Note on:
"Tradeoff Between Source and Channel Coding *"

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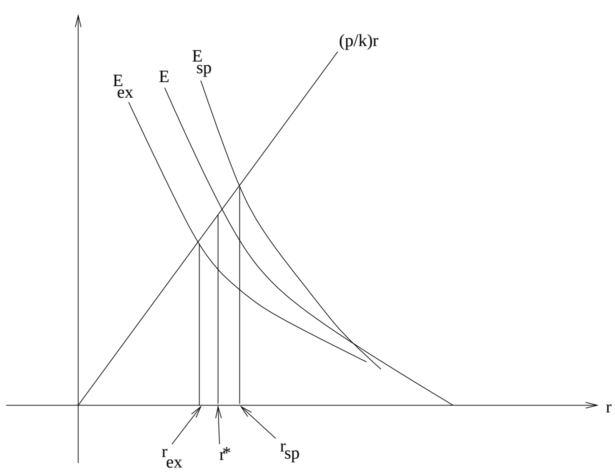
December 19, 2000

On page 1416, second column, the two lines before equation (20) could use some more clarification in order to justify equation (20). We provide more explanation here.

We will make use of the *reliability function* $E(r)$ of a channel (see the definition in the book by Gallager, "Information Theory and Reliable Communication", page 160). We know that $E_{ex}(r) \leq E(r) \leq E_{sp}(r)$ for all r, as in the figure below. The minimum probability of channel decoding



error decreases as $P_e = 2^{-kRE(r)+o(R)}$. The arguments used to obtain (4) and (14) still hold but with $E_{ex}(r)$ and $E_{sp}(r)$ each replaced by $E(r)$. This gives

$$\bar{D}_{R,r} = 2^{-pRr+O(1)} + 2^{-kRE(r)+o(R)}$$

for all r and for large R. The optimal rate r^* is found by balancing the exponents, yielding

$$E(r^*) = (p/k)r^* + o(1).$$

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*B. Hochwald and K. Zeger, *IEEE Trans. on Information Theory,* vol. 43, no. 5, pp. 1412-1424, September 1997.

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Since E_{ex} and E_{sp} are continuous, it is clear (see the figure) that

$$r_{ex} \leq r^* \leq r_{sp},$$

thus justifying equation (20) in the paper.

Here is an alternative proof of half of equation (20), namely that $r^* \geq r_{ex}$. Assume to the contrary that the channel code rate r^* that minimizes the average distortion obeys $r^* < r_{ex}$. Then from (5) and the fact that E_{ex} is a decreasing function, we have that $\frac{p}{k}r^* < E_{ex}(r^*)$. Let us denote the average distortion at transmission rate R and channel code rate r by $\bar{D}_{R,r}$ where we have added for clarification purposes the extra subscript r, and we have deleted the arguments Q and ϵ. Thus, using (4) and (14), we get

$$\begin{aligned}
\bar{D}_{R,r^*} &> 2^{-pRr^*+O(1)} + 2^{-kRE_{sp}(r^*)+o(R)} \\
&= 2^{-pRr^*+O(1)} \\
&\geq 2^{-pRr_{ex}} \\
&= 2^{-pRr_{ex}+O(1)} + 2^{-kRE_{ex}(r_{ex})+o(R)} \\
&\geq \bar{D}_{R,r_{ex}}
\end{aligned}$$

which contradicts the assumption that r^* minimizes the distortion in the limit of large R. Hence $r^* \geq r_{ex}$.