On Uplink Channel Estimation in WiMAX Systems

Yushi Shen, Microsoft, USA
Pamela C. Cosman, University of California, San Diego, USA
Laurence B. Milstein, University of California, San Diego, USA
Eduardo F. Martinez, Freescale Semiconductor Inc., USA

ABSTRACT
In this paper, channel estimation algorithms are proposed and compared for uplink WiMAX systems, which are OFDMA based. These algorithms are investigated based on a dynamic resource allocation scheme, and it is shown that each of them is suitable to specific system scenarios. For example, for a system with a bandwidth of 10MHz operating in the low frequency region (2-11GHz), a two-dimensional averaging algorithm outperforms other algorithms, such as a bilinear interpolation algorithm, because the correlations between the pilots and signals are sufficiently high in both the frequency and the time dimensions.

Keywords: 4g Communications, Channel Estimation, OFDM, OFDMA, WiMAX

INTRODUCTION
The orthogonal frequency division multiplexing (OFDM) modulation technique has various advantages in high rate data transmission, such as high bandwidth efficiency (Edfors, Sandell, Beek, Landstrom, & Sjoberg, 1996). Orthogonal frequency division multiple access (OFDMA) adds multiple access to OFDM by allowing a number of subscribers to transmit simultaneously on different subcarriers every OFDM symbol. It provides efficient control of the varying data rates of each user by changing the number of allocated subcarriers. It also provides both time and frequency diversities. OFDMA has been used in WiMAX systems for wireless metropolitan area network (MAN) communications and broadcasting. And it has been standardized in the IEEE 802.16e/D10, which specifies the air interface for fixed broadband wireless access (BWA) systems supporting multimedia services (IEEE-SA Standards Board, 2005).

Channel estimation is an important issue in any OFDM-based system for demodulation and decoding. In general, an OFDM waveform can be viewed as a two-dimensional (2D) lattice in the time-frequency plane. For pilot-assisted channel estimation techniques, where pilots refer to reference signals known at both transmitter and receiver, this 2D lattice can be
viewed as being sampled at the pilot positions, and the channel characteristics between pilots are estimated by interpolation. The two basic aspects of OFDM channel estimation are the arrangement of pilot positions, and the design of the channel estimator to interpolate between the pilots. The goal in designing channel estimators is to solve this problem with a satisfactory tradeoff between complexity and performance.

Channel estimation techniques for OFDM systems have been widely studied. In particular, Edfors, Sandell, Beek, and Wilson (1998) and Coleri, Ergen, Puri, and Bahai (2002) presented algorithms for OFDM channel estimation with a block-type pilot arrangement and a comb-type pilot arrangement, respectively, and Shen and Martinez (2006) summarized and compared these two basic channel estimation strategies. The two fundamental principles behind these algorithms are to reduce the computational complexity by adopting one-dimensional (1D) rather than two-dimensional (2D) channel estimators, and to improve the interpolation accuracy by employing second-order statistics of the fading channel in either the frequency or in the time dimension.

In WiMAX systems standardized by IEEE-SA Standards Board (2005), however, a different transmission structure and corresponding arrangement of pilot positions are used to fully employ the diversities in both the time and the frequency dimensions. The subcarriers allocated to a subscriber are both separated in frequency and hopped periodically in time. This dynamic resource allocation scheme makes it unfeasible to employ second-order statistics in either the frequency or the time dimension for uplink channel estimation, in other words, it is unfeasible to apply traditional OFDM channel estimation algorithms to WiMAX systems. However, on the other hand, because channel estimation has been constrained inside a small basic transmission unit, 2D interpolation is tolerable in terms of computational complexity Shen and Martinez (2007).

This paper is organized as follows. In Section II, the baseband model and the dynamic resource allocation scheme in an uplink WiMAX system are illustrated. In Section III, possible uplink channel estimation algorithms are proposed for the WiMAX system model described in Section II. In Section IV, the proposed algorithms are analyzed and compared under different scenarios, with respect to the system bandwidth, the center frequency, and the speed of the mobile subscriber. Simulation results and conclusions are presented in Section V.

