## TRADEOFFS OF SOURCE CODING, CHANNEL CODING AND SPREADING IN CDMA SYSTEMS \*

Qinghua Zhao Pamela Cosman Laurence B. Milstein Department of Electrical and Computer Engineering, University of California, San Diego

## ABSTRACT

We consider a CDMA system consisting of an image source coder, a convolutional channel coder, an interleaver, and a direct sequence spreading module. With different allocations of bandwidth to source coding, channel coding and spreading, the system is analyzed over a frequency selective Rayleigh fading channel. The performance of the system is evaluated using the cumulative distribution function of peak signal-to-noise ratio. We show that, among other things, given a fixed channel coding rate, allocating more bandwidth to source coding allows higher maximum image quality. At the same time, the probability of achieving this high quality is small. Allocating more bandwidth to spreading decreases the number of source information bits transmitted, thus limiting the achievable image quality, but the probability of achieving this maximum quality is high.

#### I. INTRODUCTION

Source coding, channel coding and spread spectrum are the three main components in a CDMA communication system. A number of studies have been done on the joint design of source and channel coding algorithms to yield better system throughput [1, 2, 3]. There also exists a body of research on the tradeoffs between channel coding and CDMA [4, 5, 6]. In this work, we investigate the interrelationship among all three components.

Bandwidth is the major shared resource between the three components. Source coding will free up bandwidth for both forward error correction (FEC) and spreading. Allocating more bandwidth to source coding will allow more information from the source to be transmitted. For different compression methods and rates, the bit stream coming out of the source encoder will be more or less sensitive to different types of error patterns. FEC and spreading protect the transmitted bits from noise and interference. Depending on the channel conditions and the characteristics of the source coded bit stream, the system will perform better with either more FEC or more spreading.

The paper is organized as follows. In Sections II and III, the system model and the channel model are described, respectively. Some representative results are given in Section IV, and the conclusions are given in Section V.

#### **II. SYSTEM MODEL**

The system is shown in Figure 1. We discuss each component in detail below.



Figure 1: System overview.

1. *Source coding:* The source images are encoded using a lossy compression algorithm called Set Partitioning In Hierarchical Trees (SPIHT [7]). The encoded bit stream is progressive, i.e., bits which come first can be used to reconstruct a low quality version of the source image, and bits which come later can be decoded to produce successively higher quality versions. The SPIHT algorithm has excellent compression performance, however, it is very sensitive to errors. An error in one bit may lead to complete loss of synchronization in the source decoder, rendering decoding

<sup>\*</sup>Acknowledgement: This research was partially sponsored by the Center for Wireless Communications of UCSD, and by the CoRe program of the State of California.

impossible of all subsequent bits. There is a small amount of image header information for the coded source bit stream (59 bits in most cases). This number is very small compared to the bit budget for almost all transmission rates of interest, so in all the analyses and simulations, the header is assumed to be error-free.

2. Channel coding: In Figure 2 [9], source information bits are grouped into blocks of size N. A 16-bit CRC is added to each packet. Then the packet is convolutionally encoded using a Rate-Compatible Punctured Convolutional (RCPC) [8] code. The list-based Viterbi algorithm is used to find the best candidate in the trellis. Then the CRC detects whether there is an error. If there is an error, the second best candidate is found and the CRC checked, and so on. After checking the list of paths for a predetermined number of times, if the CRC check still declares an error, the source decoder will discard that block and all subsequent blocks.



Figure 2: Source and channel coding block diagram.

