol. IT-24, pp. 349-360, May 1978.
[7] J. C. Kieffer, "A unified approach to weak universal source coding," *IEEE Trans. Inform. Theory*, vol. IT-24, pp. 674-682, Nov. 1978.
[8] P.Zador,"Asymptotic quantization error of continuous signals and quantization dimension,"*IEEE Transactions on Information Theory*, vol. IT-28, pp. 139-149, March 1982.

[9] N. M. Nasrabadi and Y. Feng, "A Dynamic Finite-State Vector Quantization Scheme," *Proceedings IEEE International Conference on Acoustics Speech and Signal Processing* (ICASSP), Albuquerque, New Mexico, pp 2261-64, May 1990.

[10] A. Gersho and M. Yano, "Adaptive Vector Quantization by Progressive Codevector Replacement," *Proceedings IEEE International Conference on Acoustics, Speech and Signal Processing* (ICASSP), May 1985.

[11] S. Panchathan and M. Goldberg, "Algorithms and Architecture for Image Adaptive Vector Quantization," *SPIE, Visual Communications and Image Processing*, vol. 1001, 1988.

[12] L. Wang, M. Goldberg, and S. Shlien, "Unified variable-Length Transform Coding and Image-Adaptive Vector Quantization," *IEEE Journal on Selected Areas in Communications*, vol. JSAC-10, No. 5, pp. 892-901, June 1992.

[13] W. Equitz, "A New Vector Quantization Clustering Algorithm," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol. ASSP-37, No. 10, pp. 1568-1575, October 1989.

[14] R. Bottemiller, "Comments on 'A New Vector Quantization Clustering Algorithm'," vol. ASSP-40, No. 2, pp. 455-456, February 1992.

[15] N. Nasrabadi and Y. Feng, "Vector Quantization of Images Based upon the Kohonen Self-Organization Feature Maps" *Proc. of the 2nd ICNN Conf.*, vol. I, pp. 101-105, 1988.

[16] E. Yair, K. Zeger, and A. Gersho, "Competitive Learning and Soft Competition for Vector Quantizer Design," *IEEE Transactions on Signal Processing*, vol. ASSP-40, No. 2, pp. 294-309, February 1992.

[17] P.-C. Chang and R. Gray, "Gradient Algorithms for Designing Predictive Vector Quantizers," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol. ASSP-34, pp. 679-690, August 1986.

[18] K. Zeger, "Corrections to 'Gradient Algorithms for Designing Predictive Vector Quantizers'," *IEEE Transactions on Signal Processing*, vol. ASSP-39, No. 3, pp. 764-765, March 1991.

[19] K. Zeger and A. Bist, "Universal Adaptive Vector Quantization with Application to Image Compression," *Proceedings IEEE Inter. Conf. on Acoust., Speech, and Sig. Proc. (ICASSP)*, San Francisco, California, March 1992.

[20] P. A. Chou and M. Effros, "Rate and Distortion Redundancies for Universal Source Coding with Respect to a Fidelity Criterion," *1993 IEEE Int. Symp. on Inf. Theory (ISIT)*, San Antonio, Texas, extended abstract.

[21] A. Gersho and R. M. Gray, "Vector Quantization and Signal Compression," Kluwer Academic Publishers, Boston, 1992.

[22] S. Even, "Algorithmic Combinatorics," Macmillan, New York, 1973.

[23] T. Cover, "Enumerative Source Coding," *IEEE Transactions on Information Theory*, vol. IT-19, No. 1, pp. 73-77, January 1992.

[24] T. Cover and J. Thomas, "Elements of Information Theory," John Wiley & Sons, Inc., New York, 1991.

[25] A. Gersho, "Asymptotically optimal block quantization," *IEEE Transactions on Information Theory*, vol. IT-25, pp. 373-380, July 1979.

[26] F. Castanie, "Signal Processing by Random Reference Quantizing", *Signal Processing*, vol. 1, pp. 27-43, 1979.

Kenneth Zeger received both the S.B. and S.M. degrees in EECS from MIT in 1984, and both the M.A. degree in mathematics and the Ph.D. in EE at the University of California, Santa Barbara, in 1989 and 1990 respectively. Dr. Zeger was an Assistant Professor of Electrical Engineering at the University of Hawaii from June 1990 to June 1992 and since July 1992 has been an Assistant Professor of Electrical Engineering at the University of Illinois at Urbana-Champaign. His present research interests include information theory, source and channel coding, speech and image compression, and computational complexity theory. He was Co-Chairman of the 1990 IEEE Communication Theory Workshop, received an NSF Research Initiation Award in 1990, and an NSF Presidential Young Investigator Award in 1991.

Anurag Bist received his B.S. degree in electrical engineering from the Indian Institute of Technology in Kanpur in 1989 and the M.S. degree in electrical engineering from Princeton in 1990. Since 1990 he has been a graduate student at the University of Hawaii.

Tamás Linder was born in Budapest, Hungary in 1964. He received the M.S. degree in electrical engineering from the Technical University of Budapest in 1988, and the Ph.D degree from the Hungarian Academy of Sciences in EE in 1992. He was a post-doctoral fellow at the University of Hawaii in 1993, and a Visiting Fulbright Scholar at the Coordinated Science Laboratory, University of Illinois at Urbana-Champaign in 1993. He is currently an Associate Researcher at the Research Group for Informatics and Electronics of the Hungarian Academy of Sciences. His research interests include information theory, vector quantization, rate-distortion theory, and statistical pattern recognition.