# Concatenated Block Codes for Unequal Error Protection of Embedded Bit Streams

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Abstract-A state-of-the-art progressive source encoder is combined with a concatenated block coding mechanism to produce a robust source transmission system for embedded bit streams. The proposed scheme efficiently trades off the available total bit budget between information bits and parity bits through efficient information block size adjustment, concatenated block coding, and random block interleavers. The objective is to create embedded codewords such that, for a particular information block, the necessary protection is obtained via multiple channel encodings, contrary to the conventional methods that use a single code rate per information block. This way, a more flexible protection scheme is obtained. The information block size and concatenated coding rates are judiciously chosen to maximize system performance, subject to a total bit budget. The set of codes is usually created by puncturing a low-rate mother code so that a single encoder-decoder pair is used. The proposed scheme is shown to effectively enlarge this code set by providing more protection levels than is possible using the code rate set directly. At the expense of complexity, average system performance is shown to be significantly better than that of several known comparison systems, particularly at higher channel bit error rates.

*Index Terms*—Concatenated block coding, embedded bit streams, fine-grain scalability, H.264, joint source–channel coding (JSCC), JPEG2000, packetization, rate allocation, set partitioning in hierarchical trees (SPIHT), unequal error protection (UEP).

# I. INTRODUCTION

**E** MBEDDED multimedia encoders produce bit streams that are progressive in nature, i.e., every successive bit contributes a certain amount of refinement after decoding. This partial decoding property comes with the cost that the usefulness of correctly received bits depends on the reliable reception of the previous bits [1]. This makes progressive bit streams vulnerable to noisy channel effects, i.e., any error renders the remaining information bits useless even if they are reliably received. Therefore, an efficient error-protection scheme is needed for the transmission of embedded bit streams over noisy channels.

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Error protection for progressive sources is usually achieved by joint source-channel coding (JSCC), in which an appropriate channel code is used to protect the bit stream to optimize some criterion, such as minimization of distortion or maximization of the useful source rate. Unequal error protection (UEP) using forward error-correction (FEC) coding of progressive sources is extensively studied in the literature. Early studies include [2], where cyclic redundancy check (CRC) codes are cascaded with FEC coding to protect set partitioning in hierarchical trees (SPIHT) progressively coded images. This was shown to be more robust than previous image-encoding results for binary symmetric channels (BSCs). Error propagation is avoided by stopping the decoding when the first CRC failure is detected. Later, [3] used a similar idea at the packet level to provide UEP by using nonuniform channel coding throughout the bit stream. This way, a more flexible coding scheme is achieved. Rate-compatible punctured convolutional (RCPC) codes are also considered by applying different channel code rates for different kinds of bits (e.g., sign and significance bits) of SPIHT encoded data [4]. More powerful codes have been also used for the transmission of embedded sources. In particular, turbo codes are considered in [5] and [6], irregular repeat-accumulate (IRA) codes in [7], and low-density parity-check (LDPC) codes in [8] and [9]. Some of the gains reported in [5]–[9] compared with [2]–[4] can be attributed to the superiority of capacity-achieving codes over conventional coding schemes rather than strictly attributing the gains to the manner in which FEC is deployed. In addition, those studies use larger block sizes to exploit the "asymptotically good" performance of LDPC or IRA codes compared with studies that use convolutional codes. In other studies, Reed-Solomon (RS) codes are utilized for graceful degradation of image quality in packet erasure channels [10]. One of the constraints of these studies is that they use a discrete channel code set usually in the form of a mother code and an associated puncturing pattern. Therefore, only a predetermined number of protection levels are possible with this finite set. Concatenated coding can loosen this constraint significantly and provide a more flexible system.

Concatenated coding was introduced in [11], where the total bit stream is first encoded with an inner code and then the coded bit stream is further encoded with an outer code. This constructs a code that has an exponentially decreasing error probability with increasing block length [11]. It is also shown to be particularly effective in bursty environments. A typical use of concatenated codes can be found in space communications and usually involves a convolutional inner code and an RS outer code. Concatenated coding can also be used for UEP of progressive sources. In particular, product codes were used across the transmitted packets as a 2-D code structure to provide UEP

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Fig. 1. Two methods of packet formatting for progressive source transmission.

for progressive sources in fading channels [12]. Later, product codes were shown to be very effective in the transmission of JPEG2000 [13] coded bit streams, particularly in correlated-Rayleigh fading using turbo and RS codes [14], [15]. The main objective of those studies is to use concatenated coding with RS coding to help improve system performance against packet erasures due to channel fading.

In addition to the choice of channel coding, packet formatting or packetization is another important parameter in a FECbased overall system design for progressive source transmission. Packetization is typically done either by fixing the size of the information block [2], [16] or by fixing the total number of coded bits [5], [6]. These packet formats are shown in Fig. 1 and named *FixedInfo* and *FixedCoded* formats hereafter. For both methods, rate-maximization and distortion-minimization problems for various channel-coding choices are solved, and fast algorithms are proposed for the former [17]. The performance of the overall system design can vary based on the choice of the channel code set and the associated packetization format.

In this paper, we propose a novel packet formatting method along with a flexible system design for the transmission of embedded bit streams. By properly adjusting the channel codes, a flexible protection assignment strategy is achieved through the use of serially concatenated coding and random block interleavers. As is common, interleaving is added between concatenated codes to spread out the burst errors. Subject to a total bit budget constraint, we obtained notable performance improvement over some of the conventional methods. This paper focuses on BSCs similar to [2], [3], [5], [16], and [17]. In some situations, soft channel information is not available, and the only access to the data occurs after some form of hard-decision decoding takes place. In this case, a BSC is a reasonable model for any memoryless channel with a binary input such as binary input Gaussian and flat Rayleigh/Rician fading channels. Note that the proposed serial encoding scheme can be combined with RS codes similar to [12], [14], and [15] to create a 2-D product code to help mitigate the correlation among the random channel coefficients in correlated fading channels.

The remainder of this paper is organized as follows: In Section II, the proposed formatting is explained, and the problem formulation is introduced. In Section III, performance, as well as the algorithm design, is addressed, and the optimization framework is discussed. A low-complexity descent search algorithm is given as an alternative to the overall optimization problem. We also compare the decoding complexity of the proposed design to previous coding architectures. Section IV presents some of the numerical results using different sets of channel codes. We derive two conditions that restrict the search space of possible channel code rates. These two conditions lead to significant complexity reductions of the optimization process. Finally, a brief summary and conclusion follow in Section V.

