

# Energy Optimization for Hybrid-ARQ and AMC

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**Abstract**—We consider the energy optimization of a cross-layer design which combines hybrid automatic repeat request (HARQ) and adaptive modulation and coding. We consider two cases: variable alphabet size with constant transmit power, and variable alphabet size with variable transmit power. We optimize the alphabet size selection algorithm and/or the transmit power for each transmission round to minimize energy consumption, subject to an overall packet error probability constraint. Numerical results show that the variable alphabet size and variable power case significantly reduces energy consumption compared to conventional HARQ schemes. For example, at a packet error probability of  $5 \times 10^{-2}$ , energy consumption of the proposed scheme is reduced by 40% relative to a comparison scheme.

**Index Terms**—Hybrid-ARQ, AMC, energy optimization

## I. INTRODUCTION

To improve the throughput in wireless communication systems, adaptive modulation and coding (AMC) [1, 2] and hybrid automatic repeat request (HARQ) [3–10] have been studied extensively at the physical layer and data link layer, respectively. They are employed in practical systems, such as 3GPP-LTE [11, 12]. AMC selects the modulation and coding based on the channel state information (CSI). HARQ is an advanced automatic repeat request (ARQ) scheme which combines ARQ and forward error correction (FEC). HARQ can be divided into two kinds: independent decoding [13, 14], where the receiver does not combine previously received transmissions of the same packet; and soft combining [15–17], where the receiver combines the transmissions to improve the decoding performance. In this paper, we consider independent decoding HARQ schemes due to their simplicity.

In [3], from which our work is mainly inspired, the authors considered a cross-layer design which combines AMC and HARQ to maximize the throughput. However, energy is equally allocated in retransmissions and the advantage of multiple transmissions is not fully exploited. Later work [4, 5] showed that unequal energy allocation can significantly improve energy efficiency for HARQ. However, AMC was not considered. In our work, we use HARQ with AMC and consider unequal energy allocation for retransmissions.

We derive the energy optimization problem for cross-layer design for two cases: multiple alphabet sizes with constant transmit power, and multiple alphabet sizes with variable transmit power. We show numerical results for the cases when the maximum number of transmissions is two or three. We compare the performance of the proposed scheme with conventional HARQ.

The rest of the paper is organized as follows. In Section II, we introduce the assumptions and system model and derive

the optimization problem. In Section III, we consider the two cases. Numerical results are presented in Section IV and conclusions are drawn in Section V.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. Assumptions and System Model

The maximum transmit power is constrained, and the instantaneous channel gain  $\gamma$  is assumed to be Rayleigh distributed. The pdf of the instantaneous received SNR  $\Gamma$ ,  $f(\Gamma)$ , does not change with time, where  $\Gamma = \gamma^2 P_0 T_s / N_0$ ,  $P_0$  is the transmit power,  $T_s$  is the symbol duration and  $N_0$  is the spectral density of the Gaussian noise. The fades of different transmissions are assumed to be independent. Perfect CSI is available at transmitter.

The symbol duration of the system,  $T_s$ , is kept constant, and a rectangular pulse shape is used. The receiver is a matched filter. All packets contain  $L = 780$  bits (including FEC, but excluding CRC bits and tail bits). Since the number of CRC and tail bits is small compared to the number of bits in a packet, we neglect the influence of CRC and tail bits on the rate. The number of symbols in an M-PSK packet is  $L / \log_2(M)$ , and the number of information bits in a packet is  $Lr$ , where  $r$  is the FEC rate. Fixed rate turbo coding is used, where the rate can be 1/2, 1/3 or 1/5. Available modulations are BPSK, QPSK, 8PSK and 16PSK, and the duration of a BPSK packet is  $T_0 = LT_s$ .  $N$  is the maximum number of transmissions. The system block diagram is shown in Fig. 1.

