2.4 Ideal Amplifiers

Of fundamental importance in the study of electric circuits is the **ideal voltage amplifier**. Such a device, in general, has two inputs, v_1 and v_2, and one output, v_o. The relationship between the output and the inputs is given by $v_o = A(v_2 - v_1)$, where A is called the **gain** of the amplifier. The ideal amplifier is modeled by the circuit shown in Fig. 2.13*a*, which contains a dependent voltage source. The resistance R is the **input resistance** of the amplifier. Note that since the input resistance $R = \infty\ \Omega$ for an ideal voltage amplifier, when such an amplifier is connected to any circuit, no current will go into the input terminals. (In general, however, there will be a current going into or coming out of the output terminal.) Also, since the output v_o is the voltage across an ideal source, we have that $v_o = A(v_2 - v_1)$ regardless of what is connected to the output. For the sake of simplicity, the ideal amplifier having gain A is often represented as shown in Fig. 2.13*b*. (Note that the reference node is not explicitly displayed in this figure.) We refer to the input terminal labeled "−" as the **inverting input** and the input terminal labeled "+" as the **noninverting input**.

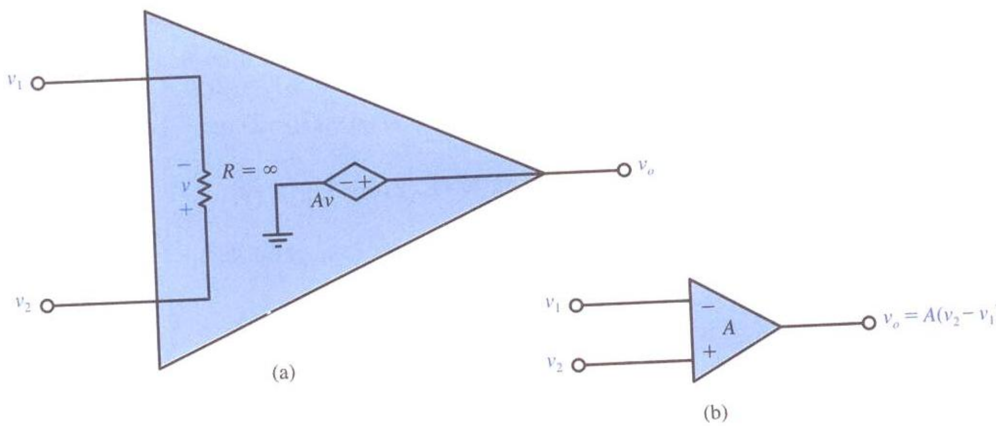

Fig. 2.13 Model and circuit symbol of an ideal amplifier having gain *A*.

Example 2.7

Let us find v_o for the ideal amplifier circuit shown in Fig. 2.14*a*. The explicit form of this circuit is shown in Fig. 2.14b.

In the circuit given in Fig. 2.14, the noninverting input is at the reference potential, that is, $v_2 = 0$ V. Furthermore, since node v_s and node v_o are constrained by voltage sources (independent and dependent, respectively), in using nodal analysis we sum

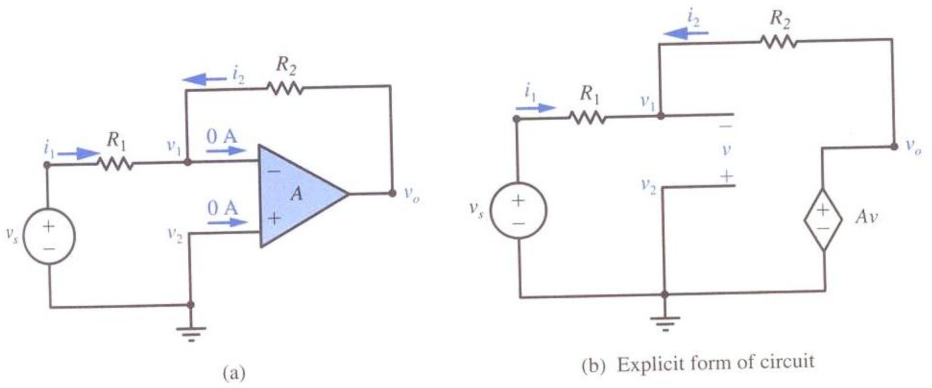

Fig. 2.14 (*a*) Ideal-amplifier circuit, and (*b*) explicit form of the circuit.