#### II. PROPOSED FORMATTING AND SYSTEM MODEL

Similar to previous studies [2], [3], error detection is achieved by appending a CRC to every information block  $\mathcal{I}_i$ ,  $1 \le i \le M$ . Once an error is detected, all additional bits are excluded from the reconstruction process.

Consider the proposed M-codeword scheme shown in Fig. 2 and assume that a discrete convolutional code set C is chosen. First, we take  $b_1$  source information bits (source block  $\mathcal{I}_1$ ) and derive two bytes of CRC<sup>1</sup> ( $N_c = 16$  bits) based on  $b_1$  bits to be concatenated with  $\mathcal{I}_1$  for bit error detection. In addition, z zero tail bits are appended to end the trellis in the all-zero state. These bits together constitute payload  $\mathcal{P}_1$  of the first codeword. The number of bits in the payload is  $|\mathcal{P}_1| = b_1 + N_c + z$ . These bits are encoded using channel code rate  $r_1 \in C$  to produce codeword  $c_1$ . In the next encoding stage,  $c_1$  is concatenated with the second information block  $\mathcal{I}_2$ ,  $N_c$  CRC bits, and z parity bits to produce the second payload  $\mathcal{P}_2$ , where  $|\mathcal{P}_2| =$  $[(b_1 + N_c + z)/r_1] + b_2 + N_c + z$  and  $|\mathcal{I}_2| = b_2$ . In the second encoding stage,  $N_c$  CRC bits are derived based on those  $b_2$  bits. Since the errors out of a maximum-likelihood sequence estimator are bursty and convolutional codes show poor performance when channel errors are not independent and identically distributed (i.i.d.) [18], we use random block interleavers to break up the long burst noise sequences. We use  $\pi(x)$  to denote the random block interleaving function that chooses a permutation table randomly, with a uniform distribution, and permutes the entries of x bitwise based on this table. We choose the size of the random permutation table to be equal to the length of the payload size in each encoding stage except the first. After interleaving, the bits in  $\pi(\mathcal{P}_2)$  are encoded using code rate  $r_2 \in \mathcal{C}$ to produce codeword  $c_2$ . This recursive process continues until we encode the last codeword  $c_M$ . Codeword  $c_M$  is sent over the BSC.

The receiver obtains the noisy version of codeword  $c_M$ , i.e.,  $\hat{c}_M$ . At the decoder, the sequential encoding operations of the encoding stage are performed in reverse order. In other words, the noisy received version  $\hat{c}_M$  is decoded first using the ordinary Viterbi decoder. Then, the deinterleaver is invoked to ob-

<sup>&</sup>lt;sup>1</sup>Here, a CRC polynomial is chosen that gives a sufficiently small undetected error probability [19], and the same CRC polynomial is used for all information blocks. The selected CRC polynomial is  $X^{16} + X^{12} + X^5 + X$ .



Parity Bits #i : Total number of parity bits produced after the i-th encoding stage.

Fig. 3. Each payload in the embedded bit stream is exposed to a BSC with different crossover probabilities.

tain  $\pi^{-1}(\pi(\widehat{\mathcal{P}}_M)) = \widehat{\mathcal{P}}_M$ . The CRC of the *M*th information block, i.e.,  $\mathcal{I}_M$ , is performed to label  $\mathcal{I}_M$  as useful or not for the reconstruction process. Thus,  $\mathcal{I}_M$  is associated with a label and peeled off from  $\widehat{\mathcal{P}}_M$ . In the next decoding stage,  $\widehat{c}_{M-1}$  is decoded and deinterleaved in the same manner to get  $\widehat{\mathcal{P}}_{M-1}$ . Based on the CRC,  $\mathcal{I}_{M-1}$  is determined to be useful or not in the reconstruction. The decoding operation is finalized after decoding codeword  $c_1$  and assigning a label to  $\mathcal{I}_1$ . Assuming that the first label with a CRC failure is associated with  $\mathcal{I}_l$ , then only the information blocks up to, but not including, block l are used to reconstruct the source.

# III. ALGORITHM DESIGN AND OPTIMIZATION

We define two sets  $\mathcal{R} = \{r_1, \ldots, r_M\}$  and  $\mathcal{B} = \{b_1, \ldots, b_M\}$ , where  $r_i \in C$  is the code rate used to protect the *i*th payload  $\mathcal{P}_i$ . Those sets are optimized to minimize distortion, as will be explained in Section III-A. The length of codeword  $n(|c_n|), 1 \leq n \leq M$ , can be found as

$$|c_n| = \left(\cdots \left(\left((b_1 + N_c + z)\frac{1}{r_1} + b_2 + N_c + z\right)\frac{1}{r_2}\right)\right)$$

$$+ b_3 + N_c + z \bigg) \cdots \bigg) \frac{1}{r_n}$$
$$= \sum_{i=1}^n \left(\prod_{j=i}^n r_j\right)^{-1} (b_i + N_c + z). \tag{1}$$

The length of codeword  $c_M$  is equal to the total allowed bit budget B, i.e.,  $|c_M| = B$ . We send the total bit stream over a BSC with crossover error probability  $\epsilon_0$ , as shown in Fig. 3(a). Because of the embedded nature of M convolutional codes, the various test statistics are dependent; thus, a closed-form analysis does not appear to be feasible. Therefore, for the purpose of algorithm design, we will ignore the dependence and set up the algorithm assuming statistical independence. The net result of this is that we approximate the sequence of data bit decisions at the output of each decoding stage as arising from a BSC, whereby the crossover probability of each BSC is functionally related to the crossover probability of the previous BSC, as well as the rate of the currently used channel encoder, as shown in Fig. 3(b). Note that, since our final results on performance are obtained by simulations, the dependence that was ignored in the algorithm design will be automatically present in the performance results.