### B. Optimization Problem

Suppose the source and FEC encoder generate  $n$  chunks of  $L$  coded bits. We use the term *chunk* to denote a set of information bits and their associated parity bits, while the term *packet* denotes any of the possibly multiple waveforms with different alphabet sizes that arise when the chunk gets transmitted or retransmitted. We use “packet error rate/probability” to denote the probability of error of any single transmission or retransmission of a chunk, and “overall packet error rate/probability” to denote the probability of error after  $N$  transmissions of a chunk. Define the following variables for chunk  $i$ ,  $i = 1, 2, \dots, n$ :

$P_{ei}(\{\Gamma\}_i)$ : overall packet error probability of chunk  $i$ , i.e., the probability that the chunk cannot be successfully decoded after  $N$  transmissions,

$\mathcal{E}_i(\{\Gamma\}_i, \{X\}_i)$ : energy consumption for chunk  $i$ , where  $\{\Gamma\}_i = (\Gamma_{i1}, \Gamma_{i2}, \dots, \Gamma_{iM})$  is the set of channel states for  $M$  transmissions of chunk  $i$ , where  $M \leq N$ .  $\{X\}_i = (X_{i1}, X_{i2}, \dots, X_{iM})$  is the set of outcomes for  $M$

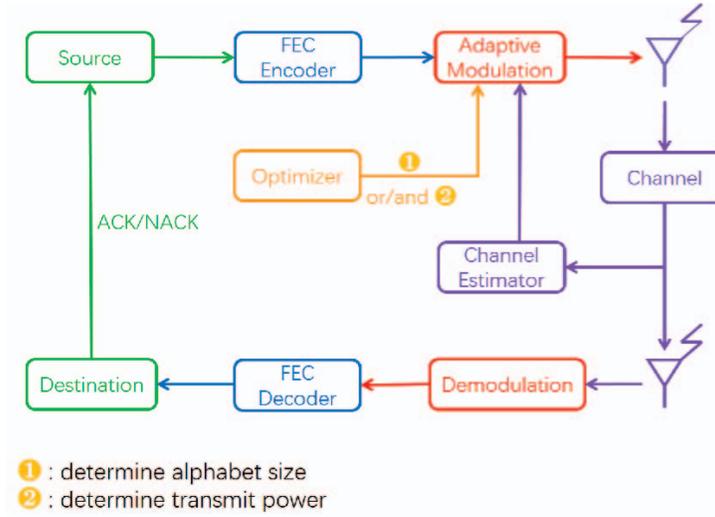


Fig. 1: System Block Diagram.

transmissions of chunk  $i$ .  $X_{ij} = 1$  means the  $j$ -th transmission of chunk  $i$  is successful, and  $X_{ij} = 0$  means the transmission is not successful. We have the following equations:

$$P_{ei}(\{\Gamma\}_i) = \mathcal{P}(X_{i1} = 0, X_{i2} = 0, \dots, X_{iN} = 0), \quad (1)$$

$$\mathcal{E}_i(\{\Gamma\}_i, \{X\}_i) = \begin{cases} \sum_{j=1}^k \mathcal{E}_{ij}(\Gamma_{ij}) & \text{if } k \text{ is the smallest number such that} \\ & X_{ij} = 1, k \leq N \\ \sum_{j=1}^N \mathcal{E}_{ij}(\Gamma_{ij}) & \text{if } X_{ij} = 0 \text{ for all } j \in [1, N-1] \end{cases} \quad (2)$$

where  $\mathcal{E}_{ij}(\Gamma_{ij})$  is the energy consumption of the  $j$ -th transmission of chunk  $i$ .

Since the channel states  $\{\Gamma\}_i$  are random and the transmission outcomes  $\{X\}_i$  are random,  $P_{ei}(\{\Gamma\}_i)$  and  $\mathcal{E}_i(\{\Gamma\}_i, \{X\}_i)$  are random. For simplicity of notation, we use  $P_{ei}(\{\Gamma\}_i)$  and  $P_{ei}$  and  $\mathcal{E}_i(\{\Gamma\}_i, \{X\}_i)$  and  $\mathcal{E}_i$  interchangeably.