**BASEBAND MODEL OF WIMAX SYSTEM**

We consider the uplink of an OFDMA system. Assume there are $K$ subcarriers, among which $K_u$ are active subcarriers used for data and pilot transmission, and the others are null subcarriers used for guard bands and a DC carrier. The active frequency bands ($K_u$ subcarriers) are allocated among multiple users, and each subcarrier is assigned to a unique mobile station (MS). As shown in Figure 1, the MS of the desired user inserts its information bits at the subcarriers allocated to it, inserts zeros at the active subcarriers allocated to other users, and adds the null subcarriers. Then, a $K$-point IFFT is used to transform the data sequence into the time domain. A cyclic prefix (CP), which is chosen to be larger than the maximal expected delay spread, is inserted to avoid inter-symbol and inter-carrier interferences. At the base station (BS), the arriving waveform is given by the superposition of the signals from all active users, each of which experience independent fading and additive white Gaussian noise (AWGN). The demodulation is the inverse process of the OFDMA modulation process.

Let the vector $X=[X_1, \ldots, X_K]$ and the vector $Y=[Y_1, \ldots, Y_K]$ denote the input data of IFFT block at the transmitter and the output data of FFT block at the receiver, respectively (see Figure 1). Let $H=[H_1, \ldots, H_K]$ denote the corresponding frequency domain elements of the sampled impulse response of the channel experienced by the desired user, and let $N=[N_1, \ldots, N_K]$ denote the vector of noise samples. Define the input matrix $X=\text{diag}(X)$. It is shown [3-6] that, under
the assumption that the CP is chosen to be larger than the expected delay spread, $Y = XH + N$, which demonstrates that the OFDMA system is equivalent to the transmission of data over a set of parallel channels, and the fading channel of the OFDMA system can be reduced to a 2D lattice in the time-frequency plane.

In IEEE 802.16e/D10 (IEEE-SA Standards Board, 2005), the Wireless MAN OFDM PHY (physical layer) is specified for NLOS (non-line-of-sight) operation for channel bandwidths (denoted by $BW$) no less than 1MHz, $K = 2048$ and $K_u = 1680$. Three consecutive OFDM symbols in time form a slot, and a tile is defined as a band of four contiguous frequency subcarriers for each slot, each containing a pilot at each of its four corners, as shown in Figure 2, where $H_i$'s ($i = 1, \ldots, 8$) donate the channel characteristics at data subcarriers and $P_j$'s ($j = 1, \ldots, 4$) those at pilot subcarriers. Thus, there are $K_u/4 = 420$ tiles. The IEEE standard divides them into six groups, each consisting of 70 contiguous tiles. For each time slot, six subcarriers are pseudo-randomly selected, one from each group, and make up a single sub-channel (note that there are 70 sub-channels in total, numbered from 0 to 69), with one or more sub-channels given to each user based on that user’s application requirement. Across time slots, a rotation scheme of the sub-channels is applied at each OFDMA slot-duration. That is, the sub-channel index number(s) allocated to a subscriber, and thus the location of the tiles and the corresponding physical locations of the subcarriers, change on a slot basis during the transmission. A typical illustration of the subcarrier allocation over time is given in Figure 3, where the sub-channel #0 is assumed allocated to the desired user at the start of the transmission.

**CHANNEL ESTIMATION ALGORITHMS**

In pilot-based channel estimation techniques, reference signals known at both transmitter and receiver are transmitted periodically. As described in Section II, the subcarriers allocated to a user are separated in frequency in any time slot, and change locations from slot-to-slot. This dynamic allocation scheme takes advantage of potential system diversities in both time and frequency, due to the expected de-correlation in both the time and frequency dimensions. For example, a deep narrow-band fade usually affects only a fraction of subcarriers in each sub-channel, and a deep long-term channel fade may affect a given user for only a short period of time due to the hopping. Also, this scheme provides better capacity performance than the traditional OFDM systems, due to the flexible user transmission start-time and duration of transmission length. However, such a complicated allocation scheme makes accurate channel estimation difficult. It is often not feasible to use the channel correlation across tiles in the frequency dimension, because the tiles can be separated beyond the coherence bandwidth of the channel, and thus, the correlation is weak. It also might not be feasible to employ the channel correlation across tiles in the time dimension, because the locations of tiles change every
time slot, and thus depending upon the channel coherence time, it might be difficult to track/estimate the channel conditions. Based on the above considerations, channel estimation for uplink WiMAX systems can be effectively accomplished by using information within a single tile. This kind of estimation will experience less precision, because we can neither average across tiles nor employ second-order statistics of the channel (correlation, delay, etc). That is why a relatively large number of pilot bits (four out of twelve) is employed in the tile structure.

