TABLE II
POSITION OF THE SPECTRAL PEAKS (EXAMPLE 2)

Iteration	f_{11}	f_{12}	f_{21}	f_{22}
0	0.1875	0.453125	0.28125	0
1	0.203125	0.140625	0.25	0.34375
2	0.203125	0.140625	0.25	0.34375
5	0.203125	0.140625	0.25	0.34375
10	0.203125	0.140625	0.25	0.34375
Direct Inversion	0.203125	0.140625	0.25	0.359375
True Values	0.20	0.15	0.25	0.35

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Corrections to "Gradient Algorithms for Designing Predictive Vector Quantizers"

Kenneth Zeger

***Abstract*—An error in a paper by Chang and Gray is pointed out and corrected. The error invalidates their observation that the generalized Lloyd algorithm is a gradient descent technique but the generalized Lloyd algorithm is a member of the related class of coordinate descent techniques. Convergence rate analysis of gradient descent algorithms for vector quantizer design is provided.**

I. INTRODUCTION

In the above paper,[1] there is an error regarding Chang and Gray's observation that the generalized Lloyd algorithm (GLA) is a steepest descent algorithm. The Lloyd algorithm is instead seen to be a member of the related class of "coordinate descent" algorithms. Furthermore, an analysis of convergence rate properties of gradient descent techniques for vector quantizer design is developed.

The notation in Chang and Gray's paper[1] will be adopted. Specifically, let $\{x_n\}$ be a training sequence of L vectors taking on values in k-dimensional Euclidean space R^k, e_i the input to the quantizer, ϵ_i the reproduction codeword with index i, and L_i the number of training vectors which are mapped into codeword i.

The steepest descent formula to update each quantizer codeword is given as follows by Chang and Gray,[1] p. 681:

$$\epsilon_{i,m+1} = \epsilon_{i,m} - \mu_q \nabla_{\epsilon_i} D(x_n, \hat{x}_n)$$

or equivalently in terms of a training sequence as

$$\epsilon_{i,m+1} = \epsilon_{i,m} - \mu_q \lim_{L\to\infty} \frac{1}{L} \sum_{j:\gamma(e_j)=u_i} \nabla_{\epsilon_i} D(e_j, \epsilon_{i,m}).$$

The above equation, given by Chang and Gray, was incorrect in using the quantity L_i for both occurrences of L. As a result, (3)–(5) in Chang and Gray should be corrected by replacing L_i by L, or, equivalently, by multiplying each occurrence of μ_q by the quantity $p_i = L_i/L$, the ith partition region probability. Similarly, the equation following (5) should be correctly rewritten as

$$\epsilon_{i,m+1} = (1 - 2\mu_q p_i)\epsilon_{i,m} + 2\mu_q p_i \frac{1}{L_i} \sum_{j:\gamma(e_j)=u_i} e_j$$

$$i = 1, \cdots, 2^R.$$

The subsequent analysis on p. 682 concerning the convergence rate and damping of the codeword update formula is thus no longer applicable.

Specifically, Chang and Gray's conclusion that the Lloyd algorithm is a gradient descent algorithm does not hold. The Lloyd algorithm can be written in terms of a codeword update equation as

$$\epsilon_{i,m+1} = \frac{1}{L_i} \sum_{j:\gamma(e_j)=u_i} e_j, \qquad i = 1, \cdots, 2^R$$

where the right-hand side is the centroid (denoted by c_i in what follows) of the nearest neighbor region of ϵ_i. There does not in general exist any value of μ_q which will yield the Lloyd algorithm from the gradient descent codeword update equation. However, if we allow different constants μ_i for the update equation of each codeword ϵ_i (thus violating the definition of gradient descent), then the Lloyd algorithm can be obtained by setting $2\mu_i p_i = 1$, for $i = 1, \cdots, 2^R$. This technique belongs to the class of algorithms named coordinate descent algorithms [1]. In general, a coordinate descent iteratively updates a single coordinate (codeword in this case) at a time. In this correspondence, however, it is assumed for convenience that all of the codewords are synchronously updated.

A gradient descent for codebook design is thus a special case of coordinate descent (where an iteration consists of a single update of each codeword). The Lloyd algorithm can be seen to be the fastest converging of all coordinate descent algorithms (it converges in one iteration), and is hence faster converging than any gradient descent algorithm. As in Chang and Gray, this discussion assumes that the encoder and training sequence are fixed.

In practice, the encoder changes after each iteration thus making possible further descent by way of additional codeword updates. In this case, since the gradient algorithm locally travels in a direction of steepest descent, it may prove to be a useful alternative to the Lloyd algorithm. To the best knowledge of the author, a theoretical comparison of the convergence rates of the gradient descent and Lloyd algorithms is currently an open problem. A descent algorithm for vector quantizer design based on conjugate gradient techniques is presented in [2]. Some preliminary results show that the

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The author is with the Department of Electrical and Computer Engineering, University of California, Santa Barbara, CA 93106.

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gradient descent performs at about the same quality and speed as the GLA, while the conjugate gradient algorithm can often outperform the GLA in both attributes.

