Name ________________________________

Your UCSD ID Number ________________________________

Signature ________________________________

**INSTRUCTIONS**

This exam is closed book and closed notes, with the one exception that you are allowed notes on both sides of one sheet of 8.5 x 11.0 inch paper (not tape or stick-ons allowed). No calculators, laptop computers, or other electronic devices are allowed.

Write your answers in the spaces provided. Show all your work. If you need extra space, please use the back of the previous page. Partial credit will be given only for substantial progress on a problem. Zero credit will be given for correct answers that lack adequate explanation of how they were obtained. There is a maximum total of 40 points on this exam. Simplify your answers as much as possible and leave answers as fractions, not as decimal numbers.

**GRADING**

1. 20 points ______
2. 20 points ______
TOTAL (40 points) ______
Problem 1 (20 points)
The circuit shown below has two resistors, one capacitor, one inductor, a voltage source $v_i(t) = 2\sqrt{2}\cos(5t + (\pi/4))$, and two current sources, $i_1(t) = 2\cos(5t + (\pi/3))$ and $i_2(t) = \sqrt{3}\sin(5t)$. Find the sinusoidal steady-state output voltage $v_o(t)$ indicated in the diagram. Simplify your answer as much as possible and make sure your answer is purely real-valued. If an expression cannot be simplified by hand, it can be left in a simplified form with square roots, trigonometric functions, etc.
Problem 1 (continued)

SOLUTION: Consider the more general circuit diagram with complex impedances:

\[ \begin{align*}
\omega &= 5 \\
V_i &= 2\sqrt{2}e^{j\pi/4} = 2\sqrt{2} \left( \frac{1 + j}{\sqrt{2}} \right) = 2 + 2j \\
I_1 &= 2e^{j\pi/3} = 2 \left( \frac{1}{2} + \frac{\sqrt{3}}{2} \right) = 1 + j\sqrt{3} \\
I_2 &= \sqrt{3}e^{-j\pi/2} = -j\sqrt{3} \\
Z_1 &= 1 + \frac{1}{j5(1/(5\sqrt{3}))} = 1 - j\sqrt{3} \\
Z_2 &= 2 + 5(1/5) = 2 + j \\
V_2 &= V_o + I_1Z_1 \\
V_i &= V_2 + (I_1 + I_2)Z_2 \\
&= V_o + I_1Z_1 + (I_1 + I_2)Z_2 \\
V_o &= V_i - I_1Z_1 - (I_1 + I_2)Z_2 \\
&= (2 + 2j) - (1 + j\sqrt{3})(1 - j\sqrt{3}) - (1 + j\sqrt{3} - j\sqrt{3})(2 + j) \\
&= (2 + 2j) - 4 - (2 + j) \\
&= -4 + j \\
\therefore v_o(t) &= \sqrt{17} \cos(5t + \pi - \tan^{-1}(1/4))
\end{align*} \]
Problem 2 (20 points)

Suppose a linear, time-invariant system has real-valued frequency response \( H(\omega) \) shown below:

Let the input signal \( f(t) \) to the system be even and periodic, with a formula for it on the interval \([0, \frac{1}{2}]\) shown below:

Find the output \( y(t) \) of the system. (Express your answer as a real-valued function, i.e. with no \( j \)'s.)
**Problem 2** (continued)

**SOLUTION:** Compute the Fourier Series of \( f(t) \) with period \( T = 2\pi/\omega_0 \):

\[
f(t) = e^{-A|t|} \quad \text{on} \quad [-B, B]
\]

\[
F_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-j\omega_0 nt} \, dt
\]

\[
= \frac{1}{T} \int_{-B}^{B} e^{-A|t|} e^{-j\omega_0 nt} \, dt
\]

\[
= \frac{1}{T} \int_{-B}^{0} e^{t(A-j\omega_0 n)} \, dt + \frac{1}{T} \int_{0}^{B} e^{-t(A+j\omega_0 n)} \, dt
\]

\[
TF_n = \frac{1 - e^{-B(A-j\omega_0 n)}}{A-j\omega_0 n} + \frac{1 - e^{-B(A+j\omega_0 n)}}{A+j\omega_0 n}
\]

\[
TF_n(A^2 + (\omega_0 n)^2) = (1 - e^{-B(A-j\omega_0 n)})(A + j\omega_0 n) + (1 - e^{-B(A+j\omega_0 n)})(A - j\omega_0 n)
\]

\[
= 2A - e^{-AB} ((A + j\omega_0 n)e^{jB\omega_0 n} + (A - j\omega_0 n)e^{-jB\omega_0 n})
\]

\[
= 2A - e^{-AB} (2A \cos(B\omega_0 n) - 2\omega_0 n \sin(B\omega_0 n))
\]

\[
\therefore F_n = \frac{A - e^{-AB} (A \cos(B\omega_0 n) - \omega_0 n \sin(B\omega_0 n))}{(T/2)(A^2 + (\omega_0 n)^2)}
\]

We know \( T = 4 \) and \( \omega_0 = \pi/2 \). The frequency response has equation \( H(\omega) = 4 - |\omega| \) when \( 3 \leq |\omega| \leq 4 \). Thus, \( 3 \leq n\omega_0 \leq 4 \) only when \( n = 2 \) and \(-4 \leq n\omega_0 \leq -3\) only when \( n = -2 \) so \( H(n\omega_0) = 0 \) for all integers \( n \), except when \( n = \pm 2 \). Also, note that \( F_n = F_{-n} \) and \( 2B\omega_0 = \pi/2 \), so \( \cos(2B\omega_0) = 0 \) and \( \sin(2B\omega_0) = 1 \). The output is

\[
y(t) = \sum_{n=-\infty}^{\infty} F_n H(n\omega_0) e^{j\omega_0 nt}
\]

\[
= F_2 H(2\omega_0) e^{2j\omega_0 t} + F_{-2} H(-2\omega_0) e^{-2j\omega_0 t}
\]

\[
= F_2 H(2\omega_0) \cdot 2 \cos(2\omega_0 t)
\]

\[
= 4 \left( \frac{A + 2\omega_0 e^{-AB}}{T(A^2 + 4\omega_0^2)} \right) (4 - 2\omega_0) \cos(2\omega_0 t)
\]

We have \( A = 3, B = 1/2, T = 4, \omega_0 = \pi/2 \), so

\[
y(t) = \frac{3 + \pi e^{-3/2}}{9 + \pi^2} \cdot (4 - \pi) \cos(\pi t)
\]
Problem 2 (continued)