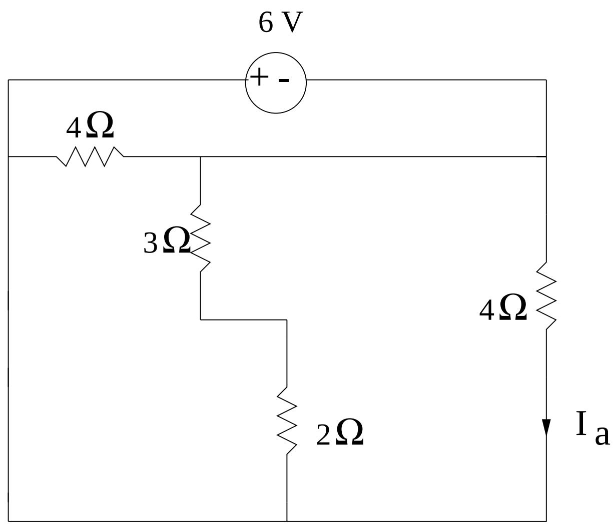
ECE 53a	Quiz #2 SOLUTIONS	Pamela Cosman	2/20/08

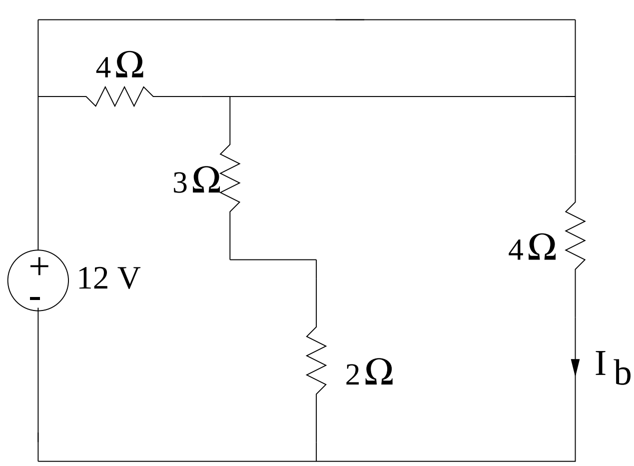
Problem 1: We first consider the 6V source alone. The current source is replaced by an open circuit, and the 12V source becomes a wire. The circuit then looks like this:



The 6V source is then directly across the 4Ω resistor where we are trying to find the current. So

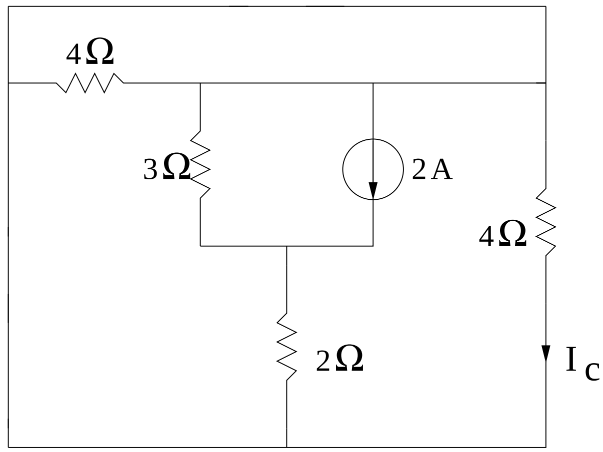
$$I_a = \frac{-6}{4} = -1.5A$$

Next we take the 12V source alone:



$$I_b = \frac{12}{4} = 3A$$

Lastly we look at the current source alone:



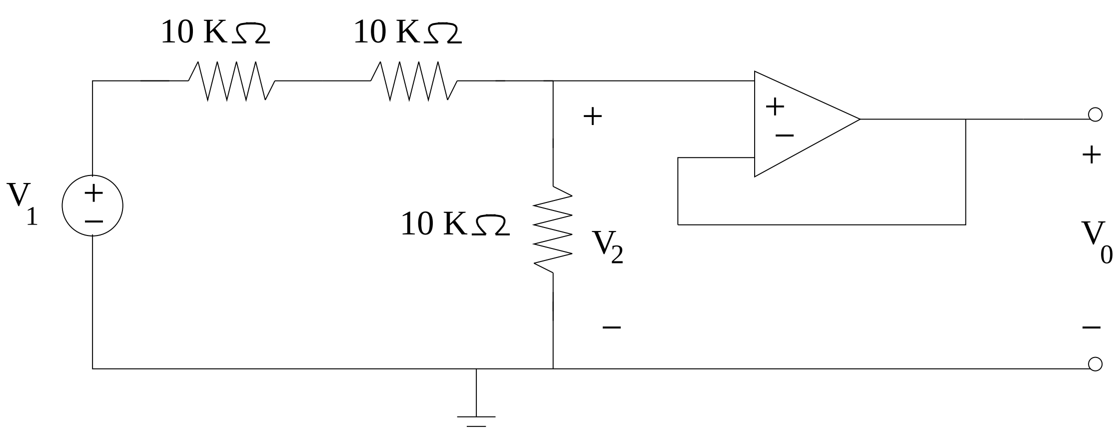
We see that the 4Ω resistor where we are finding the current is being shorted out (it is in parallel with a wire). So no current will flow across it.