currents only at node v_1 (the inverting input). Since the amplifier inputs draw no current, by KCL,

$$i_1 + i_2 = 0$$

By Ohm's law

$$\frac{v_s - v_1}{R_1} + \frac{v_o - v_1}{R_2} = 0 \quad \Rightarrow \quad R_2 v_s = (R_1 + R_2)v_1 - R_1 v_o \tag{2.23}$$

But due to the amplifier, $v_o = A(v_2 - v_1) = -Av_1$, so

$$v_1 = \frac{-v_o}{A} \tag{2.24}$$

and substituting this into Eq. 2.23, we get

$$R_2 v_s = (R_1 + R_2)\frac{-v_o}{A} - R_1 v_o = -\left[\frac{1}{A}(R_1 + R_2) + R_1\right] v_o$$

Thus,

$$v_o = \frac{-R_2 v_s}{R_1 + (1/A)(R_1 + R_2)} \tag{2.25}$$

Drill Exercise 2.7

For the ideal-amplifier circuit given in Fig. 2.14, suppose that $R_1 = 1\ \text{k}\Omega$, $R_2 = 10\ \text{k}\Omega$, $A = 100{,}000$, and $v_s = 1$ V. (a) Find v_o, v_1, i_1, and i_2. (b) Find the power absorbed by each resistor, the independent voltage source, and the ideal amplifier (i.e., the dependent voltage source).

ANSWER (a) − 10.0 V, 0.10 mV, 1.0 mA, −1.0 mA;
(b) 1.0 mW, 10.0 mW, −1.0 mW, −10.0 mW

For a circuit such as that in Fig. 2.14, let us consider the case that the gain A becomes arbitrarily large. When $A \to \infty$, from Eq. 2.25 we have that

$$v_o = -\frac{R_2}{R_1} v_s$$

We see that although the gain of the amplifier is infinite, for a finite input voltage v_s, the output voltage v_o is finite (provided, of course, that $R_1 \neq 0$). Inspection of Eq. 2.24 indicates why the output voltage remains finite—as $A \to \infty$, then $v_1 = -v_o/A \to 0$ V. This result occurs because there is a resistor connected between the output and the negative input. Such a connection is called **negative feedback**.

The Operational Amplifier

An ideal amplifier having gain $A = \infty$ is known as an **operational amplifier**, or **op amp.** In an op-amp circuit, because of the infinite gain property, we must have a feedback resistor and must not connect a voltage source directly between the amplifier's input terminals. For the circuit given in Fig. 2.14, the corresponding op-amp circuit is usually drawn as shown in Fig. 2.15. Using the fact that $v_1 = 0$ V, and summing the currents at the inverting input (node v_1), we get

$$\frac{v_s}{R_1} = -\frac{v_o}{R_2} \quad \Rightarrow \quad v_o = -\frac{R_2}{R_1} v_s \tag{2.26}$$

and the gain of the overall circuit is

$$\frac{v_o}{v_s} = -\frac{R_2}{R_1} \tag{2.27}$$

This circuit is called an **inverting amplifier**.

Notice how simple the analysis of the op-amp circuit in Fig. 2.15 is when we use the fact that $v_1 = 0$ V. Although this result was originally deduced from Eq. 2.25,

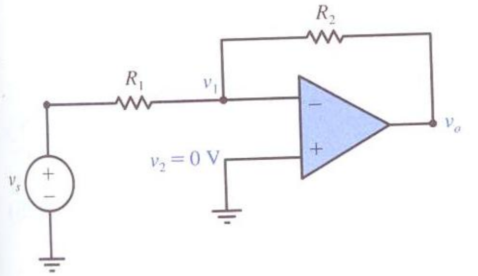

Fig. 2.15 Op-amp circuit—an inverting amplifier.

the combination of infinite gain and feedback constrains the voltage applied to the op-amp (between terminals v_1 and v_2) to be 0 V. In other words, we must have that $v_1 = v_2$.