3. Spreading: The channel coded data stream is spread, using direct sequence with a long spreading code, by a factor of M (the processing gain). Then the signal is transmitted using BPSK modulation. Assume there are K simultaneously active users in the system. The signature sequences of different users have a common chip rate of  $1/T_c = W$ , where  $T_c = T/M$ , W is the spread bandwidth and 1/T is the data bit rate. Let  $a_k(t)$  denote the signature sequence waveform of the  $k^{th}$  user, and let  $a_j^{(k)}$  be the corresponding sequence elements, where  $a_j^{(k)} \in (+1, -1)$ . Then

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_j^{(k)} P_{T_c}(t-jT_c) ,$$

where  $P_{T_c} = 1$  for  $0 \le t \le T_c$  and equals zero otherwise. Similarly, the data signal may be written as

$$b_k(t) = \sum_{j=-\infty}^{\infty} b_j^{(k)} P_T(t-jT) .$$

Therefore, the transmitted signal for the k-th user is

$$s_k(t) = Re[S_k(t)e^{j\omega_c t}],$$

where

$$S_k(t) = \sqrt{2P} a_k(t) b_k(t) e^{j\theta_k}$$

*P* is the average transmitted power, assumed to be the same for all users,  $\omega_c$  is the common carrier frequency and  $\theta_k$  is the phase of the  $k^{th}$  user. Assuming asynchronous operation, the delay of user k relative to the reference user (user 0) is  $\tau_k, k = 1, ..., K - 1$ . The composite signal at the input to the channel is

$$s_T(t) = Re[S_T(t)e^{j\omega_c t}],$$

where

$$S_T(t) = \sum_{k=0}^{K-1} \sqrt{2P} \, a_k(t- au_k) b_k(t- au_k) e^{j \phi_k} \, ,$$

 $\phi_k = \theta_k - \omega_c \tau_k$ ,  $\theta_0 = \tau_0 = 0$ ,  $(\phi_k)_{k=1}^K$  are independent identically distributed (iid) random variables, uniformly distributed in  $[0, 2\pi)$ , and  $(\tau_k)_{k=1}^K$  are iid random variables, uniformly distributed in [0, T).

#### **III. CHANNEL MODEL**

Figure 3 shows a finite-length tapped delay line model for a frequency selective multipath channel for the  $k^{th}$  user. In the figure, L is the number of resolvable multipaths in the channel, and  $c_i, i = 1, ..., L$ , are the complex gains of the different paths. Note that  $c_i$  takes the form of  $\alpha_i e^{j\theta_i}$ , in which  $\alpha_i$  is Rayleigh distributed, i.e.,  $p(\alpha) = \frac{\alpha}{\sigma^2} e^{\frac{-\alpha^2}{2\sigma^2}}$ , and  $\theta_i$  is uniformly distributed. We assume a flat Multipath Intensity Profile (MIP), which means the parameter  $\sigma^2$  in the Rayleigh density is not a function of *i*.



Figure 3: Tapped delay line model of frequency-selective channel.



Figure 4: RAKE receiver model.

We use the RAKE receiver shown in Figure 4 to resolve the multipath. The received signal is fed into a matched filter for despreading before it goes into the RAKE receiver. In the RAKE receiver, maximal-ratio combining is used to produce the optimal result. Note that  $\hat{c}_i(t) = c_i^*(t)$  under perfect channel estimation.

Fading in the wireless channel is correlated in time. The fading pattern depends on the mobile speed through the normalized Doppler value. The maximum Doppler shift is given by

$$f_D = f_{max\_Doppler} = f_c \frac{v}{c},$$

where  $f_c$  is the carrier frequency, v is the mobile speed, and c is the speed of light. Considering a scenario where  $f_c$  is 900 MHz, and the data rate is 29 K bits/sec, we have the results presented in Table 1. The Jakes' model [10, 11] is used to generate time-correlated Rayleigh fading parameters  $c_i(t)$  for each path (for each i) and independent fading between different paths. Asynchronized interfering users<sup>1</sup> and Additive White Gaussian Noise (AWGN) are added to the desired signal at the output of the channel.

Scenario	Mobile speed	$f_D$ (Hz)	$f_D \cdot T$
pedestrian	4mph	5.36	1.85e-4
local	30mph	40.2	1.39e-3
highway	70mph	93.9	3.24e-3

Table 1: Normalized Doppler and mobile speed.

