Optimal Bandwidth Allocation for Source Coding, Channel Coding and Spreading in a CDMA System *

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ABSTRACT
This paper investigates the tradeoffs between source coding, channel coding and spreading in a CDMA system, operating under a fixed total bandwidth constraint. We consider a system consisting of a uniform source with a uniform quantizer, a channel coder, an interleaver, and a direct sequence spreading module. The system uses binary phase-shift keyed (BPSK) modulation. Rate-compatible punctured convolutional (RCPC) codes and soft decision Viterbi decoding are used for channel coding. The system is analyzed over both an additive white Gaussian noise (AWGN) channel and a flat Rayleigh fading channel. A tight upper bound for frame error rate is derived for non-terminated convolutional codes. The performance of the system is evaluated using the end-to-end mean squared error (MSE). We show that, for a given bandwidth, an optimal allocation of that bandwidth can be found using the proposed method.

KEY WORDS
Bandwidth allocation, DS-CDMA, Rayleigh fading, frame error rate, convolutional codes, multiuser system.

1 Introduction

Source coding, channel coding and spread spectrum are three of the main components in a CDMA communication system. They compete for the major shared resource – bandwidth. Source coding frees up bandwidth for both forward error correction (FEC) and spreading. Allocating more bandwidth to source coding allows more information from the source to be transmitted, but reduces the bandwidth available for both FEC and spreading. For different compression methods and rates, the bit stream coming out of the source encoder will be more or less sensitive to different types of error patterns. FEC and spreading protect the transmitted bits from noise and interference. Depending on the channel conditions and the characteristics of the source coded bit stream, the system will perform better with either more FEC or more spreading.

Related studies in the literature are limited to the tradeoff between either source coding and channel coding, e.g., [1, 2], or channel coding and spreading, e.g., [3, 4]. In each case, research topics included analyzing a given system to find the optimal bandwidth allocation to each module as in [1, 3], and joint design of coding algorithms or transmitter/receiver schemes for each category [2, 4].

In [5], we studied the bandwidth allocation tradeoff for a direct sequence CDMA system that incorporated an image coder, a RCPC [6] channel coder, and a RAKE receiver. Due to the complexity of the system, we obtained most of the results through simulations. In this paper, we will investigate the tradeoffs analytically.

Let r_s, r_c and N_s denote the source code rate (in bits-per-source symbol), the channel code rate, and the processing gain, respectively. If the source produces U symbols per second, for a given bandwidth of C chips per second, we have the following constraint:

$$U \cdot r_s \cdot \frac{1}{r_c} \cdot N_s \leq C \Longrightarrow r_s \cdot \frac{1}{r_c} \cdot N_s \leq C_0 \ , \qquad (1)$$

where $C_0 = C/U$ is a constant. We will find the bandwidth allocation $(\hat{r}_s, \hat{r}_c, \hat{N}_s)$ that optimizes system performance. We will also address the question of how sensitive is the optimal allocation to changes in the channel conditions, transmission rate, and bandwidth constraint.

The paper is organized as follows. Section 2 gives an overview of the system and our approach to the bandwidth allocation problem. Sections 3 and 4 analyze the system for both AWGN and flat Rayleigh fading channels. Some representative results of tradeoffs among the three components are presented in Section 5, and the conclusions are given in Section 6.

2 System overview

We consider a multiuser scenario. The system for each user is similar and is shown in Figure 1. The source input vector $X \in \mathbf{R}^k$ has closed bounded support. Each component of the vector is considered to be one source symbol. The output of the source encoder is an m-bit binary index, and the source code rate $r_s = m/k$. The source encoder is a