We define the following averages:

$$\bar{P}_e = \frac{1}{n} \sum_{i=1}^n E_{\{\Gamma\}_i} [P_{ei}], \quad (3)$$

$$\bar{\mathcal{E}} = \frac{1}{n} \sum_{i=1}^n E_{\{\Gamma\}_i, \{X\}_i} [\mathcal{E}_i]. \quad (4)$$

Since the channel statistics and our strategy are both constant within the transmission duration of the entire message, let  $P_e$  be the overall packet error probability for any chunk and  $\mathcal{E}$  be the energy consumption per information bit for any chunk. Then

$$\bar{P}_e = \frac{1}{n} \sum_{i=1}^n E_{\{\Gamma\}_i} [P_{ei}] = E_{\{\Gamma\}} [P_e], \quad (5)$$

$$\bar{\mathcal{E}} = \frac{1}{n} \sum_{i=1}^n E_{\{\Gamma\}_i, \{X\}_i} [\mathcal{E}_i] = E_{\{\Gamma\}, \{X\}} [\mathcal{E}], \quad (6)$$

As will be discussed in more detail in the next section, the optimization problem we want to solve is to minimize  $\bar{\mathcal{E}}$  subject to an error constraint  $\bar{P}_e \leq P_{req}$  and a power constraint  $P = P_{max}$ .

### III. MULTIPLE ALPHABET SIZES & CONSTANT TRANSMIT POWER

In this section, we derive  $\bar{\mathcal{E}}$  and  $\bar{P}_e$  for the multiple alphabet sizes and constant transmit power case and we will explain how to solve the problem in Section IV.

#### A. Packet Error Probability Fitting and Alphabet Size Mapping

Since the packet error probability for turbo codes cannot be expressed analytically, we fit the packet error probability as in [3]:

$$\psi_M(\Gamma) = \begin{cases} 1, & 0 < \Gamma < \Gamma_{M,\min} \\ a_{\log_2 M} e^{-b_{\log_2 M} \Gamma}, & \Gamma \geq \Gamma_{M,\min} \end{cases}, \quad (7)$$

where  $\psi_M(\Gamma)$  is the conditional single transmission packet error probability, conditioned on  $\Gamma$ , for M-PSK, and  $a_i > 0, b_i > 0, i = 1, 2, 3, 4$ .  $\Gamma_{M,\min}$  is a parameter from curve-fitting. The simulated packet error rate for a rate 1/3 turbo code is shown in Fig. 2.

We map alphabet sizes as in [3], by setting a desired upper bound packet error rate,  $p_u$ , for each transmission. Although we want the actual packet error rate to be less than  $p_u$ , when the channel is so bad that  $p_u$  is not achievable, we just use the smallest alphabet size (BPSK) and accept that the goal is not achieved. The  $p_u$  for different retransmissions might be different, and the SNR boundaries ( $\Gamma^{(1)}, \Gamma^{(2)}, \Gamma^{(3)}, \Gamma^{(4)}, \Gamma^{(5)}$ )

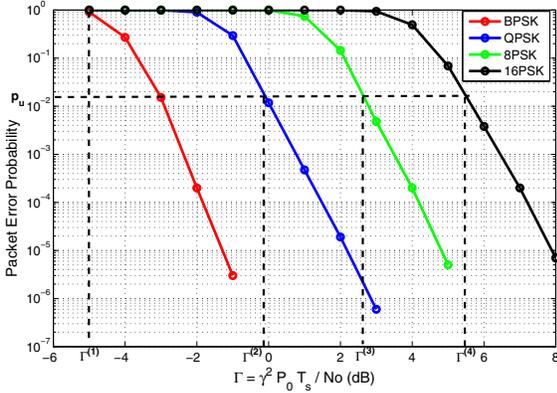


Fig. 2: Packet error rate vs. instantaneous received SNR.

in Fig. 2 are determined by  $p_u$  for each transmission as follows, where  $\Gamma^{(5)} = \infty$ :

$$\begin{aligned} \Gamma^{(1)} &= \Gamma_{2,\min}, \quad i = 1, \\ \Gamma^{(i)} &= \frac{1}{b_i} \ln\left(\frac{a_i}{p_u}\right), \quad i = 2, 3, 4. \end{aligned} \quad (8)$$

Then M-PSK is used when  $\Gamma^{(\log_2 M)} \leq \Gamma < \Gamma^{(\log_2 M + 1)}$ . Since  $N$  transmissions are allowed, there are  $N$   $p_u$ 's:  $p_{u1}, p_{u2}, \dots, p_{uN}$ , so the optimization problem can be formally stated as:

$$\begin{aligned} \min \quad & \bar{\mathcal{E}} \\ \text{s.t.} \quad & \bar{P}_e \leq P_{req} \\ & P = P_{max} \end{aligned} \quad (9)$$

where  $p_{u1}, p_{u2}, \dots, p_{uN}$  are the variables.