After decoding the noisy codeword  $\hat{c}_M$  and deinterleaving, we assume that the decoded bit stream will have a bit error rate (BER)  $\epsilon_1 < \epsilon_0$ . In light of the previous paragraph, the decoded error rate  $\epsilon_1$  is a function of  $\epsilon_0$  and channel code rate  $r_M$  and is denoted by  $\epsilon_1(\epsilon_0, r_M)$  hereafter. Next, the noisy codeword  $\widehat{c}_{M-1}$  is decoded, etc. In general, the channel that  $\mathcal{P}_i$   $\{1 \leq i \leq i \leq j\}$ *M*} experiences can be approximated as a BSC with crossover probability  $\epsilon_{M-i+1}(\epsilon_{M-i}, r_i)$ . Since  $\mathcal{P}_i \supseteq \mathcal{I}_i$ , the BER for  $\mathcal{I}_i$  is  $\epsilon_{M-i+1}(\epsilon_{M-i}, r_i)$ . In this paper,  $\{\epsilon_{M-i+1}\}_{i=1}^M$  are found by simulating the system for a given channel raw error rate  $\epsilon_0$ and code rates  $r_i, r_{i+1}, \ldots, r_M$ . As shown, the packet error rate (PER) appears in the expression for the expected distortion of the system. In principle, it is possible to determine the PERs directly by simulation. In practice, however, an information block can have any integer number of bits (up to the total length of the stream), and each such block size  $b_i$  corresponds to a different PER (PER<sub>i</sub>). Therefore, in the optimization process, it would have been computationally intractable to find all possible PERs corresponding to all possible  $b_i$  values. For this reason, we instead use the simulation to find the BER and use that and the various  $b_i$  values to compute the PERs. Having approximated each channel as a BSC with some crossover probability, we approximate the PER of the *i*th information block, i.e.,  $\mathcal{I}_i$ , by [20]

$$\operatorname{PER}_{i} \approx 1 - (1 - \epsilon_{M-i+1})^{b_{i}}.$$
(2)

The described channel modeling is illustrated in Fig. 3(b). Using (2), the expected distortion of the progressive bit stream using the proposed formatting and the independence assumption is approximated by [21]

$$\mathbb{E}[\mathcal{D}] = \sum_{l=0}^{M} \operatorname{PER}_{l+1} \prod_{i=0}^{l} (1 - \operatorname{PER}_{i}) d_{l}$$
(3)

$$= d_0 - \sum_{l=1}^{M} \prod_{i=1}^{l} (1 - \text{PER}_i) \Delta_l$$
 (4)

$$\approx d_0 - \sum_{l=1}^M \Delta_l \prod_{i=1}^l (1 - \epsilon_{M-i+1})^{b_i}$$
  
$$\stackrel{\Delta}{=} d_0 - \overline{D}_M(\mathcal{B}, \mathcal{R}) \ge 0 \tag{5}$$

where  $d_0$  is the distortion when the source decoder reconstructs the source without using any of the transmitted information,  $d_l$ is the distortion when the decoder decodes only  $\sum_{j=1}^{l} b_j$  information bits, and  $\Delta_l = d_{l-1} - d_l \ge 0$ . We also assume  $\text{PER}_0 = 0$  and  $\text{PER}_{M+1} = 1$ , as the zeroth and (M + 1)th information blocks do not exist. Minimization of  $\mathbb{E}[\mathcal{D}]$  is equivalent to maximizing the average distortion reduction  $\overline{D}_M(\mathcal{B}, \mathcal{R})$ . Note that the above expression corresponds to the expected distortion when all errors are assumed to have been caused by statistically independent events, as explained above. To help ensure that the errors are as uncorrelated as possible, we have introduced interleavers at the transmitter and corresponding deinterleavers at the receiver before each decoding stage.

#### A. Optimization

For the proposed scheme, the design objective is to find M, rate allocation  $\mathcal{R}$ , and information block size set  $\mathcal{B}$  such that

 $\overline{D}_M(\mathcal{B},\mathcal{R})$  is maximized. In other words, we intend to find set  $\{M^*,\mathcal{R}^*,\mathcal{B}^*\} = \arg \max_{\{M,\mathcal{R},\mathcal{B}\}} \overline{D}_M(\mathcal{B},\mathcal{R})$  subject to bit budget constraint B, where \* denotes the optimal values. For a given M, we need to optimize the overall parameter set  $\{\mathcal{B},\mathcal{R}\} = \{b_1, b_2, \ldots, b_M, r_1, r_2, \ldots, r_M\}$ , which is subject to the total bit budget B. The problem can be stated as

$$\max_{\mathcal{B},\mathcal{R}} \overline{D}_M(\mathcal{B},\mathcal{R}) \quad \text{subject to} \quad \sum_{i=1}^M \frac{1}{\prod_{j=i}^M r_j} (b_i + N_c + z) = B.$$

There are only either 9 RC-LDPC or 13 RCPC code rates in our set C; therefore, including the uncoded case, we have either 10 or 14 possible values for  $r_i$ , whereas  $b_i$  has a vastly larger set of possible values (in theory,  $b_i$  can take on any value from 1 up to the total number of source bits). First, we assume a fixed channel code rate set  $\mathcal{R}$  and optimize information block size set  $\mathcal{B}$  using a descent search algorithm [22] by initializing  $\mathcal{B}^* = \{b, b, \dots, b\}$  for some b satisfying Mb < B.

This constrained optimization problem is reduced to solving an unconstrained optimization problem by using a line-search method.<sup>2</sup> We employ gradient vector  $\nabla \overline{D}_M(\mathcal{B}, \mathcal{R})$  as the descent direction for fixed M and  $\mathcal{R}$ . Finally, we note that the elements of set  $\mathcal{B}$  can only take discrete values (the unit is in *bits*). Thus, the components of the gradient vector are not analytically available. We use the "gradients by forward finite differences" as our approximation, as given in [22, Sec. 2.3.1.5]. Then, we carry out this optimization procedure for all possible channel code rates. Finally, for a given M, we choose set  $(\mathcal{B}^*, \mathcal{R}^*) =$  $\arg\{\max_{\mathcal{R}}\{\max_{\mathcal{B}}\overline{D}_M(\mathcal{B}, \mathcal{R})\}\}$  to be the optimal point. For ease of reference, we present a functional flow diagram of the optimization procedure in Fig. 4 to determine the optimum parameters  $(M^*, \mathcal{B}^*, \mathcal{R}^*)$ .

