Variance-Aware Adaptive Modulation for OFDM-based Multiple Description Progressive Image Transmission

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Abstract—Average throughput maximization has commonly been used as an alternative to image or video average distortion minimization. However, as the throughput-distortion curve is non-linear, throughput maximization can be very different from distortion minimization if the variance of the throughput is large. Under high channel quality variability, the variance of the throughput will be high. Multiple description coding with unequal error protection is a promising technique for the transmission of progressive images when there exists a high variability of the channel conditions. In this paper, we consider both the variance and the average of the throughput when deciding the constellation size for adaptive modulation in an OFDM system used for transmitting progressively-coded images with multiple description coding. Simulation results show that in an adaptive modulated image transmission system, taking into account both moments of the throughput achieves better performance than a system considering average throughput alone.

Index Terms—Adaptive modulation, cross layer design, multiple description coding, Orthogonal Frequency Division Multiplexing (OFDM), progressive image transmission, variance.

I. INTRODUCTION

With limited spectrum available, there is a growing demand for higher spectrum efficiency in the mobile device industry, and thus a great interest in wireless resource optimization techniques. Adaptive modulation is one technique to increase spectrum efficiency, as rate can be allocated to take advantage of favorable channel conditions.

In this paper, we investigate the switching-threshold of adaptive modulation by taking into account both moments of the throughput for image transmission. Specifically, we consider an OFDM system used for transmitting progressively-coded images with multiple description coding [1]. Each description is mapped into one of the subcarriers of the OFDM waveforms. Contrary to the temporal coding seen in most of the literature [2], Reed Solomon (RS) erasure coding is used

across the subcarriers (descriptions) [3] to provide unequal error protection for the descriptions. A cyclic redundancy check (CRC) is used to check the validity of each description, and erase descriptions that do not pass the CRC.

In most of the literature, either spectral efficiency maximization meeting a specified bit error rate (BER) [4, 5] or throughput maximization [6] has been widely used to decide on the constellation size. It is well known that the throughput for a given constellation size tends to flatten out for large SNR. Switching to a higher constellation size at the appropriate threshold achieves higher throughput. The switching thresholds are the points where the throughput of the lower choice of constellation size, 2^{2n} cross with those of the higher choice of constellation size, $2^{2(n+1)}$.

Particularly for image or video transmission, throughput maximization has been used as an alternative to distortion minimization [7, 8] due to its simplicity. When instantaneous channel quality information is perfectly known, throughput maximization is close to distortion minimization. However, in scenarios where the variance of the throughput is large, throughput maximization can be very different from distortion minimization. Generally, throughput variance will be high when large channel quality variability is present. Hence, in this paper, we propose new objective function that considers both the variance and the average throughput for determining the switching thresholds of adaptive modulation.

To achieve minimal distortion for the FEC-based multiple description block, we need to optimize the constellation sizes and code rates jointly. However, due to the complexity of joint optimization, we decompose the problem into two sub-problems. First, we decide the constellation size of each subcarrier based on the throughput prior to RS decoding, then we decide the code rates to minimize distortion.

The remainder of this paper is organized as follows: Section II outlines the channel and system models. Section III presents the RS unequal error protection framework. Section

IV presents the adaptive modulation based on throughput maximization and the proposed variance-aware method for switching thresholds. Section V presents results and conclusions.

II. CHANNEL AND SYSTEM MODELS

A. Channel Model

In a frequency-selective OFDM channel, the entire frequency band of B_T Hz containing L subcarriers is assumed to be divided into N_c independent channels (blocks). Each of the N_c independent channels has bandwidth approximately equal to the coherence bandwidth of the channel, Δf_c, and consists of M_c identically correlated subcarriers. We assume that Δf_c is greater than or equal to the signal bandwidth of an individual subcarrier, and thus each subcarrier experiences flat fading. Furthermore, we assume Rayleigh fading that is sufficiently slow so that the fade remains constant over the length of a packet (description).

B. System Model

The system model is as follows [3]:

1) An embedded bitstream is converted into L descriptions using an FEC-based multiple description coder as shown in Fig. 1.
2) Reed-Solomon (RS) encoding is used across the descriptions to provide unequal error protection for the bistream. For a progressively encoded bitstream, the data is progressively less important in going from the beginning of the data stream to the end of the data stream. Thus, for unequal error protection, the rates of the RS codes are non-decreasing when used to protect the progressive bistream from the most important segment of data stream to the least important segment of data stream.
3) A cyclic redundancy check (CRC) is appended to each description for error detection.