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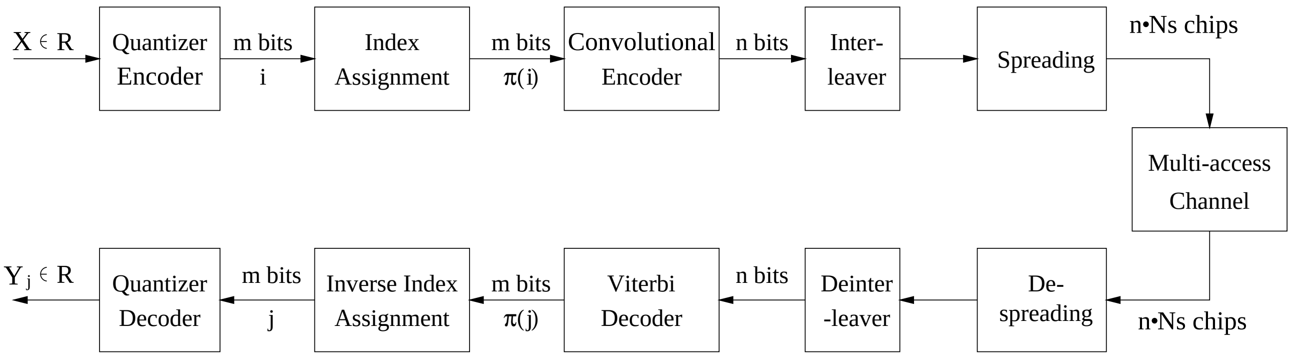


Figure 1. System overview.

quantizer with distortion

$$D_m = \sum_{i=0}^{2^m-1} \int_{S_i} ||x - y_i||^p f(x)dx \ , \qquad (2)$$

where $\{S_i\}_{i=0}^{2^m-1}$ is a partition of $\mathbf{R}^k$ into disjoint regions, each of which is represented by code vector $y_i \in \mathbf{R}^k$, $|| \cdot ||^p$ represents the p^{th} power of the usual Euclidean l_2 norm, and $f(x)$ is the probability density function of X.

In our system, we take X to be a one-dimensional ($k = 1$) uniform source over $[0, 1]$. We also take $p = 2$, so that the end-to-end distortion is the mean-squared error. The quantizer we use here is designed to be optimal for a noiseless channel, and it has partitions and code points

$$S_i = [i \cdot 2^{-m}, (i+1) \cdot 2^{-m}), \quad y_i = \frac{2^{-m}}{2}(2i+1), \quad (3)$$

respectively, where $i = 0, \cdots, 2^m - 1$, and y_i is the centroid of S_i . In [7], we prove that even though the analysis is done for a uniform source, the results can be applied to a wide variety of source distributions. Since $r_s = m/k = m$, we will use m and r_s interchangeably.

The m-bit binary representation, $i \in \{0, 1, \ldots, 2^m - 1\}$, of a source symbol is mapped to an m-bit index $\pi(i) \in \{0, 1, \ldots, 2^m - 1\}$ by the index assignment block [1]. Its purpose is to rearrange the indices so that those with small Hamming distances between them represent quantization levels which are close. This way, the distortion caused by the most probable errors is small, and thus the total distortion is small. There are many different index assignment schemes possible for a scalar quantizer [8]. We pick a random index assignment [1], where the mapping $\pi(\cdot)$ is a one-to-one mapping from indices 0 through $2^m - 1$ to a random permutation of the indices 0 through $2^m - 1$. Since the permutation is random, the index assignment can be good or bad. To measure its distortion, we must average over all possible permutations, i.e., we use the expectation of the distortion to evaluate its performance. The use of random indexing simplifies analysis, although the method proposed in this paper will work for any specific index assignment.

[1]The index assignment block is a part of the source coding. We separated it out for ease of analysis.

A channel encoder with rate $r_c = m/n$ encodes the indices and passes them to the interleaver. The interleaver output is multiplied by the spreading sequence assigned to the given user, with spreading factor N_s. The output of the spreading is modulated and passed to the channel. Here we consider DS-CDMA systems, with channel symbol rate $1/T_s$, chip rate $1/T_c$, and processing gain $N_s = T_s/T_c$. The system has K active users, with the 0^{th} user taken as the reference user.

At the receiver, the received signal is demodulated, despread, and decoded by the channel decoder to m-bit indices. These indices are mapped by the inverse index assignment block and decoded by the quantizer decoder. By comparing the reconstructed source with the original source symbol, we can calculate the end-to-end distortion.

From [9], the expected mean-squared error of a system with a uniform source, a uniform scalar quantizer, and a random index assignment, is

$$D(m, P_e) = \frac{2^{-2m}}{12} + \frac{P_e}{6}(1 + 2^{-m}) \approx \frac{2^{-2m}}{12} + \frac{P_e}{6} \ , \quad (4)$$

where P_e is the probability of index error, i.e., at least one bit of the m-bit index is in error, so the index is incorrect. In earlier work, without proof, [10] gives a similar result for an uncoded memoryless binary symmetric channel. Equation (4) works for all channel codes and channels.