Let us fix code rate set  $\mathcal{R} = \{r_i\}_{i=1}^M \in \mathcal{C}$ . First, we realize that  $\Delta_1 = d_0 - d_1$  depends on  $b_1$  because  $d_0$  is a fixed real number. Similarly,  $\Delta_2 = d_1 - d_2$  depends on both  $b_1$  and  $b_2$ . Thus,  $\Delta_l = d_{l-1} - d_l$  depends on  $b_1, b_2, \ldots, b_l$ . To signify the functional dependence, they will be denoted by  $\Delta_1(b_1)$ ,  $\Delta_2(b_1, b_2)$ , and  $\Delta_l(b_1, b_2, \ldots, b_l)$  (see Fig. 5). For this general case, our constraint equation given by (6) becomes

$$\sum_{i=1}^{M} \frac{b_i}{\prod_{j=i}^{M} r_j} = B - (N_c + z) \sum_{i=1}^{M} \left(\prod_{j=i}^{M} r_j\right)^{-1}.$$
 (7)

Letting  $\widehat{B} \stackrel{\Delta}{=} B - (N_c + z) \sum_{i=1}^{M} (\prod_{j=i}^{M} r_j)^{-1}$ , we have for the *l*th information block

$$b_l = \left(\widehat{B} - \sum_{i=1, i \neq l}^M \frac{b_i}{\prod_{j=i}^M r_j}\right) \prod_{j=l}^M r_j.$$
(8)

<sup>2</sup>The line-search algorithms are summarized in [22, Sec. 2.1.1]. We used the golden section method (explained in [22, Sec. 2.2.1]) as our 1-D line-search approach. We found that a sufficient number of iterations for convergence is almost linear in M. The complexity of the optimization problem will be also discussed in the simulations section.



Fig. 4. Functional flow diagram of the proposed optimization and transmission scheme.



Fig. 5. Distortion versus number of source bits illustrating  $\Delta_1, \Delta_2, \ldots, \Delta_l$  as functions of  $b_1, b_2, \ldots, b_l$ .

This means that one of the parameters, i.e.,  $b_l$ , can be expressed in terms of the other parameters. For example, using (8),  $b_M = r_M \widehat{B} - r_M \sum_{i=1}^{M-1} (b_i / \prod_{j=i}^M r_i)$ , and

$$\Delta_M\left(b_1, b_2, \dots, b_{M-1}, r_M\left(\widehat{B} - \sum_{i=1}^{M-1} \frac{b_i}{\prod_{j=i}^M r_j}\right)\right) \quad (9)$$

is now only a function of  $b_1, \ldots, b_{M-1}$ . Thus, using (5) and (9), we obtain

$$\overline{D}_M\left(\left\{b_1,\ldots,b_{M-1},r_M\widehat{B}-r_M\sum_{i=1}^{M-1}\frac{b_i}{\prod_{j=i}^M r_i}\right\},\mathcal{R}\right)$$
$$=\sum_{l=1}^{M-1}\Delta_l(b_1,\ldots,b_l)\prod_{i=1}^l(1-\epsilon_{M-i+1})^{b_i}$$
$$+\Delta_M\left(b_1,b_2,\ldots,b_{M-1},r_M\left(\widehat{B}-\sum_{i=1}^{M-1}\frac{b_i}{\prod_{j=i}^M r_j}\right)\right)$$

$$\times (1 - \epsilon_1)^{r_M \left(\widehat{B} - \sum_{i=1}^{M-1} \overline{\prod}_{j=i}^{b_i} r_i\right)} \prod_{i=1}^{M-1} (1 - \epsilon_{M-i+1})^{b_i}$$
(10)

which is only a function of  $\{b_1, \ldots, b_{M-1}\}$ . The constrained optimization problem in (6) becomes unconstrained and has now M-1 parameters subject to optimization. We use the gradient descent algorithm to find a maximum  $\{b_1^*, \ldots, b_{M-1}^*\}$  for this problem [22]. Finally, we calculate  $b_M^* = r_M \widehat{B} - r_M \sum_{i=1}^{M-1} (b_i^* / \prod_{j=i}^M r_i)$ .

A simple example that is easy to visualize (the concavity of  $\overline{D}_M(\mathcal{B}, \mathcal{R})$  for a given  $\mathcal{R}$ ) occurs when M = 2. In this particular case,  $\Delta_2(b_1, b_2) = \Delta_2(b_1, r_2\hat{B} - (b_1/r_1))$  is now only a nonincreasing function of  $b_1$ . After absorbing the constraint equality into the cost function, we will have an unconstrained optimization problem with the following cost function to maximize:

$$\overline{D}_{2}\left(\left\{b_{1}, r_{2}\widehat{B} - \frac{b_{1}}{r_{1}}\right\}, \mathcal{R}\right) \\
= (1 - \epsilon_{2} \left(\epsilon_{1}(\epsilon_{0}, r_{2}), r_{1}\right))^{b_{1}} \Delta_{1}(b_{1}) \\
+ (1 - \epsilon_{2} \left(\epsilon_{1}(\epsilon_{0}, r_{2}), r_{1}\right))^{b_{1}} \left(1 - \epsilon_{1}(\epsilon_{0}, r_{2})\right)^{r_{2}\widehat{B} - b_{1}/r_{1}} \\
\times \Delta_{2} \left(b_{1}, r_{2}\widehat{B} - \frac{b_{1}}{r_{1}}\right).$$
(11)

Several examples are shown in Fig. 6 for different values of  $\epsilon_0$  and  $r_{\rm tr}$ , where we take  $B = 0.3 \times 512 \times 512$  bits for a 512  $\times$  512 *Lena* image using the RCPC code set with memory 6 from [23].

### B. Concatenated Coding With LDPC Codes

The concatenated coding mechanism presented in this paper can be used with any type of error-correction code, i.e., we can employ a class of "asymptotically good" codes such as LDPC codes. However, there might be some minor changes in the proposed design when it is used with LDPC codes. For example, we would not need to consider the CRC codes in our design because of the inherent error-detection property of LDPC codes [26]. On the other hand, using LDPC codes increases both the computational complexity and memory requirements of the encoder. In the proposed design, the length of the codewords increases after each encoding stage. To enable linear encoding complexity, we resort to the semirandom LDPC encoding structure given in [27]. We optimize the system using the cost function, in which the approximate PER values are found using the analysis of LDPC codes given in [28] and the algorithm presented in [29], for finding the decoding thresholds of the semirandom rate-compatible LDPC (RC-LDPC) code family.

#### C. Complexity of the Proposed Scheme

As far as encoding of the convolutional codes is concerned, the encoders use the same amount of memory elements both in the proposed system and in the comparison systems of [2], [3], and [16]. Although convolutional encoding has a minor effect on the overall complexity, Viterbi decoding is a more complex operation. We focus on the decoding complexity at the receiver.



Fig. 6. For several different sets  $\mathcal{R}$ ,  $\epsilon_0$ , and  $r_{tr}$ ,  $\overline{D}_2(\{b_1, r_2\hat{B} - (b_1/r_1)\}, \mathcal{R})$  is plotted for a SPIHT source encoder. As shown, it is a concave function of  $b_1$  and has a single maximum point.