Particularly, this paper considers the system where only the average channel state information of each subcarrier is known at the transmitter. For adaptive modulation, square M-QAM modulation is assumed in which the constellation size of M is restricted to 2^η, where η is an even number varying from 2 to N_b. This paper assumes that $N_b = 6$ is the largest number of bits/symbol, thus the resulting constellation choices are 4-QAM, 16-QAM and 64-QAM. We use a $GF(2^{10})$ RS code so that each RS code symbol contains 10 bits. Five 4-QAM, 16-QAM and 64-QAM modulated symbols are grouped as one, two, and three RS code symbols, respectively. Details about the mapping from modulated symbols to RS code symbols can be found in [8].

III. RS UNEQUAL ERROR PROTECTION

For the transmission of a progressively coded image with multiple descriptions, an embedded bitstream is first converted into L descriptions using multiple description coding. Descriptions for an embedded bitstream are then protected by J RS codewords with different code rates. Fig. 1 illustrates
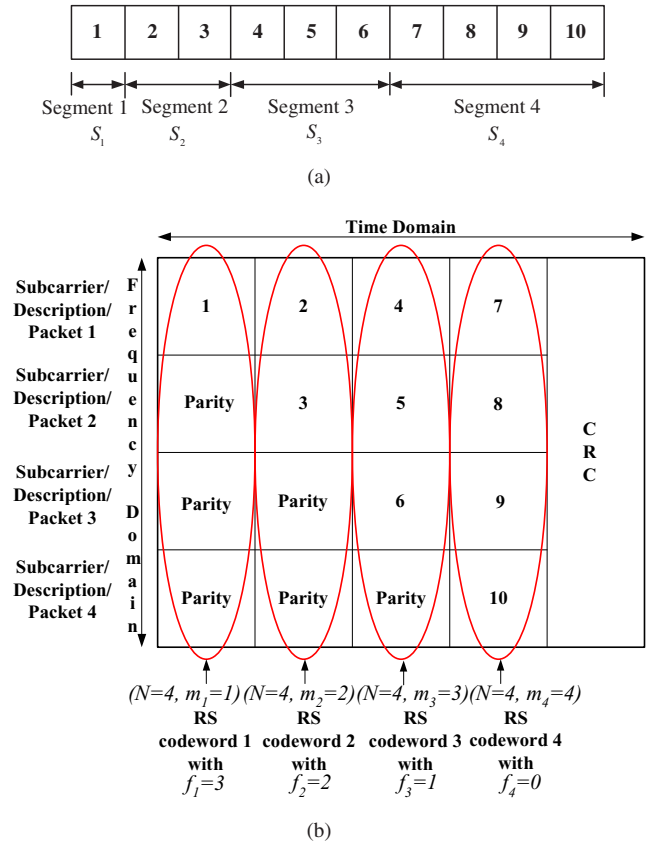

Fig. 1. An embedded bitstream is converted into multiple descriptions with $L = 4$ descriptions and $J = 4$ RS codewords. (a) An embedded bitstream partitioned into 4 segments. (b) Mapping of the descriptions to RS codewords.

a mechanism for mapping an embedded bitstream from a source encoder into multiple descriptions with unequal error protection. For a progressively coded source, prefixes of the bitstream can be used to reconstruct the source with a certain fidelity. However, any error along the bitstream will result in the subsequent bits being useless. As shown in Fig. 1, each codeword contains a segment ($S_j, j \in [1, J]$) of the information stream. Each segment consists of $m_j \in \{1, 2, ..., N\}$ source symbols. We let one source symbol correspond to one RS code symbol.

Let $f_j = N - m_j$ denote the number of RS parity symbols that protects segment S_j with the constraints $f_1 \geq f_2 \geq ... \geq f_J$ and $f_j \in \{1, 2, ..., N-1\}$. If no more than f_j RS code symbols are erased, then the receiver can recover at least the first j segments.

For a given $\underline{F} = (f_1, ..., f_J)$, the distortion, $D(\underline{F})$ can be written in a form given by [9]

$$D(\underline{F}) = \sum_{j=0}^{J} P_j(\underline{F}) \phi(T_j(\underline{F})) \tag{1}$$

where

$$P_j(\underline{F}) = \begin{cases} \text{Prob}(X > f_1) & j = 0 \\ \text{Prob}(f_{j+1} < X \leq f_j) & j = 1, ..., J-1 \\ \text{Prob}(X \leq f_J) & j = J, \end{cases} \tag{2}$$

X is the number of erased RS code symbols, ϕ represents the operational rate-distortion function of the source encoder, and $T_j(\underline{F}) = \sum_{k=1}^{j} m_k$ is the number of source symbols in the first j segments. Note that $T_0(\underline{F}) = 0$, and thus $\phi(T_0(\underline{F})) = \phi(0)$ corresponds to the distortion when no transmitted source symbol is decoded correctly.

