INSTRUCTIONS
This exam is closed book and closed notes, with the one exception that you are allowed notes on both sides of one sheet of 8.5 x 11.0 inch paper (not tape or stick-ons allowed). No calculators, laptop computers, or other electronic devices are allowed.

Write your answers in the spaces provided. Show all your work. If you need extra space, please use the back of the previous page. Partial credit will be given only for substantial progress on a problem. Zero credit will be given for correct answers that lack adequate explanation of how they were obtained. There is a maximum total of 40 points on this exam. Simplify your answers as much as possible and leave answers as fractions, not decimal numbers.
**Problem 1 (20 points)**
The circuit shown below has one resistor, one capacitor, a voltage source $v_i(t) = -\cos(2t + (\pi/3))$, and a current source, $i(t) = 2\cos(2t)$. Find the sinusoidal steady-state output voltage $v_o(t)$ indicated in the diagram. Simplify your answer as much as possible and make sure your answer is purely real-valued. If an expression cannot be simplified by hand, it can be left in a simplified form with square roots, trigonometric functions, etc.
Problem 1 (continued)

SOLUTION: Consider the more general circuit diagram with complex impedances:

![Circuit Diagram]

We have the following phasor equations:

\[
I = I_1 + I_2 \\
V_i + I_1 Z_1 = I_2 Z_2 = (I - I_1)Z_2 \\
\therefore I_1 = \frac{IZ_2 - V_i}{Z_1 + Z_2} \\
I_2 = I - I_1 = \frac{IZ_1 + V_i}{Z_1 + Z_2} \\
V_o = I_2 Z_2 = \frac{Z_2(IZ_1 + V_i)}{Z_1 + Z_2}
\]
$$\omega = 2$$

$$V_i = -e^{j\pi/3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}j$$

$$I = 2$$

$$Z_1 = R = 1$$

$$Z_2 = \frac{1}{Cj\omega} = \frac{1}{j} = -j$$

$$\therefore V_o = \frac{-j((2)(1) - \frac{1}{2} - \frac{\sqrt{3}}{2}j)}{1 - j} = \frac{1 - j}{2} \cdot \frac{3 - j\sqrt{3}}{2} = \frac{3 - \sqrt{3} - (3 + \sqrt{3})j}{4}$$

$$|V_o| = \frac{1}{4} \sqrt{(3 - \sqrt{3})^2 + (3 + \sqrt{3})^2}$$

$$= \frac{1}{4} \sqrt{9 - 6\sqrt{3} + 3 + 9 + 6\sqrt{3} + 3}$$

$$= \sqrt{3}/2$$

$$\angle V_o = -\tan^{-1} \left( \frac{3 + \sqrt{3}}{3 - \sqrt{3}} \right)$$

$$= -\tan^{-1} \left( \frac{2 + \sqrt{3}}{2} \right)$$

$$v_o(t) = \sqrt{3/2} \cdot \cos \left( 2t - \tan^{-1} \left( \frac{2 + \sqrt{3}}{2} \right) \right)$$
Problem 1 (continued)
Problem 2 (20 points)

Suppose \( f(t) \) is the function \( \sin(3t) \), except that all of its negative parts are zeroed out, as shown in the figure below. Compute the Fourier Series of \( f(t) \) in trigonometric form. Your answer should be purely real (i.e. no \( j \)'s in it).

![Graph of f(t)](image)

SOLUTION: We have \( \omega = 2\pi/T \) and

\[
f(t) = \sum_{n=-\infty}^{\infty} F_n e^{j\omega t}
\]

\[
F_n = \frac{1}{T} \int_{0}^{T/2} \sin(\omega t) e^{-j\omega t} dt
\]

\[
= \frac{1}{2jT} \int_{0}^{T/2} (e^{j\omega t} - e^{-j\omega t}) e^{-j\omega t} dt
\]

\[
= \frac{1}{2jT} \int_{0}^{T/2} (e^{j\omega t(1-n)} - e^{-j\omega t(1+n)}) dt
\]

If \( n \neq 1 \) and \( n \neq -1 \), then

\[
F_n = \frac{-1}{2\omega T} \left[ \frac{1}{1-n} \left( e^{j\omega (T/2)(1-n)} - 1 \right) + \frac{1}{1+n} \left( e^{-j\omega (T/2)(1+n)} - 1 \right) \right]
\]

\[
= \frac{-1}{4\pi} \left[ \frac{1}{1-n} \left( e^{j\pi(1-n)} - 1 \right) + \frac{1}{1+n} \left( e^{-j\pi(1+n)} - 1 \right) \right]
\]

\[
= \frac{-1}{4\pi} \left[ \frac{1}{1-n} \left( (-1)^{1-n} - 1 \right) + \frac{1}{1+n} \left( (-1)^{1+n} - 1 \right) \right]
\]

\[
= \frac{-1}{4\pi} \left[ \frac{1}{1-n} \left( (-1)^{n} - 1 \right) + \frac{1}{1+n} \left( (-1)^{n} - 1 \right) \right]
\]

\[
= \frac{1}{2\pi} \left\{ \begin{array}{ll}
\frac{1}{1-n} + \frac{1}{1+n} & \text{if } n \text{ is even} \\
0 & \text{if } n \text{ is odd}
\end{array} \right.
\]

\[
= \frac{1}{\pi} \left\{ \begin{array}{ll}
\frac{1}{n} & \text{if } n \text{ is even} \\
0 & \text{if } n \text{ is odd}
\end{array} \right.
\]
Problem 2 (continued)

Also,

\[
F_1 = \frac{1}{2jT} \int_0^{T/2} (1 - e^{-2j\omega t}) t \, dt = \frac{1}{2jT}((T/2) - 0) = \frac{1}{4j}
\]

\[
F_{-1} = \frac{1}{2jT} \int_0^{T/2} (e^{2j\omega t} - 1) t \, dt = \frac{1}{2jT}(0 - (T/2)) = -\frac{1}{4j}
\]

Thus,

\[
f(t) = F_0 + F_1 e^{j\omega t} + F_1 e^{-j\omega t} + \sum_{n \text{ even}, n \neq 0} F_n e^{jnt}
\]

\[
= \frac{1}{\pi} + \frac{1}{4j} e^{j\omega t} - \frac{1}{4j} e^{-j\omega t} - \frac{1}{\pi} \sum_{n \text{ even}, n \geq 2} \frac{e^{jnt} + e^{-jnt}}{n^2 - 1}
\]

\[
= \frac{1}{\pi} + \frac{1}{2} \sin(\omega t) - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(2n\omega t)}{4n^2 - 1}
\]
Problem 2 (continued)