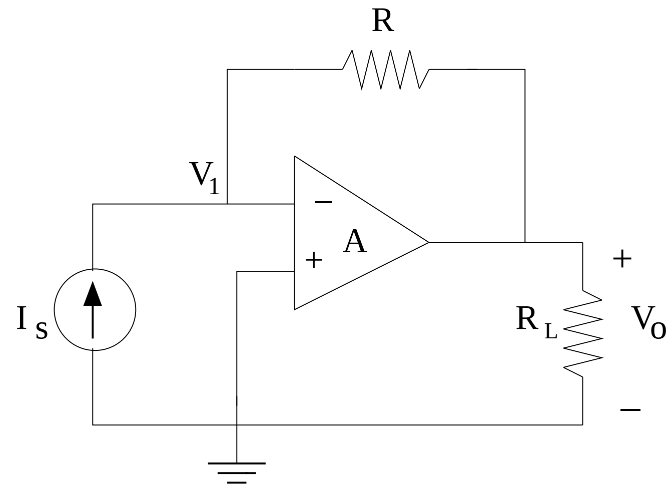
Solutions for Quiz #2

Problem 1: Since this is a finite gain amplifier, we cannot use the assumption that the two input terminals are at the same voltage. We name the voltage at the upper terminal V_1:



and then the amplifier equation is:

$$V_o = A(0 - V_1)$$

Since no current goes into the input terminals of the amplifier, the current I_s that comes from the left has to keep on going through the top resistor, which allows us to write:

$$I_s = \frac{V_1 - V_o}{R}$$

We can solve this equation for V_1, and then substitute for V_1 in the amplifier equation to obtain:

$$V_o = -A[I_s R + V_o]$$

and we can solve for V_o

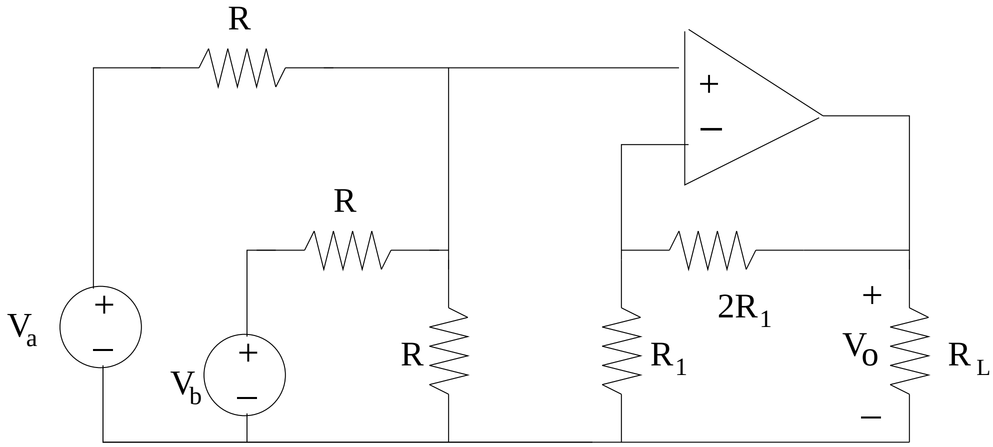
$$V_o = \frac{-A I_s R}{1 + A}$$

Now we want to have 0.1 V per μA of input current I_s. In the equation we just found, if we plug in $V_o = 0.1$ and $I_s = 1\mu$A, and we've been given A=100, so we can solve for R

$$V_o = 0.1 = \frac{100}{101} R(10^{-6})$$

$$R = \frac{101}{100} \times 10^6 \times 0.1 = 101 k\Omega$$

Problem 2: In the following ideal op amp circuit, we name the voltage at the op amp input terminals V_c.



We can then write a KCL equation for the top (non-inverting) input terminal:

$$\frac{V_a - V_c}{R} + \frac{V_b - V_c}{R} + \frac{-V_c}{R} = 0$$

which simplifies to

$$V_a + V_b = 3V_c$$

We can also write a KCL equation for the bottom (inverting) input terminal:

$$\frac{V_c}{R_1} + \frac{V_c - V_o}{2R_1} = 0$$

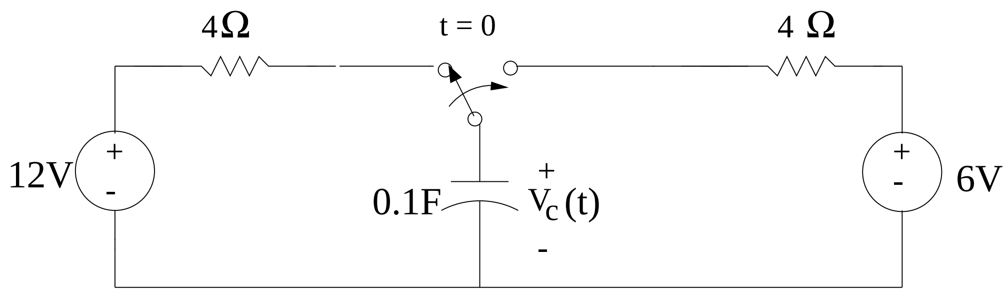
which simplifies to

$$3V_c = V_o$$

and these two simplified equations lead to

$$V_o = V_a + V_b$$

Problem 3: For the following circuit, assume that the switch is to the left since $t = -\infty$.
(a) Suppose the switch switches over to the right at $t = 0$. Find the voltage across the capacitor, $V_C(t)$, for all t.



We assume the form:

$$V_c(t) = K_1 + K_2 e^{-t/\tau}$$

and we try to find the three constants K_1, K_2, and τ.
The time constant τ is given by

$$\tau = R_{th}C = 4 \times 0.1 = 0.4$$

Before time zero, the capacitor acts like an open circuit since it is in steady state. So, no current flows, and the capacitor voltage equals the source voltage.

$$V_C(O^-) = V_C(0) = V_C(0^+) = 12 = K_1 + K_2$$

After the switch goes over, and time goes to $= \infty$, we are back in DC steady state. Again the capacitor acts like an open circuit, no current flows, and the capacitor again equals the source voltage (that is, the source voltage on the right).

$$V_C(\infty) = 6V = K_1$$

So,

$$K_2 = 12 - K_1 = 12 - 6 = 6$$

$$V_C(t) = 6 + 6e^{-t/0.4}V$$

(b) Suppose the switch, after having moved over to the right at $t = 0$, were to go back to the left at $t = 0.2s$. Find an expression for the voltage across the capacitor, $V_C(t)$, for $t > 0.2s$.

For $t > 0.2$, we assume a solution of the form

$$V_C(t) = K_3 + K_4 e^{-(t-0.2)/\tau}$$

and this is the same τ as before since the capacitor is the same as before, and the Thevenin equivalent resistance seen by the capacitor looking to the left portion of the circuit is the same as the Thevenin equivalent resistance seen by the capacitor looking to the right.
We use our previous expression to evaluate $V_C(0.2)$

$$V_C(0.2^+) = V_C(0.2) = V_C(0.2^-) = 6 + 6e^{-0.2/0.4} = 6 + 6e^{-0.5} = K_3 + K_4$$

And, as time goes towards $+\infty$ the capacitor again acts like an open circuit, and its voltage becomes equal to the source voltage (this time, the source voltage on the left)

$$V_C(\infty) = 12 = K_3$$

Therefore

$$K_4 = 6 + 6e^{-0.5} - 12 = -6 + 6e^{-0.5}$$

and the final expression for $V_C(t)$ for $t > 0.2$ is

$$V_C(t) = 12 + (-6 + 6e^{-0.5})e^{-(t-0.2)/0.4}$$