In order to use a gradient descent with a changing encoder, it is convenient to assume a zero-boundary condition, which ensures the validity of the gradient derivation. Namely, it is assumed that every training vector at each iteration has exactly one nearest codeword, so that infinitesimally small changes in the codewords ϵ_i do not change the nearest neighbor encoder mapping γ. While we have no theoretical guarantee that for a fixed training set the zero-boundary condition will hold after an arbitrary algorithmic change to the codewords, it is known to be a necessary condition for global optimality of a vector quantizer.

II. Gradient Descent Convergence

To find the steepest descent algorithm with the fastest convergence rate, define $p_{\min} = \min_i \{p_i\}$ and $p_{\max} = \max_i \{p_i\}$, and compute

$$\begin{aligned}\min_{\mu} \max_{i} &|1 - 2\mu_q p_i| \\ &= \min_{\mu} \max \{2\mu_q p_{\max} - 1,\ 1 - 2\mu_q p_{\min}\} \\ &= \min_{\mu} \begin{cases} 2\mu_q p_{\max} - 1 & \text{if } \mu_q \geq (p_{\min} + p_{\max})^{-1} \\ 1 - 2\mu_q p_{\min} & \text{if } \mu_q \leq (p_{\min} + p_{\max})^{-1} \end{cases} \\ &= \frac{p_{\max} - p_{\min}}{p_{\max} + p_{\min}}\end{aligned}$$

where the minimization was achieved by setting $\mu_q = (p_{\min} + p_{\max})^{-1}$.

We can also compute the value of μ_q which yields the greatest decrease in mean-square distortion for an iteration of the gradient descent algorithm. Denote the weighted mean-square distortion as a function of μ_q after a single update of the gradient descent by

$$D(\mu_q) = \sum_i \sum_{j:\gamma(e_j)=u_i} (e_j - \epsilon_{i,m+1})^T W (e_j - \epsilon_{i,m+1}).$$

We substitute into the equation above the gradient descent equation

$$\epsilon_{i,m+1} = \epsilon_{i,m} + \mu_q p_i (W + W^T)(c_i - \epsilon_{i,m})$$

and set the derivative of D with respect to μ_q equal to zero. Solving for μ_q then yields

$$\mu_q = \frac{\sum_i p_i^2 (c_i - \epsilon_{i,m})^T (W + W^T)^2 (c_i - \epsilon_{i,m})}{\sum_i p_i^3 (c_i - \epsilon_{i,m})^T (W + W^T)^3 (c_i - \epsilon_{i,m})}$$

which in the simple mean-square case reduces to

$$\mu_q = \frac{\sum_i p_i^2 \|c_i - \epsilon_{i,m}\|^2}{2 \sum_i p_i^3 \|c_i - \epsilon_{i,m}\|^2}.$$

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High Resolution Two-Dimensional ARMA Spectral Estimation

Xian-Da Zhang and Jie Cheng

Abstract—**This correspondence presents a "practical" algorithm for estimating the power spectrum of a 2-D homogeneous random field, based on 2-D autoregressive moving average (ARMA) modeling. This algorithm is a two-step approach: first the AR parameters are estimated by solving a new version of the 2-D modified Yule–Walker (MYW) equation, for which some existing efficient algorithms are available; then the MA spectrum parameters are obtained via simple computations. The potential capability and the high resolution performance of the algorithm are demonstrated by using some numerical examples.**

I. Introduction

In recent years, considerable progress has been made in developing high resolution 2-D spectral estimations, especially the maximum entropy (ME) method [1]–[6].

The 1-D ME method is known to be equivalent to autoregressive (AR) linear prediction modeling. Since any prediction model of 2-D random fields fails to be connected to the ME estimation [3], and the computation of the 2-D ME estimate is highly nonlinear, iterative algorithms have been developed by Woods [1] and Lim and Malik [2] in order to implement the 2-D ME method. However, on the one hand, such spectrum estimates need not exist [7]; and, on the other hand, it is well known that for any iterative algorithm, its convergence rate and the accuracy of the estimate crucially depend on the selection of algorithm parameters. To overcome these fundamental difficulties, Kimura and Honoki [6] recently proposed a hybrid approach, i.e., a "linearized" ME estimation. Their algorithm seems to be conceptually simple and computationally attractive and practical. Unfortunately, the final spectrum estimate given by this hybrid approach does not coincide with the true ME estimate even for the case of cyclic and skew-cyclic Teoplitz covariance matrices, as Zhang and Cui [8] have recently shown.

From the viewpoint of modeling, Cadzow and Ogino [9] have developed a procedure for generating a 2-D autoregressive moving average (ARMA) model. However, there are some difficulties associated with their procedure, including weighting coefficient selection and heavy computational burden for estimating AR parameters. In addition, the use of the smoothed periodogram method for determining the numerator dynamics of power spectrum would result in reduction of the frequency resolution.

In this correspondence, we propose a new 2-D ARMA spectral estimator which is based on a two-step approach: first the AR parameters are estimated; then the moving average (MA) spectrum

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X.-D. Zhang was with the Changcheng Institute of Metrology and Measurement (CIMM), Beijing 100095, China. He is now with the Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093.

J. Cheng was with the CIMM, Beijing 100095, China. He is now with the Department of Electronics, China College of Metrology, Hangzhou, Zhejiang Province, China.

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