The value of P_e depends on the channel code, modulation scheme, channel conditions and receiver structure. Generally, finding the expression for P_e is no trivial task. In this paper, we will find a close upper bound for P_e, and thus an upper bound for the distortion $D(m, P_e)$, as a function of the three parameters r_s, r_c, and N_s. We denote this upper bound by $D_U(r_s, r_c, N_s)$, and find the optimal bandwidth allocation $(\hat{r}_s, \hat{r}_c, \hat{N}_s)$ for D_U. By operating the system with this bandwidth allocation, we can guarantee a system performance better than $D_U(\hat{r}_s, \hat{r}_c, \hat{N}_s)$.

Since we use non-trivial channel codes, P_e, and thus, $D(m, P_e)$, are decreasing functions of $1/r_c$, i.e., if both r_s and N_s are given, the larger $1/r_c$ (for a given level of complexity) is, the better the performance of the system. Therefore, we can replace the inequality constraint, Equation (1), by an equality constraint:

$$r_s \cdot \frac{1}{r_c} \cdot N_s = C_0 \ . \quad (5)$$

Hence, the problem we need to solve is to minimize $D_U(r_s, r_c, N_s)$ under constraint (5).

3 System

The bit stream out of the index assignment block is encoded by a non-terminated convolutional encoder with code rate r_c. At the receiver, a soft decision Viterbi decoder decodes the noise-contaminated bit stream to indices. The output of the interleaver is multiplied by a long pseudo-random sequence assigned to the given user and transmitted using BPSK modulation.

Since we transmit the indices by sequentially passing them through a non-terminated convolutional code, the m-bit index error rate is also the frame error rate of this convolutional encoder. A frame of size m consists of m consecutive information bits. The error rate of an information frame of size m is the probability that at least one of the m bits in the frame is decoded incorrectly. In [11], an upper bound for the frame error probability was given heuristically, but a requirement of very large m/r_c was imposed. In [7], we derive a tight upper bound for frame error rate for any coded frame lengths which are larger than the constraint length:

$$P_e = P_f(m) \leq \sum_d \left((l_m - 1)a_d + b_d\right) P_1(d) \ , \quad (6)$$

where m is the information frame size, l_m is the number of branches of the trellis that are in a frame, $P_1(d)$ is the pairwise probability of two sequences that have Hamming distance d, a_d is defined, as in [6], by $a_d \triangleq \sum_s T_{d,s}$, and

$$b_d \triangleq \sum_s s T_{d,s} = \frac{\partial \sum_s T_{d,s} x^d y^s}{\partial y}|_{x=y=1} \ . \quad (7)$$

In Equation (7), $T_{d,s}$ are the coefficients of the *complete path eumerator* [12]: $T(x, y) = \sum_{d,s} T_{d,s} x^d y^s$, where d is the Hamming weight of the encoder output of a path, s is the length of a path, and d, s both go from 1 to $+\infty$. Values of b_d for the RCPC codes in [6] are listed in [7]. Figure 2 compares the bound in (6) with simulation results for the rate 1/2 code in [6] with memory 4. From the plot, we can see that the theoretical upper bound is quite tight. For both AWGN and Rayleigh fading channels, we calculate $P_1(d)$ and then optimize the end-to-end distortion of the system.

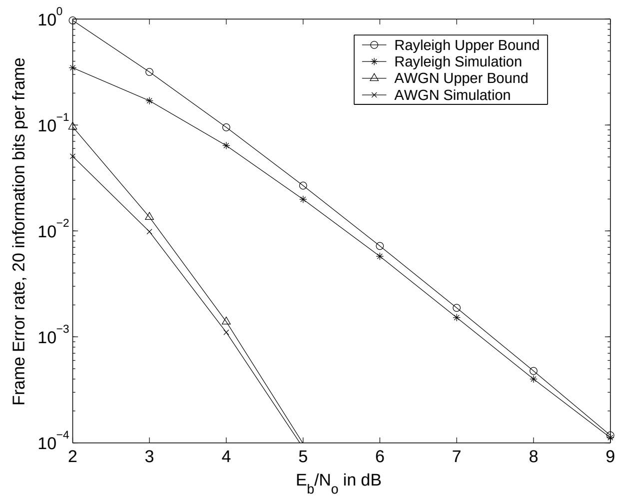


Figure 2. Upper bound on frame error rate for rate 1/2, convolutional code with memory 4.