The least squares (LS) estimator is a suitable and efficient technique for WiMAX systems (Edfors, Sandell, Beek, & Wilson, 1998; Coleri, Ergen, Puri, & Bahai, 2002; Shen & Martinez, 2006; Shen & Martinez, 2007), and minimizes the parameter \(E\{(Y- XH)H(Y- XH)\}\), where \((\cdot)^H\) means the conjugate transpose operation. It has been shown (Edfors, Sandell, Beek, & Wilson, 1998) that the LS estimator is given by \(H_{LS} = X^{-1}Y\). If we assume unit amplitude with zero phase is employed to transmit pilot signals, the corresponding received signal itself will be the LS estimate of the channel characteristics. As illustrated in Figure 2, inside a WiMAX tile, the fading channel characteristics are sampled at pilot subcarriers on the corners \(P_j, j=1, \ldots,\)

Figure 2. Illustration of uplink “tile” structure specified in IEEE 802.16e/D10

Figure 3. Illustration of the sub-channel allocation scheme specified in IEEE 802.16e/D10, where \(T\) is the OFDMA slot time duration
4), and the channel characteristics at the data subcarriers \( H_i, i=1, \ldots, 8 \) are estimated by interpolation. In general, the interpolation can be expressed as a weighted sum of the channel values at the pilot subcarriers, that is,

\[
H_i = \sum_{j=1}^{8} \alpha_j P_j, \quad \text{where } i = 1, \ldots, 8
\]  

(1)

In this section, several specific algorithms for channel estimation are presented.

A. 2D Averaging Algorithm

In the 2D averaging algorithm, the average of the four pilot subcarriers is used for the channel estimator for all the data subcarriers inside the tile, that is,

\[
H_i = \frac{1}{4} \sum_{j=1}^{4} P_j, \quad \text{where } i = 1, \ldots, 8
\]  

(2)

This algorithm is a simple channel estimation technique for WiMAX systems. It is also efficient in reducing effects of noise, since the variance of noise will be reduced by a factor of 4. It is suitable for a channel where the fading is slow, and where the neighboring frequency subcarriers are highly correlated.

B. Bilinear Interpolation Algorithm

Another possible interpolation algorithm is bilinear interpolation (Hildebrand, 1987), where the \( H_i \)’s are estimated by horizontal or vertical 1D linear interpolation, based upon fitting straight lines between the samples. The details are shown in Table 1. As discussed in Section IV, this algorithm will be appropriate if the channel is fast fading, and the neighboring frequency bands are weakly correlated. Note that, because the input data used for interpolation have only 2 points in any given row or column ([\( P_1, P_2 \), etc.]), other polynomial-based interpolation methods that produce smoother results in general, such as cubic spline interpolation and piecewise cubic Hermite interpolation, actually end up with the same interpolation results as the bilinear interpolation for the particular case in this context.

C. Frequency-Averaging-Time-Interpolation (FATI) Algorithm

The frequency-averaging-time-interpolation (FATI) algorithm is a combination of the averaging algorithm and the bilinear interpolation algorithm. It functions by averaging in the frequency dimension and linearly interpolating in the time dimension. The formulae to calculate \( H_i \)’s are as follows:

\[
\begin{align*}
H_i &= \frac{P_1 + P_3}{2}, \quad i = 1, 2 \\
H_i &= \frac{(H_i + H_{i+2})}{2} = \frac{(P_1 + P_2 + P_3 + P_4)}{4}, \quad i = 3, 4, 5, 6
\end{align*}
\]  

(3)