### B. Single Transmission (No ARQ)

Consider the case of only one transmission, i.e.,  $N = 1$ . The average PER is

$$\begin{aligned} \bar{P}_e &= E_{\{\Gamma\}}[P_e] = \int_0^\infty P_e(\Gamma) f(\Gamma) d\Gamma \\ &= \sum_{i=1}^4 \int_{\Gamma^{(i)}}^{\Gamma^{(i+1)}} \psi_{2^i}(\Gamma) f(\Gamma) d\Gamma \end{aligned}, \quad (10)$$

where  $P_e(\Gamma)$  is the conditional packet error probability, conditioned on  $\Gamma$ , and  $\psi_M(\Gamma)$  is the conditional packet error probability, conditioned on  $\Gamma$ , for M-PSK. That is,

$$P_e(\Gamma) = \psi_{2^i}(\Gamma) \quad \Gamma^{(i)} < \Gamma < \Gamma^{(i+1)}. \quad (11)$$

The average energy consumption is

$$\begin{aligned} \bar{\mathcal{E}} &= E_{\{\Gamma\}}[\mathcal{E}] = \int_0^\infty \mathcal{E}(\Gamma) f(\Gamma) d\Gamma \\ &= \sum_{i=1}^4 \int_{\Gamma^{(i)}}^{\Gamma^{(i+1)}} \frac{PT_s}{i} f(\Gamma) d\Gamma \end{aligned}, \quad (12)$$

where  $\mathcal{E}(\Gamma)$  is the energy consumption when the SNR is  $\Gamma$ , and equals

$$\frac{PT_s}{i} \quad \text{for} \quad \Gamma^{(i)} < \Gamma < \Gamma^{(i+1)}. \quad (13)$$

### C. HARQ

Let the maximum number of transmissions  $N = 2$ . Increasing the number of transmissions follows a similar analysis. Since the two transmissions of a chunk are independent, we have

$$f_{\Gamma_1, \Gamma_2}(\Gamma_1, \Gamma_2) = f_{\Gamma_1}(\Gamma_1) f_{\Gamma_2}(\Gamma_2) \quad (14)$$

where  $f_{\Gamma_1, \Gamma_2}(\Gamma_1, \Gamma_2)$  is the joint pdf of  $\Gamma_1$  and  $\Gamma_2$ . Suppose the SNR boundaries for the first and second transmissions are  $(\Gamma_1^{(1)}, \Gamma_1^{(2)}, \Gamma_1^{(3)}, \Gamma_1^{(4)}, \Gamma_1^{(5)})$  and  $(\Gamma_2^{(1)}, \Gamma_2^{(2)}, \Gamma_2^{(3)}, \Gamma_2^{(4)}, \Gamma_2^{(5)})$ , respectively, where  $\Gamma_1^{(1)} = \Gamma_2^{(1)} = \Gamma_{2,\min}$ ,  $\Gamma_1^{(5)} = \Gamma_2^{(5)} = \infty$ .

The average overall PER is

$$\begin{aligned} \bar{P}_e &= E_{\Gamma_1, \Gamma_2}[P_e] = \int_0^\infty \int_0^\infty P_e(\Gamma_1, \Gamma_2) f_{\Gamma_1, \Gamma_2}(\Gamma_1, \Gamma_2) d\Gamma_1 d\Gamma_2 \\ &= \int_0^\infty p_{e1} f(\Gamma_1) d\Gamma_1 \int_0^\infty p_{e2} f(\Gamma_2) d\Gamma_2 \\ &= \bar{P}_{e1} \bar{P}_{e2} \end{aligned} \quad (15)$$

where  $p_{ei}(\Gamma)$  is the conditional packet error probability, conditioned on  $\Gamma$ , for the  $i$ -th transmission of a chunk, and  $\bar{P}_{ei}$  is the average PER in the  $i$ -th transmission. With the alphabet size mapping in Section III-A, we have

$$p_{e1}(\Gamma) = \psi_{2^i}(\Gamma) \quad \Gamma_1^{(i)} < \Gamma < \Gamma_1^{(i+1)}, \quad i = 1, 2, 3, 4, \quad (16)$$

