Problem 2.1:
Let $S_n$ be the set of all Huffman trees with $n$ leaves that are not balanced and do not contain a self-synchronizing string.

(a) For each $n \leq 5$, prove that $S_n$ is empty.

(b) Find a tree in $S_9$ that is a prefix-suffix code. Explain why.

(c) Find a tree in $S_9$ that is not a prefix-suffix code. Explain why.

(d) Prove for all $n \geq 3$ that $S_{n^2}$ is non-empty.

Problem 2.2:
Prove that no prefix-suffix Huffman codes have a self-synchronizing string.

Problem 2.3:
Suppose $k$ and $n$ are positive integers such that $2 \leq k < n$.
(a) Prove that there exists a prefix-suffix code with $2^{k-1}$ codewords of length $k$ and $2^{n-2}$ codewords of length $n$.

(b) Find a prefix suffix code with these lengths in the special case when $k = 3$ and $n = 5$.

Problem 2.4:
(a) Find all shortest self-synchronizing strings for the following Huffman code: $\{0^{49}, 1^{49}\} \cup \bigcup_{n=1}^{48} \{0^n1, 1^n0\}$.

(b) Find all self-synchronizing strings of length at most 6, for the following prefix code: $\{10, 010, 011, 110, 111\}$.