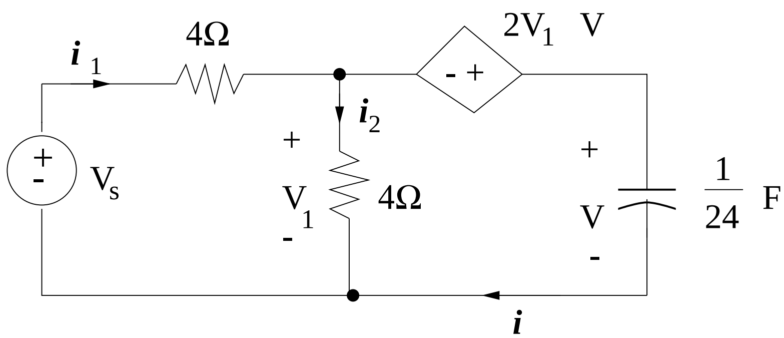
ECE 53a	Quiz #3 SOLUTIONS	Instructor: Pamela Cosman	3/3/08

Problem 1: This problem is directly from homework set 4, and the solutions for that were already posted.

Problem 2: The input V_s has value 12 V for $t < 0$ and value 0 for $t \geq 0$.



Before $t = 0$, the circuit is in steady state, and the capacitor acts as an open circuit. So current is just flowing around the loop on the left. The 12V of the voltage source is divided in half between the two 4Ω resistors. Therefore

$$V_1 = 6V \text{ before } t = 0$$

By KVL around the loop on the right:

$$V = V_1 + 2V_1 = 18V \text{ before } t = 0$$

So the initial condition is

$$V(0^-) = V(0) = V(0^+) = 18V$$

After $t = 0$, we can write KCL at the top node:

$$i_1 = i_2 + i$$

We use Ohm's Law and the capacitor law to say:

$$i_1 = \frac{-V_1}{4} \qquad \text{and} \qquad i_2 = \frac{V_1}{4} \qquad \text{and} \qquad i = C\frac{dV}{dt}$$

Substituting these into the KCL equation, we get:

$$\frac{-V_1}{4} = \frac{V_1}{4} + C\frac{dv}{dt}$$

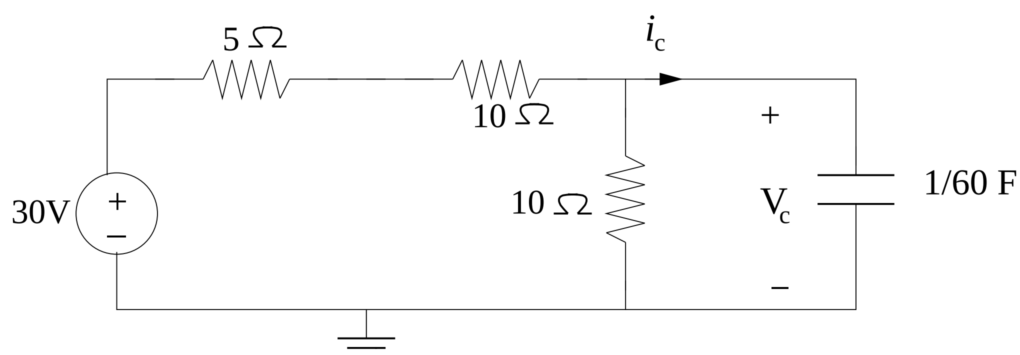
$$-V_1 = V_1 + 4\frac{1}{24}\frac{dV}{dt} \qquad \text{and we regroup terms to get} \qquad 0 = 2V_1 + \frac{1}{6}\frac{dV}{dt}$$

Substituting for V_1 ($V_1 = V/3$) and multiplying through by 6, we get

$$\frac{dV}{dt} + 4V = 0$$

$$V(t) = V(0)e^{-4t} = 18e^{-4t} \text{ for } t \geq 0$$

Problem 3: Prior to time zero, the switch is closed, which effectively shorts out everything in the circuit to the right of the switch. That is, all voltages and currents in the circuit to the right of the switch are zero. The 30V voltage source is directly across the 5Ω resistor.
So, $V_c(0^-) = V_c(0^+) = V_c(0) = 0\text{V}$
For $t > 0$, the circuit looks like this:



We can write one node voltage (KCL) equation:

$$\frac{30 - V_c}{15} = \frac{V_c}{10} + i_c$$

and then use the capacitor law

$$i_c = C\frac{dV_C(t)}{dt}$$

to write

$$\frac{30 - V_c}{15} = \frac{V_c}{10} + \frac{1}{60}\frac{dV_C(t)}{dt}$$

This can be rearranged to obtain:

$$\frac{dV_C(t)}{dt} + 10V_c = 120$$

The solution is

$$V_c = 12 - 12e^{-10t}$$

to satisfy the initial condition on $V_c(0) = 0$.
So, we can write the expression for v(t) for all time as either

$$v(t) = 0 \text{ for } t \leq 0 \text{ and } v(t) = 12 - 12e^{-10t} \text{ for } t \geq 0$$

or as

$$v(t) = (12 - 12e^{-10t})u(t) \text{ for all time}$$

(b) The voltage across the capacitor must be continuous, and this is also the voltage across the resistor. Since the current i_R through a resistor is proportional to the voltage across it, the current i_R must be continuous as well.
The current i_c through the capacitor, on the other hand, does not have to be continuous. And in this case, it has a jump discontinuity. We can see that by saying

$$i_c = C\frac{dV_C(t)}{dt}$$

and so $i_c = 0$ for $t < 0$ and $i_c = \frac{1}{60}120e^{-10t}$ for $t > 0$. So $i_c(0^-) = 0$ and $i_c(0^+) = 2$ so there is a jump discontinuity.

(c) We normally consider that the circuit is back in steady state after five time constants have elapsed.
The time constant for this circuit is $T = \frac{1}{10}$, so at $5T$ or one half second, the effect of throwing the switch has settled down, and we are back in steady state.