#### **IV. RESULTS**

The choice of a good system depends on the performance measure and the channel conditions. For a given system, both the fades and the noise in the channel are random processes. Therefore, the output from the source decoder will not be the same for different trials. We measure the performance of the system by looking at the output for many independent trials. The cumulative distribution function (CDF) of PSNR (Peak Signal-to-Noise Ratio<sup>2</sup>) of the decoded image is used to evaluate the performance [12].

#### 1. Tradeoffs of bandwidth

For all the curves, we kept the ratio of energy-per-source-bit to noise power spectral density,  $E_b/N_o$ , constant, where  $N_o/2$  is the two sided noise power spectral density.

In Figure 5, the channel coding rate is fixed at 0.72. The number of users K = 10, and  $E_b/N_o$  is 4dB. Parameters which are varied are processing gain and source coding rate, represented by (processing gain, source coding rate) in the plots. For the top-most curve, we see that there is a high probability that the output image has a low PSNR. But since more bandwidth is allocated to source coding, there is a small, but nevertheless non-zero, probability of achieving very good PSNR results, (the right end of the curve reaches a PSNR greater than 34dB). In contrast, for the lowest curve, there is a high probability of achieving high PSNR. But since less bandwidth is allocated to source coding, the best PSNR achievable is limited to 32.5dB, lower than the corresponding values of the other curves.



Figure 5: CDF plots: Tradeoff between source coding rate and processing gain. Uncorrelated fading, channel coding rate 0.72.

<sup>&</sup>lt;sup>1</sup>Chip synchronization is assumed.

<sup>&</sup>lt;sup>2</sup>Defined as  $10 \log \frac{peak image energy}{noise image energy}$ , where noise image = received image - original image

We can see that, in Figure 5, there are crossovers between the curves. If there were no crossovers, it would be easy to say that the lowest curve represents the best system. When the curves cross, a given system may be superior for one application but not for another. Comparison of the curves may then involve looking at the area under the curve, perhaps with some weighting (e.g., all PSNRs less than a certain amount may be considered equally bad, and all PSNRs above a certain amount may be considered equally good). The application requirements can sometimes be summarized by saying that a given image quality must be present at least a specified fraction of the time. Some curves may then be inadmissible. These issues are discussed in [12].



Figure 6: CDF plots: Tradeoff between channel coding rate and processing gain. Local fading pattern, interleaver 10 by 10.

Figure 6 shows the tradeoff between channel coding rate and processing gain. The number of users K = 10 and  $E_b/N_o = 4dB$ . The parameters varied are processing gain and channel coding rate. The source coding rate is fixed, therefore the best achievable image quality is the same for all curves, and there is no crossover. Note that for decreasing channel coding rate r = 0.80, 0.46, 0.36, 0.30, approximately 16%, 64%, 92%, 84% of the decoded images have PSNR larger than 30dB. It is easy to see that in this scenario, the system first improves when more bandwidth is allocated to the channel coding (channel coding rate decreases), and then deteriorates when too much bandwidth is allocated to channel coding. There are two counterbalancing effects to the system when more bandwidth is allocated to channel coding instead of spreading. As the channel code rate, r, gets lower, the coding gain increases which benefits the system. At the same time, the processing gain decreases and this cause both loss of some diversity enhancement and a decrease in interference suppression.

#### 2. Effects of interleaving

The source coding algorithm, the channel coding algorithm and the interleaver might cause significant delays in the system, especially for time critical applications such as voice and video transmissions. Here we will discuss the effects of the interleaver.