3.1 AWGN channel

For a direct sequence CDMA system with a large number of users, the pairwise error probability for the AWGN channel is approximately given by

$$P_1(d) = Q\left(\sqrt{\frac{E_s \cdot d}{g(K-1)E_s/N_s + N_o/2}}\right) = Q\left(\sqrt{\frac{E_c \cdot dN_s}{g(K-1)E_c + N_o/2}}\right) \triangleq Q\left(\sqrt{HdN_s}\right) \ , \quad (8)$$

where

$$H \triangleq \frac{E_c}{g(K-1)E_c + \eta_o/2} = \frac{1}{g(K-1) + \eta_o/2E_c} \ . \quad (9)$$

Also E_s is the energy per channel bit; $\frac{N_o}{2}$ is the power spectral density of the white Gaussian noise, g is a constant which depends on the pulse shape, and equals $2/3$ when we use square-shaped chips, K is the total number of users; N_s is the processing gain, and $E_c = \frac{E_s}{N_o}$ is the energy per chip (which is kept constant). Substituting (8) into (6) and then into (4), we have

$$D(m, r_c, N_s) \leq \frac{1}{12} \cdot 2^{-2m} + \frac{1}{6} \sum_d \left((l_m - 1)a_d + b_d\right) \times Q\left(\sqrt{HdN_s}\right) \triangleq D_U(m, r_c, N_s) \ . \quad (10)$$

3.2 Flat Rayleigh fading channel

Assume $E[\alpha^2] = 1$, where α is the fade amplitude and has a Rayleigh density, and assume the fading seen by the channel decoder is uncorrelated from bit to bit. For a direct sequence CDMA system, the conditional pairwise error probability, conditioned on the fading parameters, is given by

[13]

$$
\begin{aligned}
P_2(d|\{\alpha_i\}) &= Q\left(\sqrt{\frac{E_s\sum_0^{d-1}\alpha_i^2}{g(K-1)E_s/N_s+\eta_0/2}}\right) \\
&= Q\left(\sqrt{\frac{E_c\cdot N_s\sum_0^{d-1}\alpha_i^2}{g(K-1)E_c+\eta_0/2}}\right) \\
&\triangleq Q\left(\sqrt{HN_s\sum_0^{d-1}\alpha_i^2}\right) .
\end{aligned}
$$

Averaging $P_2(d|\{\alpha_i\})$ over the distribution of all α_i, $i = 0,\ldots,d-1$ yields the pairwise error probability [13, 14.4.15]

$$
P_1(d) = \left(\frac{1-\mu}{2}\right)^d \sum_{i=0}^{d-1}\binom{d-1+i}{i}\left(\frac{1+\mu}{2}\right)^i , \quad (11)
$$

where

$$
\mu = \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}} ,
$$

and $\bar{\gamma}_c = \frac{1}{2}HN_sE[\alpha_i^2]$. When $\bar{\gamma}_c \gg 1$, $P_1(d)$ can be further simplified to

$$
\begin{aligned}
P_1(d) &\approx \left(\frac{1}{4\bar{\gamma}_c}\right)^d\cdot\binom{2d-1}{d} = \left(\frac{1}{2HN_s}\right)^d\binom{2d-1}{d} \\
&= \left(\frac{1}{2H}\right)^d\binom{2d-1}{d}\cdot N_s^{-d} \triangleq h(d)\cdot N_s^{-d} ,
\end{aligned} \quad (12)
$$

where

$$
h(d) = \left(\frac{g(K-1)+\eta_0/2E_c}{2}\right)^d\binom{2d-1}{d} . \quad (13)
$$

Substituting (12) into (6) and then into (4), we have

$$
\begin{aligned}
D(m,r_c,N_s) \leq& \frac{2^{-2m}}{12} + \frac{1}{6}\sum_d((l_m-1)a_d+b_d) \\
&\times h(d)N_s^{-d} \triangleq D_U(m,r_c,N_s) . \quad (14)
\end{aligned}
$$

