1. \( P(A \cup (B^c \cup C^c)^c) = P(A \cup BC) \) by DeMorgan’s theorem.
   
   (a) \( P(BC) = 0 \), and therefore \( P(A \cup BC) = P(A) = 1/2 \)
   
   (b) \( P(A \cup BC) = P(A) + P(BC) - P(ABC) = 1/2 + 1/4 - 1/6 = 7/12 \)
   
   (c) \( P(A \cup BC) = P(A) + P(BC) - P(ABC) = 1/2 + 1/3 - 0 = 5/6 \)
   
   (d) \( (A^c \cap (B^c \cup C^c))^c = A \cup (B^c \cup C^c)^c \) by DeMorgan’s theorem. Hence,
   
   \[ P(A \cup (B^c \cup C^c)^c) = 1 - P(A^c \cap (B^c \cup C^c)) = 1 - 0.7 = 0.3. \]

2. Let \( B \) be the event that the two dice show the same number, and let \( A \) be the event that the sum is greater than seven. We want \( P(B|A) \). We have that \( |A| = 15 \), since 15 outcomes of two dice sum to eight or greater; therefore \( P(A) = 15/36 = 5/12 \). Also, \( AB = \{(4, 4), (5, 5), (6, 6)\} \), since these are the outcomes that show the same number \textit{and} sum to eight or greater. Therefore, \( P(AB) = 3/36 = 1/12 \), and it follows that \( P(B|A) = P(AB)/P(A) = 1/5 \).

3. By definition, \( P(AB|(A \cup B)) = P(AB \cap (A \cup B))/P(A \cup B) \). Note that \( AB \cap (A \cup B) = AB \). Therefore \( P(AB|(A \cup B)) = P(AB)/P(A \cup B) \). Also observe that \( A \subset A \cup B \), and hence \( P(A \cup B) \geq P(A) \). Therefore, \( P(AB|A \cup B) \leq P(AB)/P(A) \). The right hand side of this inequality is \( P(B|A) \). But \( P(B|A) = P(AB|A) \), so the inequality is proven.

The source of the inequality was \( A \subset A \cup B \), which implied that \( P(A \cup B) \geq P(A) \). We have equality if and only if \( A = A \cup B \), or, equivalently, \( B \subset A \).

4. Let \( A \) and \( B \) denote respectively the events that your first choice and your final choice is the curtain concealing the prize. \( P(A) = 1/3 \), \( P(A^c) = 2/3 \).

   (a) If you always switch, then \( P(B|A) = 0 \), while \( P(B|A^c) = 1 \). Hence,
   
   \[ P(B) = P(B|A)P(A) + P(B|A^c)P(A^c) = 2/3. \]
   
   (b) If you never switch, then \( P(B|A) = 1 \), while \( P(B|A^c) = 0 \). Hence, \( P(B) = 1/3 \).
   
   (c) If you decide at random, then \( P(B|A) = P(B|A^c) = 1/2 \) and \( P(B) = 1/2 \) also. Monty is correct in his assertion. (Would he lie to you? Besides, it was on TV, so it must be true!!!!)

5. Let \( B \) be the event that the target is hit and let \( A \) be the event that there is a gust of wind.

   (a) \( P(B) = P(B|A)P(A) + P(B|A^c)P(A^c) = (0.4)(0.3) + (0.7)(0.7) = 0.61 \)
   
   (b) This is asking for \( P(A^c|B^c) = P(A^cB^c)/P(B^c) = [P(B^c|A^c)P(A^c)]/P(B^c) \). From part (a), \( P(B^c) = 0.39 \). \( P(B^c|A^c) = 1 - P(B|A^c) = 0.3 \). Therefore, \( P(A^c|B^c) = (0.3)(0.7)/0.39 = 0.54 \).
6. (a) If $A$ is an event independent of itself, then $P(A) = P(AA) = P(A)P(A) = [P(A)]^2$. This can happen only if $P(A) = 0$ or $P(A) = 1$.

(b) $P(A \cup B) = P(A) + P(B) - P(AB)$. If $A$ and $B$ are independent, then

$$P(A \cup B) = P(A) + P(B) - P(A)P(B) = 0.3 + 0.4 - (0.3)(0.4) = 0.58.$$ If $A$ and $B$ are disjoint, then $P(AB) = 0$, so $P(A \cup B) = 0.7$. If $P(A)$ were 0.6 and $P(B)$ were 0.8 then the events could be independent, but they could not be disjoint since then $P(AB)$ would be zero, and you would have to conclude that $P(A \cup B) = 1.4$, which is nonsense.