Table 1. Illustration of Bilinear Interpolation Algorithm

<table>
<thead>
<tr>
<th>( H_i )</th>
<th>( P_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_i = (1/2)(P_1 + P_3) )</td>
<td>( P_1 )</td>
</tr>
<tr>
<td>( H_i = (1/3)(P_1 + P_2 + P_4) )</td>
<td>( P_2 )</td>
</tr>
<tr>
<td>( H_i = (1/6)(P_1 + P_2 + P_3 + P_4) )</td>
<td>( P_3 )</td>
</tr>
<tr>
<td>( H_i = (1/3)(P_1 + P_3) )</td>
<td>( P_4 )</td>
</tr>
<tr>
<td>( H_i = (1/3)(P_1 + P_2 + P_3 + P_4) )</td>
<td>( P_5 )</td>
</tr>
<tr>
<td>( H_i = (1/6)(P_1 + P_2 + P_3 + P_4) )</td>
<td>( P_6 )</td>
</tr>
<tr>
<td>( H_i = (1/3)(P_1 + P_3) )</td>
<td>( P_7 )</td>
</tr>
<tr>
<td>( H_i = (1/3)(P_1 + P_2 + P_3 + P_4) )</td>
<td>( P_8 )</td>
</tr>
</tbody>
</table>
interpolation in time that is suitable where the channel fading is relatively fast.

D. Time-Domain-Interpolation (TDI) Algorithm

In the time-domain-interpolation (TDI) algorithm, a DFT-based frequency interpolation is followed by a linear time interpolation (Fernandez-Getino Garcia, Paez-Borrallo, & Zazo, 2001). We define two 4-element vectors \( A \) and \( B \), where \( A = \text{DFT}[\text{IDFT}(P_1, P_2), 0, 0] \), and \( B = \text{DFT}[\text{IDFT}(P_3, P_4), 0, 0] \). These are called the DFT-based channel estimators, which use the time-domain transformation consisting of zero-padding to obtain the interpolation in frequency-domain. In each tile, the channel characteristics of the first and third OFDMA symbols are simply obtained from \( A \) and \( B \) directly. Let \( A(i) \) denote the \( i \)th element of vector \( A \) and similarly with \( B(i) \), then \( H_1 = A(2) \), \( H_2 = A(3) \), \( H_3 = B(2) \), and \( H_4 = B(3) \); the channel characteristics of the second OFDMA symbol are obtained by a linear time interpolation, that is, \( [H_5, H_6, H_7, H_8] = (1/2) [A + B] \).

ANALYSIS AND COMPARISON OF THE PROPOSED ALGORITHMS

In this section, we compare these algorithms to determine the situations under which each algorithm is appropriate. Of most interest here are relationships between the signal bandwidth and the channel coherence bandwidth, and between the slot time duration and the channel coherence time.

In a WiMAX system operated at a typical bandwidth \( BW=10 \text{MHz} \), the subcarrier spacing

\[
\Delta f = \text{floor}(n \cdot BW / 8000) \times 8000 / K,
\]

where \( n \) is the sampling factor with a typical value of \( 8/7 \), \( K=2048 \) is the number of IFFT/FFT points, and \( \text{floor}(x) = \lfloor x \rfloor \) is the largest integer not greater than \( x \) (IEEE-SA Standards Board, 2005). The result is \( \Delta f=5.58 \text{kHz} \), which is relatively narrow compared to typical channel coherence bandwidths, because a typical value of coherence bandwidth is in the range of 50kHz to 500kHz for many cellular systems in urban areas, and in the range of 1MHz to 3MHz for many radio channels in indoor environments (Rappaport, 1999; Tse & Viswanath, 2005; Kanodia, Sabharwal, & Knightly, 2004). Thus, the correlations among the four neighboring frequency subcarriers inside a tile will be sufficiently high, and the averaging algorithm is appropriate in the frequency dimension.

The OFDMA symbol duration is calculated

\[
T_s = (1+G) T_{b},
\]

where \( T_{b} = 1/\Delta f \), is the useful symbol time for data transmission, and \( G \) is the ratio of the CP time to the useful symbol time, and is chosen from the set \{1/4, 1/8, 1/16, 1/32\} (IEEE-SA Standards Board, 2005). So the OFDMA slot time, which is equal to three times \( T_s \), is in the range of 185\( \mu \text{s} \) to 224\( \mu \text{s} \). On the other hand, the coherence time of the channel is specified by

\[
T_{coh} = c / (v f_c),
\]

where \( c \) and \( v \) are the speed of light and the speed of the MS, respectively, and \( f_c \) is the center frequency of the waveform.