4 Optimization

In the equations for the upper bound of the distortion, (10) and (14), $D_U(m,r_c,N_s)$ is not a simple function of r_c, i.e., for a given set of RCPC codes, the spectrum, a_d and b_d, cannot be written as a function of r_c. Thus, we cannot find the optimal bandwidth allocation by taking derivatives of D_U with respect to r_c. To find the optimal triplet $(\hat{m},\hat{r}_c,\hat{N}_s)$, we first fix r_c, and find the optimal allocation (m^*,N_s^*) and the minimum distortion $D_U^*(m)$ for this r_c. Then by comparing the minimum $D_U^*(m)$'s for different r_c, we find the best triplet.

For a given channel code rate r_c, we can use the bandwidth constraint and substitute $N_s = C_0\cdot r_c/m$ into $D_U(m,r_c,N_s)$, so that the upper bound D_U becomes a function of a single variable m. We denote this new function as $D_U(m)$.

4.1 AWGN channel

Substituting $N_s = C_0\cdot r_c/m$ into (10), we have

$$
\begin{aligned}
D_U(m) = \frac{1}{12}\cdot 2^{-2m}+&\frac{1}{6}\sum_d((l_m-1)a_d+b_d) \\
&\times Q\left(\sqrt{HdC_0r_c/m}\right) . \quad (15)
\end{aligned}
$$

Differentiating $D_U(m)$ with respect to m, and setting it equal to zero, results in

$$
\begin{aligned}
0 =& 12\frac{\partial D_U(m)}{\partial m} \\
=& 2^{-2m}\cdot(-2\ln 2) + 2\sum_d a_d\cdot\frac{1}{I_b}Q\left(\sqrt{\frac{HdC_0r_c}{m}}\right) \\
&+2\sum_d((l_m-1)a_d+b_d)\cdot-\frac{e^{-\frac{HdC_0r_c}{2m}}}{\sqrt{2\pi}}\cdot-\frac{\sqrt{HdC_0r_c}}{2\sqrt{m^3}} \\
\approx& 2^{-2m}\cdot(-2\ln 2) + 2\sum_d\frac{a_d}{I_b}\frac{e^{-\frac{HdC_0r_c}{2m}}}{\sqrt{2\pi}\sqrt{\frac{HdC_0r_c}{m}}} \\
&+2\sum_d((l_m-1)a_d+b_d)\cdot\frac{e^{-\frac{HdC_0r_c}{2m}}}{\sqrt{2\pi}}\cdot\frac{\sqrt{HdC_0r_c}}{2\sqrt{m^3}} .
\end{aligned} \quad (16)
$$

The approximation in the last step of (16) is valid when $HdC_0r_c/m = HdN_s$ is large. It is easy to show that $D_U(m)$ is a convex cup, so, solving (16) numerically with any good root finding algorithm gives the optimal m^* for an AWGN channel.

4.2 Flat Rayleigh channel

Substituting $N_s = C_0\cdot r_c/m$ into (14) results in

$$
\begin{aligned}
D_U(m) = \frac{2^{-2m}}{12}+&\frac{1}{6}\sum_d h(d)(C_0r_c)^{-d}i \\
&\times((l_m-1)a_d+b_d)m^d . \quad (17)
\end{aligned}
$$

Setting the derivative of $D(m)$ equal to zero, we obtain

$$
\begin{aligned}
0 =& 12\frac{\partial D_U(m)}{\partial m} = 2^{-2m}\cdot(-2\ln 2) + 2\sum_d a_dh(d) \\
&\times(C_0r_c)^{-d}\left(\frac{m^d}{I_b}+((l_m-1)a_d+b_d)d\cdot m^{d-1}\right) .
\end{aligned} \quad (18)
$$

As was the case with (16), (18) needs to be solved numerically. Note that for large signal-to-noise ratios, $\frac{E_s}{N_o}$, we can ignore non-minimum distance error events and thus use simpler forms of $\frac{\partial D_U}{\partial m}$ for both cases above.