Example 2.8

Consider the op-amp circuit with feedback in Fig. 2.16*a*. Again, the inputs of the amplifier draw no current, and so in applying KCL at node v_1, we have

$$i_1 + i_2 = 0 \quad \Rightarrow \quad \frac{v_1}{R_1} + \frac{v_1 - v_o}{R_2} = 0$$

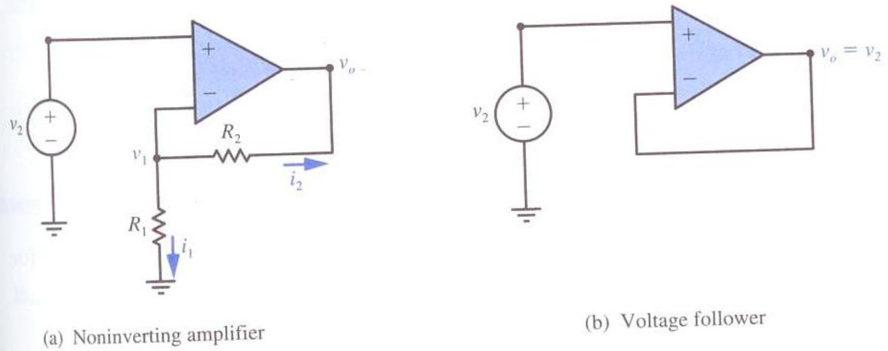

Fig. 2.16 (*a*) Noninverting amplifier, and (*b*) voltage follower.

Since the input voltage to the amplifier is 0 V, or equivalently, since both input terminals must be at the same potential, then $v_1 = v_2$, and substituting this fact into the last equation, we obtain

$$\frac{v_2}{R_1} + \frac{v_2 - v_o}{R_2} = 0 \quad \Rightarrow \quad R_2 v_2 + R_1 v_2 - R_1 v_o = 0$$

from which

$$v_o = \frac{R_1 + R_2}{R_1} v_2 = \left(1 + \frac{R_2}{R_1}\right) v_2$$

$$\frac{v_o}{v_2} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1} \tag{2.28}$$

and this is overall gain of this circuit, which is called a **noninverting amplifier**.

Note that if $R_2 = 0\ \Omega$, then $v_o = v_2$. Under the circumstance, R_1 is superfluous and may be removed. The resulting op-amp circuit shown in Fig. 2.16*b* is known as a **voltage follower**. Such a configuration is used to isolate or buffer one circuit from another.

Drill Exercise 2.8

For the noninverting amplifier given in Fig. 2.16*a*, suppose that $R_1 = 1\ \text{k}\Omega$, $R_2 = 9\ \text{k}\Omega$, and $v_2 = 1$ V. (a) Find v_o, i_1, and i_2. (b) Find the power absorbed by each resistor, the independent voltage source, and the op amp.

ANSWER (a) 10 V, 1 mA, −1 mA; (b) 1 mW, 9 mW, 0 W, −10 mW

Operational-amplifier circuits can be quite useful in some situations in which there is more than a single input.

Example 2.9

Let us determine the output voltage v_o in terms of the input voltages v_a and v_b for the op-amp circuit shown in Fig. 2.17. Since $v_2 = 0$ V, then $v_1 = 0$ V. By KCL, at the inverting input,

$$i_1 + i_2 = i_3$$

from which we get

$$\frac{v_a}{R_1} + \frac{v_b}{R_1} = -\frac{v_o}{R_2} \qquad \Rightarrow \qquad v_o = -\frac{R_2}{R_1}(v_a + v_b) \tag{2.29}$$

Since the output v_o is the sum of the input voltages v_a and v_b (multiplied by the constant $-R_2/R_1$), this circuit is called an **adder** (or **summer**). A common applica-

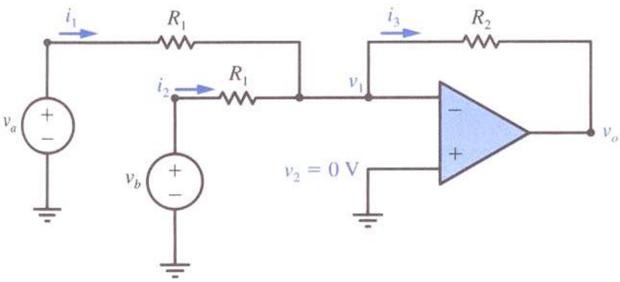

Fig. 2.17 Op-amp adder.

tion of such a circuit is as an "audio mixer," where, for example, the inputs are voltages attributable to two separate microphones and the output voltage is their sum.