The frequency range for WiMAX systems is from 2GHz to 66GHz (IEEE-SA Standards Board, 2005). If the system is operated at a low frequency in the range of, say 2.5GHz, which is typically used in second generation mobile systems, even under highway speeds (say, 90 miles/hour), \( T_{coh} \) is about 3ms, which is more than 10 times larger than the slot duration. Thus, the three OFDMA symbols can be treated as highly correlated, and the averaging algorithm in the time dimension is appropriate. Alternatively, if the system is operated in the high frequency range, say, 60GHz, \( T_{coh} \) may be as small as 100us at highway speeds, which is comparable to the slot duration, and the linear interpolation operation should be employed.

In conclusion, for a system with a bandwidth of 10MHz or smaller, if the system is operated in the low frequency region, the
simplest 2D averaging algorithm is applicable; if the system is operated in the high frequency region, the FATI algorithm should be adopted. However, for wide bandwidth systems (for example the ultra wideband (UWB) technology, which is characterized by signals with a bandwidth larger than 500MHz), the bilinear interpolation algorithm and the TDI algorithm should be employed, because the frequency spacings among the four neighboring frequency subcarriers might be smaller than or comparable to the coherence bandwidth of the typical mobile channels.

SIMULATION RESULTS

The proposed channel estimation algorithms are simulated for uplink WiMAX systems with the following typical system parameters: total number of subcarriers used $K=2048$, number of useful subcarriers is $K_u=1680$, and two sub-channels are allocated to the desired user during the transmission period. The transmitter and receiver chains are simulated as per the IEEE 802.16e/D10 (IEEE-SA Standards Board, 2005). A Rayleigh fading multipath channel model is used and is generated by the Jakes’ model, and the coherence bandwidth is 200kHz (Shen, Cosman, & Milstein, 2006a; Shen, Cosman, & Milstein, 2006b; Dent, Bottomley, & Croft, 1993). Also, we choose the length of the cyclic prefix $L_{cp}=512$ (i.e., $G=1/4$), which is greater than the maximum delay spread. The transmitted symbols are QPSK modulated without forward error correction (FEC). Perfect synchronization is assumed because our goal is to concentrate on channel estimation performance. Simulations are carried out for different signal-to-noise (SNR) ratios for different system scenarios, and the performance is measured by averaging over channel realizations by selecting the multipath delays independently.

Figure 4 shows the bit error rate (BER) plot comparing the proposed channel estimation algorithms, for a WiMAX system operated at bandwidth $BW=10$MHz and center frequency $f_c=2.5$GHz, with a pedestrian speed of MS $v=3$mph. The results show that the 2D averaging algorithm outperforms the other algorithms. This is because the correlations among the neighboring frequency subcarriers and those among adjacent time slots are both sufficiently high, thus 2D averaging efficiently reduces the effect of thermal noise.

Figure 5 shows BER performance for a WiMAX system with a bandwidth $BW=10$MHz, $f_c=60$GHz, and local vehicle speed $v=35$mph, where the FATI algorithm is shown to outperform others. Figure 6 is for a WiMAX system with a wide bandwidth $BW=500$MHz, $f_c=60$GHz, and highway speed $v=70$mph, where the bilinear interpolation algorithm is shown to have the best performance. Both observations are consistent with the analysis in Section IV.

Figures 4 to 6 illustrate that it is important to choose an appropriate channel estimation algorithm based on the system parameters. For example, by comparing the high-SNR performances of the 2D averaging algorithm and the bilinear interpolation algorithm, it is seen that the former outperforms the latter with about 5 times lower BER ($1.3 \times 10^{-3}$ vs. $7 \times 10^{-3}$) under the scenario of Figure 4, but it underperforms the latter one with more than 30 times higher BER ($1.5 \times 10^{-3}$ vs. $5 \times 10^{-3}$) under the scenario of Figure 6. We also see that the FATI algorithm is the best choice among them if the algorithm is designed for a system with unknown system parameters, because it has a relatively simple realization structure and relatively acceptable performance over a broad range of scenarios. Lastly, it is observed that the error-floor-like behavior is pronounced in all the curves. This error floor is expected to be lowered, with the use of FEC.