5 Results

Figure 3 shows the upper bound for the end-to-end distortion, D_U, versus the source code rate and channel code rate for an uncorrelated Rayleigh fading channel, under the bandwidth constraint $\frac{r_s}{r_c} \cdot N_s \leq C_0 = 800$. Here $\frac{E_c}{N_o}$ is $-6dB$, and the active number of users in the system is $K = 10$. The RCPC codes used are from Table 1 of [6]; their spectra are listed in [7]. From Figure 3, we can see that, for each given channel code rate r_c, there is an optimal source code rate m that achieves the smallest D_U for this r_c. The global optimum always falls at the smallest r_c, i.e., the strongest channel coding.

This is true for both AWGN and flat Rayleigh fading channels when no interference suppression is implemented.

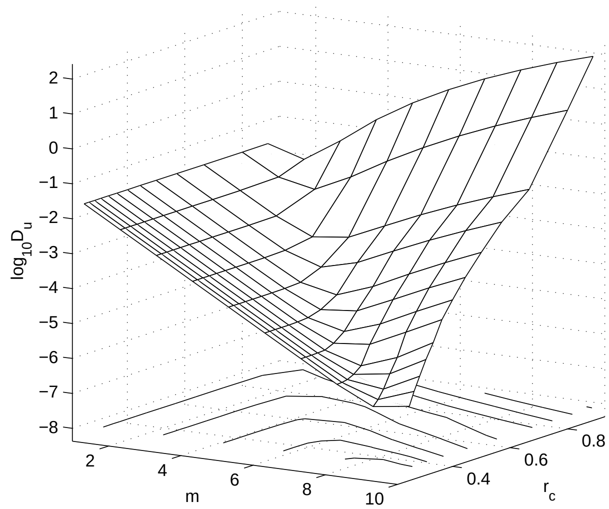


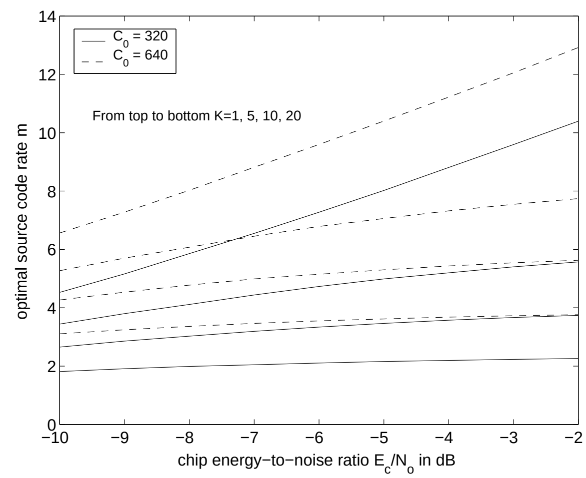
Figure 3. Distortion D_U vs. source code rate m and channel code rate r_c.

For any fixed r_c, by solving (16) and (18), we also show in the following figures how m^* and N_s^* vary when the channel conditions change.

Figure 4(a) shows the variation of the optimal m with the chip energy-to-noise ratio, $\frac{E_c}{N_o}$, and Figure 4(b) shows analogous results for the optimal value of N_s. There are two sets of curves on each figure, one for bandwidth constraint $C_0 = 320$, and the other for $C_0 = 640$. The curves are parameterized by the number of users $K = 1, 5, 10, 20$. Also, all curves correspond to the use of the memory 4, rate $1/2$ convolutional code in [6], and an uncorrelated flat Rayleigh fading channel.

For each curve in Figure 4, where the number of users, K, is fixed, we see that as $\frac{E_c}{N_o}$ increases, m^* increases, and N_s^* decreases. This is because the processing gain, N_s, has two effects on the performance of the system: 1) A larger N_s suppresses more interference from other users; 2) a larger N_s leads to a larger $E_b = \frac{N_s \cdot E_c}{r_c}$, and thus reduces the raw error rate into the channel decoder. As $\frac{E_c}{N_o}$ increases, the channel gets better, and we do not need as large an E_b, so we can decrease N_s and allocate more of the available bandwidth to source coding, i.e., increase m.

Alternatively, for each set of curves which have the same bandwidth constraint, we see that as the number of users increases, m^* decreases. This is because we need to allocate more bandwidth to spreading to suppress the multi-user interference. We can also see that the increase (decrease) of $m^*(N_s^*)$ is slower for a larger number of users. This is because with more users, the multi-user interference dominates the thermal noise, while the effects of the change of $\frac{E_c}{N_o}$ are comparably less significant.