For a fixed-constraint length, the decoding complexity of punctured convolutional codes linearly increases with the length of the codeword [24]. In addition, the maximum trellis complexity of a random block interleaver linearly increases with the length of the interleaver [25]. On the other hand, since we use the LDPC encoding structure given in [27] and the Max-product algorithm at the receiver, the computation complexity of both LDPC encoding and decoding is linear with increasing block length. If we assume that the complexity of deinterleaving is minor compared with the decoding operation, the maximum trellis complexity of the decoding procedure of the proposed scheme is found to be on average at most M times more complex than the decoding operations of the systems in [2], [3], [9], and [16].

# IV. NUMERICAL RESULTS AND DISCUSSIONS

We use the following channel code sets: 1) a convolutional code set that consists of the 13 RCPC codes with memory 6 found in [23] and used in [2] and 2) an LDPC code set that consists of the nine RC-LDPC codes used in [9]. The convolutional code is treated as a block code by using zero-tail padding with z = 6 zeros at the end of each information block.

# A. Simulations With RCPC Codes

Throughout this section, the RCPC channel code set is used. First, we show some simulation results that illustrate the role

of the interleavers in our system design. We set M = 2 and choose two pairs of codes  $(r_1, r_2)$ , i.e., (2/3, 1/2) and (4/9, 1/2), and information block size  $(b_1, b_2)$ , i.e.,  $(10^4, 10^4)$ . In Fig. 7, the decoded BER is plotted for the bits in  $\mathcal{I}_1$  as a function of crossover probability  $\epsilon_0$  with two different permutation table sizes, i.e.,  $|\mathcal{P}_1| = 1500$  and  $|\mathcal{P}_1| \approx 10^4$ . As expected, if we increase the size of the random permutation table, we get better BER performance. Also shown in the same figure is the decoded BER for the bits in  $\mathcal{I}_2$  (single code rate 1/2). The curve named non-interleaved shows that, when there is no interleaver, we obtain very minor performance improvement over the single code rate scheme. This is because correlated error patterns at the output of the Viterbi decoder significantly affect the performance of the following decoding stage. At higher crossover error probabilities, the decoded BER for  $\mathcal{I}_1$  is worse than that for the single code rate scheme. To explain this, first observe in Fig. 7(a) that toward the right-hand edge of the plot (roughly  $\epsilon_0 > 0.1$ ), the outer decoder is overwhelmed by the high raw channel error rate. As we move to the left, the outer decoder is not overwhelmed. However, the decoded BER of the outer decoder has to be low enough so that the inner encoder is not overwhelmed. Finally, toward the left-hand edge of the plot (roughly  $\epsilon_0 < 0.09$ ), neither decoder is overwhelmed and give lower decoded BERs than  $\epsilon_0$ .

Finally, we obtained the curve denoted *i.i.d.* Assump. in Fig. 7 in the following way: For any given raw channel error rate  $\epsilon_0$ , we first decoded outer code  $r_2$  and obtained decoded BER  $\epsilon_2$ 



Fig. 7. Effect of interleaving with different sizes of the random permutation tables.

by simulation. After finding  $\epsilon_2$ , we sent the inner codeword (information block encoded with  $r_1$ ) through a BSC with crossover probability  $\epsilon_2$  and obtained decoded BER  $\epsilon_1$  by simulation again (argument given in Section III). The curve *i.i.d Assump*. given in Fig. 7 is the set of  $\epsilon_1$ s for different values of  $\epsilon_0$  shown in the abscissa. It is verified that an interleaver depth of  $|\mathcal{P}_1|$  using a block length of  $10^4$  gives a very close approximation to the i.i.d. assumption.

In the rest of the simulations, we use an embedded bit stream, and we use our protection scheme to send it over a BSC. We compare four transmission schemes based on embedded bit streams.

- ShZeg: This is based on [2]. It uses a single optimal code with *FixedInfo* formatting. The information block size is 200 bits, and a single optimal code rate is chosen from the code set that minimizes the average distortion for all the transmission rates in consideration. In addition, the list Viterbi decoder (LVA) of [2] is replaced with the ordinary Viterbi algorithm.
- NosLu: This is the approach presented in [3]. It uses an optimal code rate for each information block with *FixedInfo* formatting [21]. The information blocks have size of 202 bits.
- 3) Product Code: This system was originally proposed in [12] for a correlated fading channel, but the effectiveness of the scheme was shown for a BSC as well. It uses RS codes in concatenation with convolutional codes as a 2-D code structure.
- 4) *Concatenated:* This is the proposed concatenated coded system with related optimization as defined.

We produced the embedded bit stream by encoding the grayscale  $512 \times 512$  *Lena*, *Barbara*, and *GoldHill* images using SPIHT with arithmetic coding and JPEG2000 *Part 1* (with no error-correction capability) progressive image coders. Due to space limitations, we only present the results of *Lena* encoded using SPIHT with arithmetic coding and *Goldhill* encoded with JPEG2000. Other combinations exhibited similar results. We tested two crossover error probabilities, namely,

 $\epsilon_0 = 0.01$  and  $\epsilon_0 = 0.05$ . We define the total transmission rate  $r_{\rm tr}$  in bits per pixel (bpp) for a given  $L_x \times L_y$  image as  $r_{\rm tr} = B/(L_x \times L_y)$ . Quality assessment of the decoded images is given in terms of the average peak signal-to-noise ratio (PSNR). PSNR results for all the systems are shown as a function of  $r_{\rm tr}$  in bpp, where

$$PSNR = 10 \log_{10} \left( \frac{MAX_I^2}{\mathbb{E}[\mathcal{D}]} \right)$$
(12)

and  $MAX_I = 255$  is the maximum possible intensity value of the image.

For a given BSC with crossover probability  $\epsilon_0$  and sets  $\mathcal{B}$  and  $\mathcal{R}$ , we produce the total bit stream  $c_M$ , as explained in Section II, and send it through the BSC. We obtain  $\{\epsilon_i\}_{i=1}^M$  by simulating M sequential decoding stages. Note that, given sets  $\mathcal{B}$  and  $\mathcal{R}$ , the sizes of the random permutation tables are automatically determined. We should also notice that  $\{\epsilon_i\}_{i=1}^M$  are assumed to be independent of  $\{b_i\}_{i=1}^M$ , as the decoded BER of RCPC codes only slightly changes with varying block size [30]. Finally, we calculate  $\mathbb{E}[\mathcal{D}]$  using the approximations given in (2) and (5). After optimization, we find the optimal parameters to be used in the proposed scheme. Finally, using the optimal set of parameters  $(M^*, \mathcal{B}^*, \mathcal{R}^*)$ , we simulate the system to find the expected distortion and convert it to PSNR using (12). As shown in Fig. 8, the proposed system is more effective at higher channel error rates and higher transmission rates.