Drill Exercise 2.9

For the op-amp adder shown in Fig. 2.17, suppose that $R_1 = 1$ kΩ, $R_2 = 10$ kΩ, $v_a = -0.2 \cos 2000\pi t$ V, and $v_b = 0.3 \cos 4000\pi t$ V. Find v_o, i_1, i_2, and i_3.

ANSWER $2 \cos 2000\pi t - 3 \cos 4000\pi t$ V, $-0.2 \cos 2000\pi t$ mA, $0.3 \cos 4000\pi t$ mA, $-0.2 \cos 2000\pi t + 0.3 \cos 4000\pi t$ mA

Example 2.10

Let us find the output voltage v_o of the op-amp circuit with two inputs is shown in Fig. 2.18.

Using the fact that $v_1 = v_2 = v$, by KCL at the inverting input

$$i_1 = i_3 \qquad \Rightarrow \qquad \frac{v_a - v}{R_1} = \frac{v - v_o}{R_2}$$

from which

$$R_1 v_o + R_2 v_a = (R_1 + R_2)v \tag{2.30}$$

Applying KCL at the noninverting input, we get

$$i_2 = i_4 \qquad \Rightarrow \qquad \frac{v_b - v}{R_1} = \frac{v}{R_2}$$

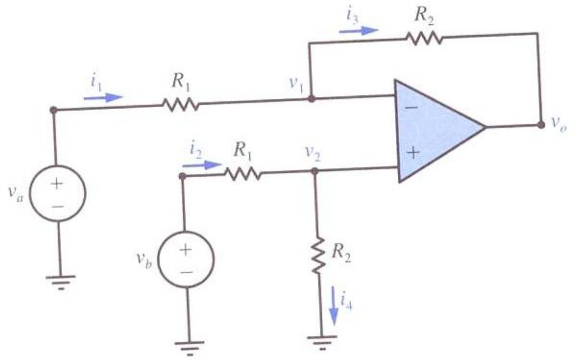

Fig. 2.18 Difference amplifier.

from which

$$R_2 v_b = (R_1 + R_2)v \tag{2.31}$$

Combining Eq. 2.30 and 2.31 results in

$$R_1 v_o + R_2 v_a = R_2 v_b \qquad \Rightarrow \qquad v_o = \frac{R_2}{R_1}(v_b - v_a) \tag{2.32}$$

Since the output is the difference of the inputs (multiplied by the constant R_2/R_1), such a circuit is called a **difference amplifier** or **differential amplifier**.

Drill Exercise 2.10

For the difference amplifier shown in Fig. 2.18, suppose that $R_1 = 1$ kΩ, $R_2 = 9$ kΩ, $v_a = -0.2 \cos 2000\pi t$ V, and $v_b = 0.3 \cos 4000\pi t$ V. Find v_o, v, i_1, i_2, i_3, and i_4.

ANSWER $1.8 \cos 2000\pi t + 2.7 \cos 4000\pi t$ V, $0.27 \cos 4000\pi t$ V, $-0.2 \cos 2000\pi t - 0.27 \cos 4000\pi t$ mA, $0.03 \cos 4000\pi t$ mA, $-0.2 \cos 2000\pi t - 0.27 \cos 4000\pi t$ mA, $0.03 \cos 4000\pi t$ mA

For a physical operational amplifier, the gain and the input resistance are large, but not infinite. A more practical model of an actual op amp is to have a resistance R_o (called the **output resistance** of the op amp) connected in series with the dependent voltage source. Yet, assuming that an op amp is ideal often yields a simple analysis with very accurate results.