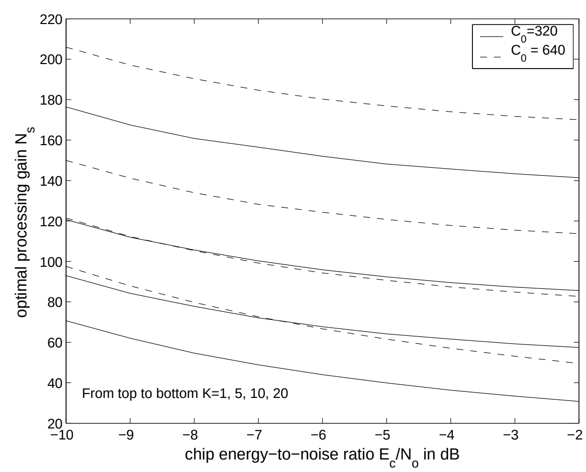


Figure 4. m^* and N_s^* vs. chip energy-to-noise ratio E_c/N_o. Flat Rayleigh fading channel.

Figure 5 illustrates how m^* and N_s^* change as the bandwidth constraint C_0 changes. The system used in this figure is the same as in Figure 4. From this figure, we see that as C_0 increases, m^* increases, and m^* increases faster when there are fewer users in the system. This is because when there is less interference, as $\frac{E_c}{N_o}$ increases, the channel condition improves faster than when there is more interference.

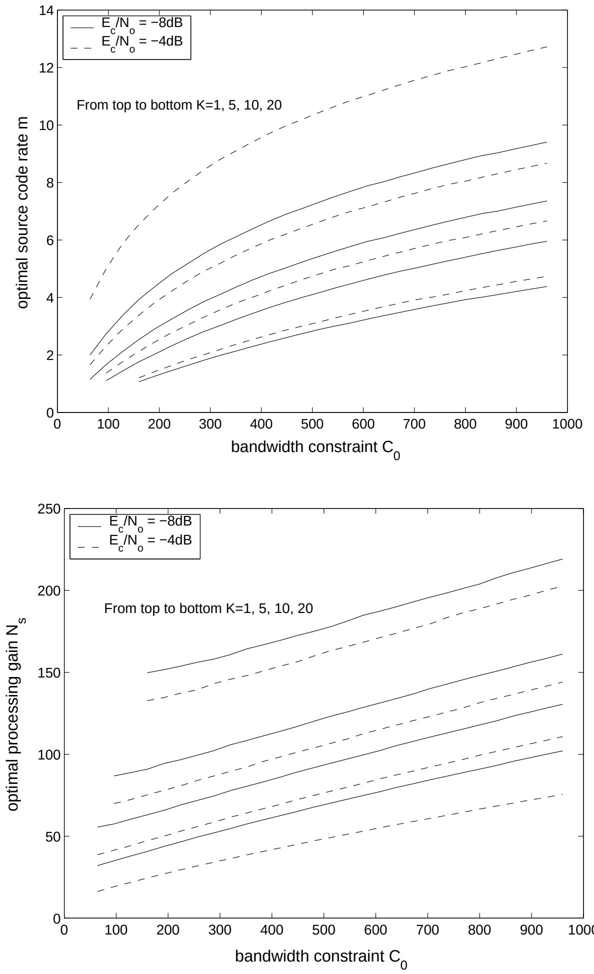


Figure 5. a) m^* and b) N_s^* vs. bandwidth constraint C_0. Flat Rayleigh fading channel.

Thus we do not need as large a processing gain N_s, and we can afford to allocate more bandwidth to the source coding. Similar results for the AWGN channel are presented in [7].

6 Conclusions

In this paper, we studied the bandwidth allocation problem for a CDMA system which employed RCPC channel coding and soft decision Viterbi decoding. Under a bandwidth constraint, we optimized the system performance by combining analytical and numerical techniques.

For the system we considered, our results show that for both AWGN and flat Rayleigh fading channels, for a given r_s, it is always beneficial to use the strongest channel code possible when the complexity of the system is not a concern. We also showed, for a given r_c, how the optimal allocation between r_s^* and N_s^* changes when the channel conditions – number of interfering users, channel noise, or bandwidth constraints – change.

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