Another observation is that, although *NosLu* optimizes channel code rates for each information block, because of the flat region of the R–D curve of the source, *NosLu* does not improve much over *ShZeg* at higher transmission rates. However, at smaller transmission rates, *NosLu* gives around 0.35-dB gain over the *ShZeg* scheme as reported in [3]. In addition, note that [16] does not use the approximation in [3] and finds the optimal rate schedule using dynamic programming instead of Lagrangian multipliers. However, we experimentally observed that the performance loss due to the approximation in [3] is minor. *Product Code* for the *Lena* image gives around 0.47-dB gain over *ShZeg* for all transmission rates. The advantage of



Fig. 8. Different systems are compared over a BSC with BER = 0.01 and BER = 0.05 using a  $512 \times 512$  Lena image.



Fig. 9. Different systems are compared over a BSC with BER = 0.01 and BER = 0.05 using a  $512 \times 512$  Goldhill image.

Product Code is its 2-D coding structure that provides the same level of protection of ShZeg using an overall higher code rate (RCPC + RS). This allows more information content in the bit budget and the potential for performance improvement. Concatenated provides more than 1-dB gain over ShZeg and 0.5-dB gain over Product Code at all transmission rates. The advantage of *Concatenated* can be attributed in part to packet size adjustment (and, thus, PER adjustment) and its flexible protection by multiple channel codes. Using multiple code rates enables protection levels that are not possible using the discrete code set. We tried information block sizes of 100, 200, 400, 600, 800, and 1000 and verified that choosing other than 200 bits as the information block size of conventional methods does not help their performance much. In Fig. 9, we show average PSNR performances of some of those systems using Goldhill encoded with JPEG2000. The results of Concatenated exhibit similar gains compared with ShZeg, i.e., they are more significant at lower  $\epsilon_0$  and higher transmission rates. The jaggedness of the curves is due to the discrete code set and the source encoder.

Next, we provide and compare results for Concatenated using Lena at various values of M, as shown in Table I. There are two observations with regard to the proposed formatting and channel code rate allocation of *Concatenated*. First, when M = $m \in \{1, 2, 3, 4, 5\}$ , we have  $r_1^{*(m)} \ge r_2^{*(m)} \ge \cdots \ge r_m^{*(m)}$ , where the superscript denotes the current value of M (we refer to this as the First Condition). This is because baseline protection is achieved by the outermost code, and then, incremental protection is added by each encoding stage. Our results show that incremental protection is enough to obtain the necessary UEP with a less powerful outermost code. Note that this result can be used to restrict the search space of our exhaustive search in finding optimal code rate set  $\mathcal{R}^*$ . It leads to complexity reductions, as shown in Table II. Second, if we consider two different values for m, namely,  $m_1$  and  $m_2$ , where  $m_2 = m_1 + 1$ , we observe that  $r_{m_2}^* (m_2) \ge r_{m_1}^* (m_1)$  (we refer to this as the Second Condition). This is because, as we increase M, we observe that the outermost code rate did not need to be more powerful than the one used in the system with one less encoding stage, since

$\epsilon_0 = 0.15, r_{tr} = 0.3$ bpp						$\epsilon_0 = 0.1, r_{tr} = 0.3 \text{ bpp}$								
M	$r_1^*$	$r_2^*$	$r_3^*$	$r_{4}^{*}$	$r_5^*$	PSNR (dB)	M	$r_1^*$	$r_2^*$	$r_3^*$	$r_{4}^{*}$	$r_5^*$	PSNR (dB)	
1	1/4	N/A	N/A	N/A	N/A	14.48	1	1/4	N/A	N/A	N/A	N/A	20.44	
2	4/7	1/4	N/A	N/A	N/A	26.96	2	2/3	1/3	N/A	N/A	N/A	28.45	
3	8/9	2/3	1/4	N/A	N/A	27.32	3	8/9	4/5	4/13	N/A	N/A	28.71	
4	1	8/9	2/3	4/15	N/A	27.38	4	8/9	8/9	4/5	1/3	N/A	28.79	
5	1	1	8/9	2/3	4/15	27.37	5	1	8/9	8/9	4/5	1/3	28.75	
		$\epsilon_0$	= 0.05,	$r_{tr} = 0$	.6 bpp		$\epsilon_0 = 0.01, r_{tr} = 0.7 \text{ bpp}$							
М	$r_1^*$	$r_2^*$	$r_3^*$	$r_4^*$	$r_5^*$	PSNR (dB)	M	$r_1^*$	$r_2^*$	$r_3^*$	$r_4^*$	$r_5^*$	PSNR (dB)	
1	1/4	N/A	N/A	N/A	N/A	31.65	1	2/5	N/A	N/A	N/A	N/A	34.60	
2	8/9	1/3	N/A	N/A	N/A	32.72	2	8/9	4/7	N/A	N/A	N/A	35.77	
3	8/9	4/5	4/9	N/A	N/A	33.10	3	8/9	8/9	2/3	N/A	N/A	36.07	
4	8/9	8/9	8/9	4/9	N/A	33.22	4	1	8/9	8/9	2/3	N/A	36.08	
5	1	8/9	8/9	8/9	4/9	33.18	5	1	1	8/9	8/9	2/3	36.06	
		$\epsilon_0$ :	= 0.005	$r_{tr} = 0$	).5 bpp		$\epsilon_0 = 0.001, r_{tr} = 0.3 \text{ bpp}$							
М	$r_1^*$	$r_2^*$	$r_3^*$	$r_4^*$	$r_5^*$	PSNR (dB)	M	$r_1^*$	$r_2^*$	$r_3^*$	$r_4^*$	$r_5^*$	PSNR (dB)	
1	1/2	N/A	N/A	N/A	N/A	33.80	1	2/3	N/A	N/A	N/A	N/A	33.02	
2	8/9	2/3	N/A	N/A	N/A	34.90	2	8/9	4/5	N/A	N/A	N/A	33.43	
3	1	8/9	2/3	N/A	N/A	34.97	3	1	8/9	4/5	N/A	N/A	33.49	
4	1	1	8/9	2/3	N/A	34.97	4	1	1	8/9	4/5	N/A	33.48	
5	1	1	1	8/9	2/3	34.96	5	1	1	1	8/9	4/5	33.45	