Although its usefulness is great, the physical size of an op amp typically is small. This is a result of the fact that op amps are commonly available on IC chips, which normally contain between one and four op amps. In our previous discussions, we depicted the op amp as a three-terminal device. In actuality, a package containing one or more op amps typically has between 8 and 14 terminals. As we have seen, three of the terminals are the inverting input, the noninverting input, and the output. However, there are also terminals which are used for applying constant voltages (called "power-supply voltages") that operate or "bias" the numerous transistors comprising an op amp, for frequency compensation, and for other details to be discussed in Chapter 10. One of the important practical electronic considerations for proper op-amp operation is that when feedback is between the output and only one input terminal, that input terminal should be the inverting input.

Example 2.11

Shown in Fig. 2.19 is a practical amplifier that uses the popular 741 op amp. For this circuit, terminal (pin) 2 is the inverting input, terminal 3 is the noninverting input, and terminal 6 is the output. Furthermore, pin 4 has a constant voltage of −15 V applied to it and pin 7 has a constant voltage of +15 V applied to it.[2] The 741 op amp typically has a gain of $A = 200{,}000$, and input resistance of $R = 2$ MΩ, and an output resistance of $R_o = 75$ Ω.

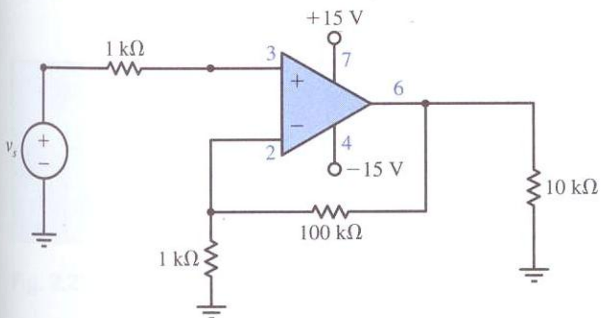

Fig. 2.19 Practical op-amp circuit.

Suppose that v_j is the voltage at terminal j, where $j = 1, 2, 3, \ldots$ Assuming that the op amp is ideal, then $v_3 = v_s$ since the inputs of an (ideal) op amp

[2] These constant voltages supply power to the actual components forming the op amp and are ignored when the op-amp circuit is analyzed.

draw no current. Also, due to feedback, $v_2 = v_3 = v_s$, and by KCL at terminal 2,

$$\frac{v_6 - v_s}{100{,}000} = \frac{v_s}{1000} \quad \Rightarrow \quad v_6 = 101v_s$$

We can get a more accurate result by not assuming that the op amp is ideal. Doing this, which requires a much greater analysis effort (see Drill Exercise 2.11), yields

$$v_6 = 100.95v_s$$

which is not significantly different from the simple approach taken above, and is even the same when rounded off to three significant digits.

Drill Exercise 2.11

For the circuit given in Fig. 2.19, suppose that the amplifier has input resistance $R = 2\ \text{M}\Omega$, output resistance $R_o = 75\ \Omega$ (which is connected in series with the dependent source Av), and gain $A = 200{,}000$. Use nodal analysis to get an accurate value for the output voltage v_6.

ANSWER $100.95v_s$

2.5 Thévenin's and Norton's Theorems

Suppose that a resistor R_L, called a **load resistor**, is connected to an arbitrary (in the sense that it contains only elements discussed previously) circuit as shown in Fig. 2.20. What value of the load resistor R_L will absorb the maximum amount of power? Knowing the particular circuit, we can use nodal or mesh analysis to obtain an expression for the power absorbed by R_L, then take the derivative of this expression to determine what value of R_L results in the maximum power absorbed by R_L. The effort required for such an approach can be quite great. Fortunately, though, a very important circuit theory concept states that as far as R_L is concerned, the arbitrary circuit shown in Fig. 2.20 behaves as though it is a single independent voltage source connected in series with a single resistance. Once we determine the values of this source and this resistance, we simply apply the results on maximum power transfer (which follows shortly) to find the value of R_L that results in the maximum power transfer to the load.