ECE 45 DC Circuits Review

Circuits Concepts:

- **Sources**: voltages in series can be combined; currents in parallel can be combined.
  The power supplied/dissipated by a component is the current multiplied by the voltage drop. If current flows from higher to lower (lower to higher) voltage, power is dissipated (supplied).

- A **branch** is two components (source, resistor, capacitor, inductor, etc.) connected end to end.
  A **node** is a connection of multiple branches.
  Nodes have a particular voltage, and branches have a particular current.

- **Kirchhoff’s Current Law**: A node has zero net current, i.e. the incoming current of a node equals the outgoing current of a node.

- **Kirchhoff’s Voltage Law**: The net voltage in a closed loop in a circuit is zero.

- These laws can be used to set up systems of equations which can be used to solve for specific voltages and currents in the circuit.

- **KCL (Node Voltage)**: (Used by SPICE)
  
  - Assign a node to be the reference ground, 0V.
  - Assign a voltage $V_i$ to all additional nodes, where $V_i$ is the voltage with respect to ground.
  - Decide on arbitrary directions for the currents in each branch. It helps to write equations for each current in terms of the voltages $V_i$ ahead of time for consistency.
  - For each node, the net current is zero, so write an equation $\sum I_{in} = \sum I_{out}$.
  - Solve the system of equations for desired values.

- **KVL (Mesh Current)**: (Generally easier if fewer loops than nodes.)
  
  - Assign each independent loop a current $I_i$.
  - Write a KVL equation for each loop such that $\sum V = 0$. It helps to write equations for each voltage in terms of the currents $I_i$ ahead of time.
  - If multiple loops include a branch, the current in that branch is the sum of the loops which include that branch.
  - Solve the system of equations.

- **Power**: The power generated or dissipated by a component is proportionate to the current through the component and the voltage across the component. $P = V I$. 

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• **Source Transformation:** Replace a current/voltage source with a voltage/current source:

\[
\frac{V_{in}}{R} \quad \equiv \quad \frac{V_{in}}{R}
\]

\[
I_{in} \quad \equiv \quad I_{in}R
\]

• **Voltage/Current Divider:** Application of KCL/KVL that allows for quick calculation of a voltage/current. For any \( n \geq 2 \).

\[
V_k = V_{in} \frac{R_k}{R_1 + R_2 + \cdots + R_n}
\]
for each \( k = 1, 2, \ldots, n \)

\[
I_k = I_{in} \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}
\]
for each \( k = 1, 2, \ldots, n \)
Circuit Examples:

1. Find the output voltage of the following circuit:

\[
\begin{align*}
V_{i1} &= 20V \\
V_{i2} &= 5V \\
R_1 &= R_2 = R_3 = 10\Omega \\
R_4 &= 5\Omega \\
R_5 &= 5\Omega
\end{align*}
\]

Labeling currents and voltages gives us:

By KCL:
\[
\sum I_{in} = \sum I_{out}
\]

By Ohm’s Law:
\[
\begin{align*}
I_1 &= (V_{i1} - V_a)/R_1 \\
I_2 &= V_a/R_2 \\
a) \ I_1 + I_3 = I_2 \\
b) \ I_5 = I_3 + I_4 \\
I_4 &= V_b/R_4 \\
I_5 &= (V_{i2} - V_b)/R_5
\end{align*}
\]

Substituting expressions for \(I_1, ..., I_5\) into \(a\) and \(b\) gives us:

\[
\begin{align*}
a) \quad \frac{V_1}{R_1} &= V_a \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - V_b \frac{1}{R_3} \\
b) \quad -\frac{V_2}{R_5} &= V_a \frac{1}{R_3} - V_b \left( \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \right)
\end{align*}
\]

Plugging in numerical values and subtracting \(a\) from \(b\) gives us

\[
V_b \frac{4}{5} = 3 \rightarrow V_b = \frac{15}{4} V \quad \text{and} \quad V_a = \frac{55}{8} V
\]

and so

\[
V_o = V_a - V_b = \frac{25}{8} V
\]
2. Find the power dissipated by \( R_6 \)

Labeling currents and voltages gives us:

\[
\begin{align*}
V_1 &= 1V \\
V_2 &= 2V \\
R_1 &= R_4 = 2\,\Omega \\
R_2 &= 1\,\Omega \\
R_3 &= R_5 = 4\,\Omega \\
R_6 &= 8\,\Omega \\
\end{align*}
\]

By KVL:

\[
\sum V_{gain} = \sum V_{drop}
\]

1) \( V_1 = V_{R_1} + V_{R_2} \)
2) \( V_{R_2} = V_{R_3} \)
3) \( V_2 = V_{R_4} + V_{R_5} \)
4) \( 0 = V_{R_3} + V_{R_4} + V_{R_6} \)

By Ohm's Law:

\[
\begin{align*}
V_{R_1} &= R_1 I_1 \\
V_{R_2} &= R_2 (I_1 - I_2) \\
V_{R_3} &= R_3 (I_2 + I_4) \\
V_{R_4} &= R_4 (I_3 + I_4) \\
V_{R_5} &= R_5 I_3 \\
V_{R_6} &= R_6 I_4 \\
\end{align*}
\]

Plugging in numerical values and substituting the expressions for \( V_{R_1}, ..., V_{R_6} \) into 1), ..., 4) yields:

\[
\begin{align*}
1) \quad & 1 = 3 I_1 - I_2 \\
2) \quad & 0 = I_1 - 5 I_2 - 4 I_4 \\
3) \quad & 2 = 6 I_3 + 2 I_4 \\
4) \quad & 0 = 4 I_2 + 2 I_3 + 14 I_4 \\
\end{align*}
\]

\[
\begin{pmatrix}
3 & -1 & 0 & 0 \\
1 & -5 & 0 & -4 \\
0 & 0 & 6 & 2 \\
0 & 4 & 2 & 14 \\
\end{pmatrix}
\begin{pmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4 \\
\end{pmatrix}
=
\begin{pmatrix}
1 \\
0 \\
2 \\
0 \\
\end{pmatrix}
\rightarrow
I_1 = 5/13 \, A \\
I_2 = -2/13 \, A \\
I_3 = 19/52 \, A \\
I_4 = -5/52 \, A \\
\]

Now that we know the values of all of the currents in the circuit, we can solve for the current through \( R_6 \) and the voltage across \( R_6 \), and so:

\[
P_{R_6} = V_{R_6} I_{R_6} = R_6 (I_{R_6})^2 = \frac{25}{338} \, W
\]