TABLE I Optimal Allocation:  $\mathcal{R}^*$  for M-Packet Transmission. Optimal Parameter  $M^*$  Is Shown in Bold

TABLE IINumber of Possibilities With and Without the First Condition. We Obtain Complexity Reductions With Increasing M. For Example,<br/>When M = 5, It is Enough to Search Only 1/62 of All Possibilities

$\forall \epsilon_0, \forall r_{tr}$	m = 1	m = 2	m = 3	m = 4	m = 5	m = 6
No condition	14	196	2744	38416	537824	7529536
$r_1^{*(m)} \ge \dots \ge r_m^{*(m)}$	14	105	560	2380	8568	27132

TABLE III

NUMBER OF POSSIBILITIES WITH TWO CONDITIONS WHEN  $\epsilon_0 = 0.01$  and  $r_{tr} = 0.7$  bpp. We Obtain More Complexity Reductions with Increasing M. For Example, when M = 5, It is Enough to Search Only 1/3055 of All Possibilities

$\epsilon_0 = 0.01,  r_{tr} = 0.7 { m bpp}$	$m_1 = 1$	$m_1 = 2$	$m_1 = 3$	$m_1 = 4$	$m_1 = 5$	$m_1 = 6$
No condition	14	196	2744	38416	537824	7529536
$r_1^{*(m_1)} \ge \dots \ge r_{m_1}^{*(m_1)}$	14	105	560	2380	8568	27132
$\{r_{m_2}^{*}{}^{(m_2)} \ge r_{m_1}^{*}{}^{(m_1)}\} \land \{r_1^{*}{}^{(m_1)} \ge \dots \ge r_{m_1}^{*}{}^{(m_1)}\}$	14	50	85	120	176	260

going from  $m_1$  to  $m_2$  encoding stages gives a certain level of additional protection for more important bits. Therefore, it is enough to choose the outermost code  $r_{m_2}^{*}{}^{(m_2)}$  to be equal to or greater than  $r_{m_1}^{*}{}^{(m_1)}$  to provide baseline protection for the total bit stream. Having a larger outermost code rate also allows more source bits into the bit stream and increases the potential for performance improvement. However, as we increase M, the CRC and tailing bits will start to overwhelm the allowed bit budget, giving rise to performance degradation due to lack of source bits. Using the condition  $r_{m_2}^{*}{}^{(m_2)} \ge r_{m_1}^{*}{}^{(m_1)}$ , we can obtain additional reductions in complexity, particularly at low  $\epsilon_0$ . One particular example is shown in Table III.

Table I also shows the average PSNR performance of *Concatenated* as a function of M. For all the cases shown, we pick up more than 0.5-dB gain using the optimal value  $M^*$ . For  $1 \leq M \leq M^*$ , we get diminishing returns as we approach  $M^*$ . When  $M > M^*$ , there can be slight degradation in the performance, because with larger M, the optimization results in a code rate of unity for some of the initial information blocks. This leads to an increase in the number of CRC bits, which, in turn, decreases the total number of source and channel coding bits in the allowed bit budget. In addition, since the sizes of the

permutation tables are chosen to be equal to the payload size in this paper, smaller information blocks mean shorter permutation table size, which significantly impact the overall performance of *Concatenated*.

Finally, we compare the proposed scheme with some of the reported results found in [2], [3], and [12]. As shown, those studies use the LVA for increased performance. Therefore, we use our proposed scheme with the LVA for comparison. In other words, the ordinary Viterbi algorithm is replaced with the LVA in the proposed formatting and system model. The LVA finds the "best path" that has the smallest accumulated metric in the trellis and satisfies the CRC at the same time. We constrained the path search depth to 70 candidate paths. If none of the 70 candidate paths satisfies the CRC, decoding failure is declared. Table IV shows some results for *Lena* and *Goldhill* images for  $\epsilon_0 = 0.1$  and  $\epsilon_0 = 0.01$ .

As a perspective on PSNR, if one sets the information rate equal to that which corresponds to the capacity of a BSC with crossover probability  $\epsilon_0$  and assumes that the data are sufficiently coded such as to produce no errors, then the results corresponding to this idealized scenario are listed in the rows entitled "error-free case." Note that the capacity of a BSC is given

TABLE IV

 $\begin{array}{l} \text{PSNR (in decidels) for 512 \times 512 Images Transmitted Over a BSC at Various Transmission Rates With LVA decoding. \\ \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.1} \approx \{0.1338, 0.2682, 0.5278\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.252, 0.505, 0.994\} \text{ bpp and } \{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.252, 0.505, 0.994\} \text{ bpp and } \{0.252, 0.505, 0.994\} \text{ bpp$ 

r <sub>tr</sub> (bpp)	Image	System	Channel raw BER	$(\epsilon_0)$	Image	System	Channel raw BER $(\epsilon_0)$	
			0.1	0.01			0.1	0.01
0.252		Proposed	29.46	32.74		Proposed	27.43	29.54
	Lena	[2]([3], [12])	28.4 (28.61, 28.88)	32.0	Goldhill	[2]	26.7	29.0
		"error-free case" 31.29		33.67		"error-free case"	28.63	30.31
0.505	Lena	Proposed	32.33	35.83		Proposed	29.24	31.79
		[2]([3], [12])	31.1 (31.27, 31.6)	35.2	Goldhill	[2]	28.6	31.2
		"error-free case"	34.39	36.91		"error-free case"	30.78	32.8
0.994		Proposed	35.27	38.84		Proposed	31.41	34.72
	Lena	[2]([3],[12])	34.2 (34.25, 34.66)	38.0	Goldhill	[2]	30.7	34.0
		"error-free case"	37.39	39.9		"error-free case"	33.4	35.98

#### TABLE V

PSNR (IN DECIBELS) FOR 512 × 512 Lena and Goldhill Images Encoded With JPEG2000 and Transmitted Over a BSC at Various Transmission Rates. "Error-Free Case" Corresponds to  $\{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.1} \approx \{0.1338, 0.2682, 0.5278\}$  bpp and  $\{0.252, 0.505, 0.994\} \times C(\epsilon_0)|_{\epsilon_0=0.01} \approx \{0.2316, 0.4642, 0.9137\}$  bpp. Bold Denotes the Winning Scheme for a Given Transmission Rate and Raw Channel Error Rate. ConRCPC Uses the Proposed Coding Paradigm and RCPC Code Family With Ordinary Viterbi Decoding, and ConLDPC Uses the Proposed Coding Paradigm and RC-LDPC Code Family With a Belief Propagation Algorithm