The average overall energy consumption is

$$\begin{aligned} \bar{\mathcal{E}} &= E_{\Gamma_1, \Gamma_2} E_{X_1}[\mathcal{E}] \\ &= E_{\Gamma_1, \Gamma_2} [\mathcal{P}(X_1 = 1) \cdot \mathcal{E}|_{X_1=1} + \mathcal{P}(X_1 = 0) \cdot \mathcal{E}|_{X_1=0}] \\ &= \int_0^\infty \int_0^\infty [e_1(\Gamma_1) + p_{e1}(\Gamma_1) e_2(\Gamma_2)] f_{\Gamma_1, \Gamma_2}(\Gamma_1, \Gamma_2) d\Gamma_1 d\Gamma_2 \\ &= \bar{\mathcal{E}}_1 + \bar{P}_{e1} \bar{\mathcal{E}}_2 \end{aligned} \quad (17)$$

where

$$e_j(\Gamma) = \frac{PT_s}{i} \quad \Gamma_j^{(i)} < \Gamma < \Gamma_j^{(i+1)}, \quad i = 1, 2, 3, 4, \quad j = 1, 2. \quad (18)$$

So far we have described how to calculate  $\bar{\mathcal{E}}$  and  $\bar{P}_e$ , given the variables  $p_{u1}$  and  $p_{u2}$ . However, this problem is not convex on  $p_{u1}$  and  $p_{u2}$ . To obtain an accurate solution, we solve the optimization problem with an exhaustive search, which is described in Section IV.

The case of multiple alphabet sizes with variable transmit power follows a similar analysis, except that two variables,  $P_1$  and  $P_2$ , which are the transmit powers for the two transmissions, are considered. The case of single alphabet size with variable transmit power was studied in [4, 5].

## IV. NUMERICAL RESULTS

We want to solve eq. (9) when  $N = 2$  or 3, and to do this, we exhaust every combination of  $p_{u1}, p_{u2}$  and determine the overall average energy consumption  $\bar{\mathcal{E}}$  and overall average PER  $\bar{P}_e$  for this combination. We just need to pick the combination  $p_{u1}, p_{u2}$  which satisfies the PER constraint and has the least energy consumption. However, because  $p_{u1}, p_{u2}$  are continuous, we cannot exhaust every combination of  $p_{u1}, p_{u2}$ ,

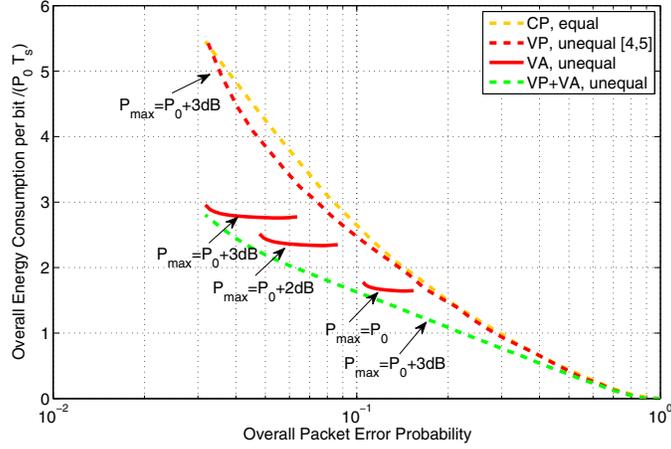


Fig. 3: Energy consumption comparison.

so we take discrete samples of  $p_{u1}, p_{u2}$ . In the simulation, the step size for  $\log_{10}(p_{u1})$  and  $\log_{10}(p_{u2})$  is 0.02.

Fig. 3 compares the proposed scheme to the existing schemes [4, 5]. The average SNR  $E[\Gamma]$  is 0dB. Both the orange dashed and red dashed are the comparison schemes and always use BPSK, whereas the power is constant across the transmissions for the orange dashed curve, which is denoted by “equal”, and power is allowed to vary with transmissions for the red dashed curve, which is denoted by “unequal”. There is a small gain for using unequal power allocation for the two transmissions. At a PER of  $5 \times 10^{-2}$ , the energy difference is about 10%. The red solid curves correspond to constant power and variable alphabet size scheme, and different curves represent different constant powers. The term “unequal” means  $p_u$  is allowed to be different for the transmissions. At a PER of  $5 \times 10^{-2}$ , energy is saved by about 36% relative to “VP, unequal”. These curves span a small extent along the x-axis because they correspond to constant power, and the only way to change the PER is by changing  $p_u$ , i.e., the alphabet size mapping strategy. Since the influence of  $p_u$  on the PER is not as the power, the range of PER is not large. The green dashed curve uses variable power and variable alphabet size, and both power and  $p_u$  are allowed to vary with each transmission. At a PER of  $5 \times 10^{-2}$ , the “VP+VA, unequal” reduces the energy consumption by 40% relative to “VP, unequal” and 9% relative to “VA, unequal”. This scheme has the advantages of both “VP, unequal” and “VA, unequal”; it gives a wide range of PER outcomes and low energy consumption.