Figure 7: System performance parameterized by interleaver size. Channel coding rate 0.80, packet size 250, pedestrian fading pattern

Generally, a larger interleaver will scatter correlated errors. However, this does not always benefit the system, especially when the system performance depends more on packet erasure rate than on bit error rate. Figure 7 shows the system performance versus interleaver size under different channel conditions. The channel coding rate is 0.80. There are K = 6active users, the processing gain is 128, and  $E_b/N_o$  is 4dB. We see that a larger interleaver size does not necessarily lead to better performance. This is because the interleaver disperses the errors, and thus more packets are affected. Note that even though that dispersion of errors results in fewer errors per packet, the number of those bit errors may still be large enough to overwhelm the decoder. For the curves shown in Fig. 7, the interleaver size has to be about 120 by 120 before the decoder functions efficiently.

### **V. CONCLUSIONS**

Given a fixed total bandwidth, each coding scenario has a different probability distribution of achieving certain PSNR values for the decoded image. Allocating more bandwidth to source coding allows us to achieve a higher maximum image quality, but the probability of achieving this quality is smaller. On the other hand, allocating more bandwidth to spreading decreases the number of source information bits transmitted and thus limits the best achievable image quality, but the probability of achieving this quality is higher.

For a given bandwidth, there are optimal allocations of bandwidth to source coding, channel coding and spreading, depending on the result one wants to achieve. Tradeoffs among the parameters allow us to tune the system performance to a particular set of requirements.

# References

- B.D. Pettijohn, K. Sayood, and M.W. Hoffman. "Joint source/channel coding using arithmetic codes," *Proceedings DCC 2000. Data Compression Conference*, p.73-82, March 2000.
- [2] G. Cheung and A. Zakhor. "Bit allocation for joint source/channel coding of scalable video," *IEEE Transactions* on *Image Processing*, vol.9, (no.3), p.340-56, March 2000.
- [3] M. Zhao, A.A. Alatan, and A.N. Akansu. "A new method for optimal rate allocation for progressive image transmission over noisy channels," *Proceedings DCC 2000. IEEE Data Compression Conference*, p.213-22, March 2000
- [4] D.J. Van Wyk, I.J. Oppermann and L.P. Linde. "Performance tradeoff among spreading, coding and multiple-antenna transmit diversity for high capacity space-time coded DS/CDMA," *Proceedings of Conference on Military Communications MILCOM 1999*, p.393-7 vol.1. Sept. 1999
- [5] I. Oppermann and B. Vucetic. "Capacity of a coded direct sequence spread spectrum system over fading satellite channel using an adaptive LMS-MMSE receiver," *IEICE Trans. Fundamentals*, vol E79-A, no. 12v pp. 2043-2049, Dec. 1996.
- [6] J.R. Foerster and L.B. Milstein. "Coding for a coherent DS-CDMA system employing an MMSE receiver in a Rayleigh fading channel," *IEEE Transactions on communication*, vol. 48, no. 6, June 2000.
- [7] A. Said and W. A. Pearlman. "A new, fast, and efficient image codec based on set partitioning in hierarchical trees," *IEEE Transactions on Circuits and Systems for Video Technology*, 6(3):243–250, June 1996.
- [8] J. Hagenauer. "Rate-compatible punctured convolutional codes (RCPC codes) and their applications," *IEEE Transactions on Communication*, vol 36, pp. 389-400, Apr. 1988.

- [9] P.G. Sherwood and K. Zeger. "Progressive Image Coding on Noisy Channels," *IEEE Signal Processing Letters*, vol. 4, no. 7, pp. 189-191, July 1997.
- [10] W. C. Jakes. Microwave Mobile Communications. Wiley-Interscience, 1974.
- [11] P. Dent, G.E. Bottomley and T. Croft. "Jakes' model revisited," *Electronics Letters*, 24th June, 1993, vol.29, No. 13.
- [12] P. C. Cosman, J. K. Rogers, P. G. Sherwood, and K. Zeger. "Combined Forward Error Control and Packetized Zerotree Wavelet Encoding For Transmission of Images Over Varying Channels," *IEEE Transactions on Image Processing*, pp. 982-993, June 2000.
- [13] J.G. Proakis. Digital Communications. McGraw-Hill, 1995.