Imaga	m (hpp)	Sustam	Cl	nannel rav	w BER (e	io)	Imaga	Channel raw BER $(\epsilon_0)$			
Innage	/tr(opp)	System	0.01	0.03	0.08	0.1	mage	0.01	0.03	0.08	0.1
		ConRCPC	31.39	30.12	28.31	27.78	Goldhill	28.77	27.96	26.86	26.44
		ConLDPC	32.83	32.59	31.29	30.86		29.74	29.41	28.63	28.39
	0.252	RC-LDPC [9]	32.77	32.42	31.03	30.48		29.79	29.35	28.49	28.09
		IRA [7]	32.75	32.09	30.24	29.92		29.74	29.23	28.01	27.90
		RCTC [5]	32.56	31.90	29.94	29.40		29.64	29.16	27.88	27.69
		"error-free case"	33.59	32.88	31.67	31.17		30.30	29.88	28.89	28.57
		ConRCPC	34.47	33.1	31.14	30.37		30.88	30.08	28.69	28.07
Lena		ConLDPC	36.12	35.67	34.31	34.08		32.21	31.79	30.69	30.41
	0.505	RC-LDPC [9]	36.08	35.41	34.00	33.32		32.28	31.65	30.52	30.13
		IRA [7]	36.18	35.48	33.37	33.13		32.10	31.65	30.12	29.99
		RCTC [5]	35.67	35.15	33.20	32.76		31.79	31.38	30.04	29.89
		"error-free case"	36.82	36.16	34.86	34.34		32.84	32.31	31.14	30.72
		ConRCPC	37.53	36.17	34.19	33.42		33.63	32.3	30.63	30.21
		ConLDPC	39.01	38.77	37.38	37.13		35.30	34.65	33.41	33.11
	0.994	RC-LDPC [9]	39.03	38.34	36.91	36.19		35.23	34.33	32.99	32.42
		IRA [7]	38.93	38.26	36.26	36.03		35.00	34.18	32.30	32.20
		RCTC [5]	38.78	37.74	36.15	35.85		34.81	34.05	32.24	32.10
		"error-free case"	39.84	39.24	37.90	37.36		36.06	35.40	33.92	33.39

by  $C(\epsilon_0) = 1 - h(\epsilon_0)$ , where  $h(\epsilon_0) = -\epsilon_0 \log_2(\epsilon_0) - (1 - \epsilon_0) \log_2(1 - \epsilon_0)$  is the binary entropy function, and  $\epsilon_0$  is the raw error rate of the channel. As shown, the proposed scheme not only improves the performance of [2] but also gives results that are close to the "error-free case."

### B. Simulations With RC-LDPC Codes

Throughout this section, the RC-LDPC channel code set is used. An extra byte (instead of the CRC) is added in each encoding stage to inform the RC-LDPC decoder about the channel coding rate used, as in [9]. Our mother code rate is 8/13, and through puncturing and extending the mother code [31], we obtain nine code rates in total, namely, 8/22, 8/20, 8/18, 8,16, 8/15, 8/13, 8/12, 8/11, and 8/10. This yields similar simulated PER performance to that of the irregular RC-LDPC construction in [9] for a BSC. In what follows, we present the performance of the proposed coding scheme using the *Lena* and *Goldhill* images (encoded with JPEG2000) with the class of RCPC (*ConRCPC*) and RC-LDPC (*ConLDPC*) codes. The results are given in Table V using a maximum of 250<sup>3</sup> iterations

 $^{3}$ Reducing the maximum number of iterations to 100 as in [9] will only reduce the reported PSNRs in Table V by about 0.15 dB.

of the Max-product algorithm at the receiver. As shown, similar gains are obtained, except that the results are now closer to the "error-free case." In addition, note that the proposed scheme is more powerful at higher crossover error probabilities, as it virtually increases the size of the available code set and allows better protection than would be possible with the strongest code rate in the code set. Finally, we note that, at  $\epsilon_0 = 0.1$ , the proposed scheme is at most only  $\approx 0.3$  dB away from the maximum achievable PSNR for all the simulations presented.

# *C. Discussion on the Advantages of the Proposed Transmission Scheme*

Progressive image reconstruction has several advantages. One of these advantages is not preserved by our coding structure, but the others are. Progressive coding allows rapid display of a low-quality version of the image at early stages of the reception of the bit stream. The receiver progressively obtains better image quality as more bits reliably arrive and can redisplay the image multiple times at increasingly higher quality, allowing one to abort the image early in the reception if one determines, based on the low quality version, that the image is not the one desired. This might allow for faster browsing of a remote image database. This feature of allowing redisplaying multiple times is not preserved by our coding structure, as decoding of the most important information chunk cannot proceed immediately when it is received, as it is interleaved with other information chunks and their parity bits. This advantage of progressive image coding does not in any case extend to scalable video coding, because a single frame of video, even if it were scalably encoded, is not redisplayed multiple times at increasing quality levels.

The other advantages of progressive image transmission (which do extend to scalable video) are supported by our encoding structure. Progressive coding allows for simple downsizing of the total bit stream in case of congestion at an intermediate router. This feature is retained. The router would need to deinterleave but would not need to do either source decoding or channel decoding in order to strip off sections of information bits and parity bits and transmit onward the beginning portions of the progressive stream. Finally, progressive encoding has the advantage of being very suitable for UEP; thus, the initial information bits are more heavily protected for graceful degradation of the transmitted source quality at the receiver. We note that, if the proposed system were used for scalable video coding, the overall delay incurred on the encoder side would be dominated by the delay of the scalable video source encoder and not by the delay of the proposed channel encoder and interleaver. Although some scalable video coders incur significant delay (such as 3-D SPIHT, which might need to buffer a group of 16 frames), for a lower delay encoder such as H.264 fine-grain scalability, if the M codewords are all part of one frame, then interleaving is all done within one frame. In this case, the proposed system will incur negligible processing delay.

# V. CONCLUSION

We have considered an embedded bit stream using concatenated block encoding in conjunction with random block interleavers. Novel formatting is introduced for embedded bit streams with better reconstruction properties at the receiver. Both the information block size and the channel code rates are subject to optimization, which makes the proposed system more flexible. A low-complexity descent search for block size adjustment and a constrained exhaustive search in finding the optimal code rates are presented. In addition, using the proposed image coding system, we use multiple code rates for the protection of information blocks, which makes additional BERs achievable beyond those possible using conventional methods with a discrete channel code rate set. The proposed scheme achieves this performance gain at the expense of loss of progressive display capability and increased complexity.

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