In conclusion, we have investigated the dynamic resource allocation scheme for uplink WiMAX systems, and illustrated that traditional OFDM channel estimation algorithms, which adopt 1D channel estimators with the usage of second-order statistics of the fading channels, are unfeasible to be employed in WiMAX.
Figure 4. Bit error rate (BER) performance versus SNR, for a WiMAX system operated with the bandwidth $BW=10\text{MHz}$ and the center frequency $f_c=2.5\text{GHz}$, with a pedestrian speed of the MS $v=3\text{mph}$

![Figure 4](image1.jpg)

Figure 5. Bit error rate (BER) performance versus SNR, for a WiMAX system operated with the bandwidth $BW=10\text{MHz}$ and the center frequency $f_c=60\text{GHz}$, with a local speed of the MS $v=35\text{mph}$

![Figure 5](image2.jpg)
systems. We then proposed, analyzed, and compared possible tile-by-tile channel estimation algorithms for uplink WiMAX operation, which adopt different types of intra-tile 2D interpolations. Each proposed channel estimation algorithm is suitable to certain system scenarios, and making the appropriate choice among them is important since it may affect the system performance dramatically.

ACKNOWLEDGMENT

This research was supported by Freescale Semiconductor Inc., by Microsoft Inc., by Ericsson Inc., by the State of California under the CoRe program, and by the California Institute for Telecommunications and Information Technology.

REFERENCES


Yushi Shen received his B.S. degree in electrical engineering from the Tsinghua University, Beijing, P. R. China, in 2001, and his M.S. and Ph.D. degrees in electrical and computer engineering, from the University of California at San Diego (UCSD), La Jolla, in 2003 and 2006, respectively. He is currently working at Digital Media Division in Microsoft Inc., Seattle. Dr. Shen is the recipient of various awards including the Chinese Government Award for Outstanding Students Abroad (2006). He is the Co-Founder and Technical Leader for the Windows Media developer team in Microsoft India Development Center (MSIDC), Hyderabad, India. And he is the Invited Keynote Speaker for the 8th International Conference on Computing, Communications and Control Technologies (CCCT 2010), and for the 14th World Multi-Conference on Systemics, Cybernetics and Informatics (WMSCI 2010).

Pamela C. Cosman received her B.S. in electrical engineering from the California Institute of Technology, Pasadena, in 1987, and her M.S. and Ph.D. degrees in electrical engineering from Stanford University, Stanford, CA, in 1989 and 1993. In 1995, she joined the department of Electrical and Computer Engineering at the University of California at San Diego (UCSD), La Jolla, where she is currently a Professor and past Director of the Center for Wireless Communications (CWC). Dr. Cosman is a member of Tau Beta Pi and Sigma Xi, and she is an IEEE Fellow. She was formerly the Editor-in-Chief and is currently a Senior Editor of the IEEE Journal on Selected Areas in Communications.

Laurence B. Milstein received Ph.D. degrees in electrical engineering from the Polytechnic Institute of Brooklyn, Brooklyn, NY. Since 1976, he has been with the Department of Electrical and Computer Engineering, University of California at San Diego (UCSD), where he is a Professor and former Department Chairman. He has also been a consultant to both government and industry in the areas of radar and communications. Dr. Milstein has been an IEEE Fellow since 1985, and was a former Chair of the IEEE Fellows Selection Committee. He has been a member of the Board of Governors of both the IEEE Communications Society and the IEEE Information Theory Society. He is a recipient of the 1998 Military Communications Conference Long-Term
Technical Achievement Award, an Academic Senate 1999 UCSD Distinguished Teaching Award, an IEEE Third Millenium Medal in 2000, the 2000 IEEE Communication Society Armstrong Technical Achievement Award, the 2002 MILCOM Fred Ellersick Award, and the Ericsson Endowed Chair in Wireless Communication Access Techniques in 2005.

Eduardo F. Martinez received his B.S.E.E. and M.S.E.E. degrees from Oklahoma State University, in 1978 and 1981, respectively. He is currently the General Manager of Digital Signal Processing (DSP) and Systems Engineering Department at Freescale Semiconductor Inc. Mr. Martinez has more than 25 years industrial developing and managing experience on areas covering software and hardware in wireless technologies, software algorithms, and video coding and communications. Prior to the current role at Freescale, he served as the Manager of Systems and Applications Engineering Department at Motorola Inc. Before that, he was the DSP Tools Product Manager at Motorola Inc.