$$I_c = 0$$

The total current is the sum of the 3 components:

$$I_0 = -1.5 + 3 + 0 = 1.5A$$

Problem 2:
(a) In class on Feb. 1, I discussed how to use op amps to scale voltages up and down, including both inverting and non-inverting configurations. This problem corresponds to case 4. The configuration we need is:



(b) Now the amplifier is no longer an ideal op amp, but rather a finite-gain amplifier with gain A=100.
It is still the case, however, that $V_2 = \frac{V_1}{3}$ where V_1 is the source voltage, because no current goes into the input terminals of the op amp, so the left hand side of the circuit is still doing voltage division.
Now we can write

$$V_0 = 100(\frac{V_1}{3} - V_0)$$

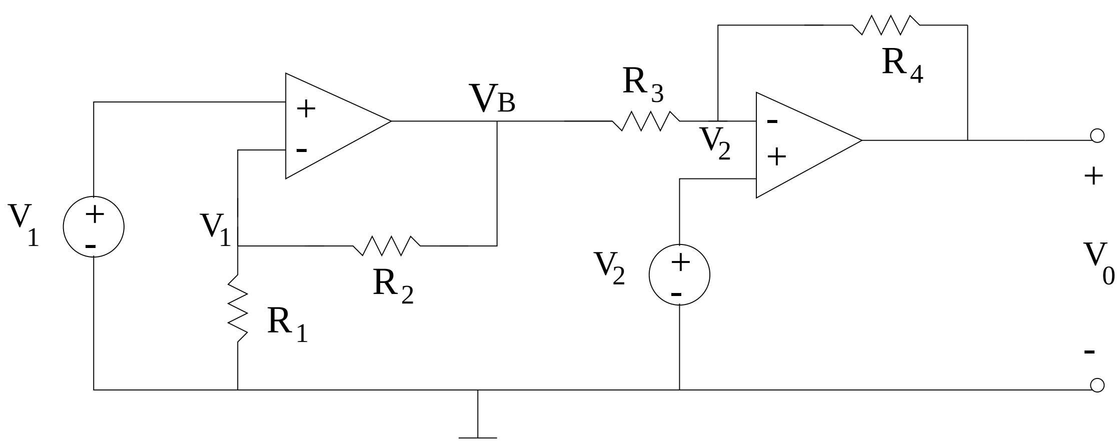
$$3V_0 = 100V_1 - 300V_0$$

$$303V_0 = 100V_1$$

$$V_0 = \frac{100}{303}V_1$$

So the output voltage is now 100/303 of the input voltage, rather than 1/3.

Problem 3:
We make the usual op amp assumption that the voltages at the two input terminals of an op amp are equal.



So we can write the following KCL equation at the inverting input of the first op amp:

$$\frac{V_1}{R_1} + \frac{V_1 - V_B}{R_2} = 0$$

which leads to

$$R_2V_1 + R_1V_1 = R_1V_B \qquad \rightarrow \qquad V_B = \left(\frac{R_2 + R_1}{R_1}\right) V_1$$

We can also write a KCL equation at the inverting input of the second op amp:

$$\frac{V_B - V_2}{R_3} = \frac{V_2 - V_{out}}{R_4}$$

which leads to

$$R_4V_B - R_4V_2 = R_3V_2 - R_3V_{out}$$

We substitute for V_B using the first equation to obtain:

$$V_{out} = V_2 + \frac{R_4}{R_3}V_2 - \frac{R_4}{R_3}\left(\frac{R_2 + R_1}{R_1}\right) V_1$$

Problem 4:

$$I(t) = C\frac{dV(t)}{dt} = 2\frac{dV(t)}{dt} = \begin{cases} 0 & \text{for} \quad t \leq 0 \\ 2 & \text{for} \quad 0 < t \leq 1 \\ 0 & \text{for} \quad t > 1 \end{cases}$$

$$I_R(t) = \frac{V(t)}{R} = \frac{V(t)}{2} = \begin{cases} 0 & \text{for} \quad t \leq 0 \\ t/2 & \text{for} \quad 0 < t \leq 1 \\ 1/2 & \text{for} \quad t > 1 \end{cases}$$

$$I_S(t) = I(t) + I_R(t) = \begin{cases} 0 & \text{for} \quad t \leq 0 \\ 2 + t/2 & \text{for} \quad 0 < t \leq 1 \\ 1/2 & \text{for} \quad t > 1 \end{cases}$$

$$W_C(t) = \frac{1}{2}CV^2(t) = V^2(t) = \begin{cases} 0 & \text{for} \quad t \leq 0 \\ t^2 & \text{for} \quad 0 < t \leq 1 \\ 1 & \text{for} \quad t > 1 \end{cases}$$

$$p_R(t) = \frac{V^2(t)}{R} = \frac{V^2(t)}{2} = \begin{cases} 0 & \text{for} \quad t \leq 0 \\ t^2/2 & \text{for} \quad 0 < t \leq 1 \\ 1/2 & \text{for} \quad t > 1 \end{cases}$$