Figs. 4 and 5 show the energy consumption and PER for the two transmissions for “VP+VA, unequal”. The first transmission has lower energy consumption and higher PER than the second transmission. If there are two transmission opportunities, the first transmission gets sent at a low cost. If it happens to succeed, then the second transmission is not necessary and energy consumption is low. However, if the first transmission fails, we need to expend additional resources on the second transmission to guarantee the PER.

For the “VP+VA, unequal” strategy, Fig. 6 compares energy

results for three different values of FEC rate, and for three different values of  $N$  (maximum number of transmissions). For a given FEC rate, a larger  $N$  has better performance because of the diversity in the transmissions. For a given  $N$ , a lower FEC rate has better performance due to the coding gain.

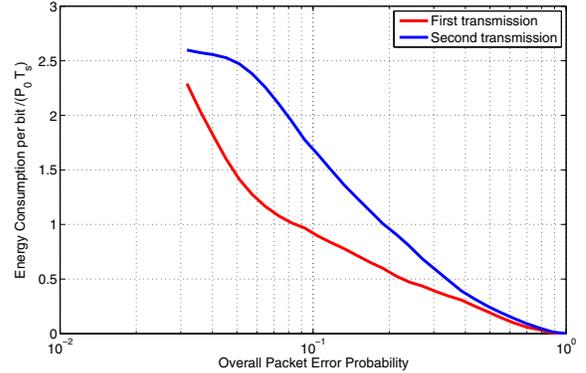


Fig. 4: Energy for the two transmissions.

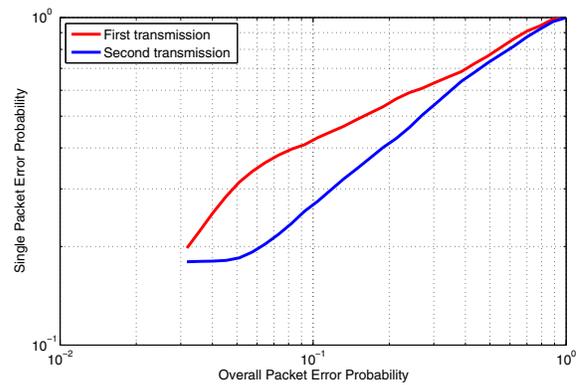


Fig. 5: Single transmission PER for the two transmissions.

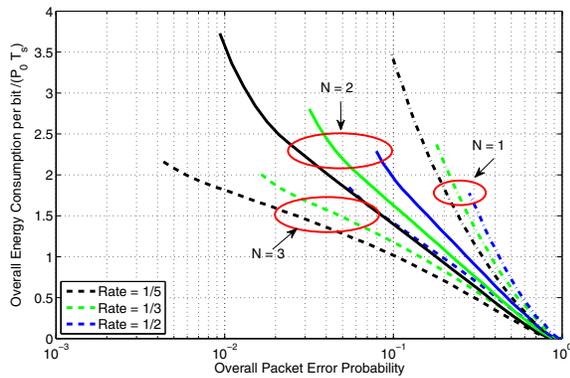


Fig. 6: Comparison among different parameters.

## V. CONCLUSIONS

In this paper, we presented a cross-layer scheme which combines HARQ and AMC. We optimized the energy allocated to retransmissions to minimize the energy consumption, subject to a packet error probability constraint. We compared the proposed schemes to the conventional HARQ schemes. Numerical results demonstrated that the unequal upper bound and power allocation can reduce energy consumption. Our future work includes extending the maximum number of transmissions and applying the results to other data types, such as video.

## ACKNOWLEDGEMENT

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