The optimization goal for FEC is to determine the set of RS parity assignments that minimize the average distortion shown in (1). Given the operational distortion-throughput function $\phi(T_j(\underline{F}))$, the problem is as follows:

$$D^* = \min_{\underline{F} \in F} \{D(\underline{F})\} = \min_{\underline{F} \in F} \left\{ \sum_{j=0}^{J} P_j(\underline{F}) \phi(T_j(\underline{F})) \right\} \quad (3)$$

where F denotes the set of J-tuples $(f_1, f_2, ..., f_J)$ such that $f_1 \geq f_2 \geq ... \geq f_J$. To find the optimal FEC assignment for the RS codewords, we adopted the hill climbing approach proposed in [10]. Note that the degree of unequal error protection increases as the number of independent channels, N_c decreases [3].

IV. ADAPTIVE MODULATION

Adaptive modulation requires determining the switching thresholds for changing the constellation sizes. We first present threshold switching based on average throughput maximization. Next, we propose an improved method which considers the variance as well as the average throughput.

A. Adaptive Modulation Based on Average Throughput Maximization

In order to maximize the system throughput prior to RS decoding, this scheme varies the constellation size at each subcarrier according to the channel conditions (i.e., maximizing the average number of successfully received bits from all subcarriers). The system throughput prior to RS decoding, T_{uc}, can be written as

$$T_{uc} = \sum_{l=1}^{L} \Gamma_{l,M}(\overline{\gamma}) \quad (4)$$

where $\Gamma_{l,M}(\overline{\gamma})$ is the average throughput for the lth subcarrier prior to RS decoding with a constellation size of M, L is the total number of subcarriers, and $\overline{\gamma}$ is the average SNR.

Let μ denote the number of unique constellation choices. The average SNR ($\overline{\gamma}$) range is divided into μ regions. When the average SNR falls into a particular region, the constellation size associated with that region is transmitted. The region boundaries $\{\overline{\gamma}_b\}$, $b = 0, 2, 4, ... N_b$ are determined from the function $\Gamma_{l,M}(\overline{\gamma})$, which is given by

$$\Gamma_{l,M}(\overline{\gamma}) = \int (1 - PER_{l,M}(\gamma))((\log_2 M) z) f_\gamma(\gamma) d\gamma \quad (5)$$

where $\log_2 M$ is the number of bits in one symbol, z is the total number of modulated symbols in one packet, and $f_\gamma(\gamma)$ is the probability density function of SNR (γ). $PER_{l,M}(\gamma)$ in (5) is the conditional packet error rate of subcarrier l, conditioned on γ, for a constellation size of M. Note that packet error rate is synonymous with the description loss probability, and is given by

$$PER_{l,M}(\gamma) = 1 - (1 - SER_{l,M}(\gamma))^z. \quad (6)$$

In (6), $SER_{l,M}(\gamma)$ is the conditional symbol error probability of subcarrier l, conditioned on γ. For a Gray-encoded M-QAM square constellation [11], this conditional symbol error probability is given by (7).

In this paper, we consider the scenario where the transmitter only knows the average channel state information of each subcarrier. The probability density function of the SNR in (5) is modelled as a Chi Square distribution with two degrees of freedom [11]:

$$f_\gamma(\gamma) \triangleq \frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}}. \quad (8)$$

Fig. 2 depicts the average throughput of one subcarrier prior to RS decoding, $\Gamma_{l,M}(\overline{\gamma})$, for $M = 4, 16, 64$. The boundaries of the switching thresholds $\{\overline{\gamma}_b\}$, $b = 0, 2, 4, .., N_b$ are the points where the throughput of the two adjacent constellations cross from constellation size of 2^{2n} to constellation size of $2^{2(n+1)}$. If the average SNR ($\overline{\gamma}$) of a particular subcarrier is in the region bounded by $\overline{\gamma}_b$ and $\overline{\gamma}_{b+2}$, then the subcarrier will be assigned a constellation size of $M = 2^{b+2}$.
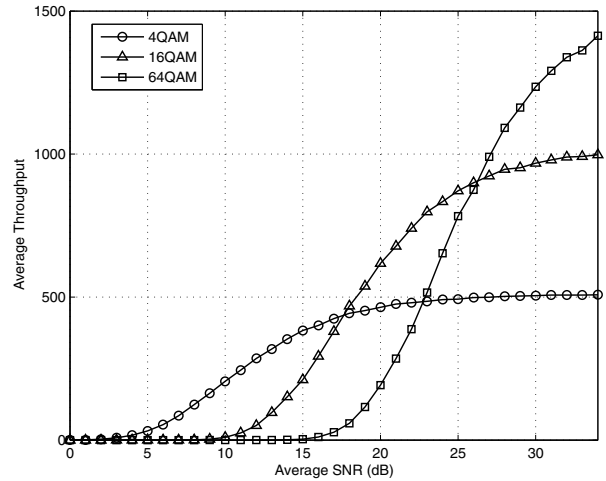

Fig. 2. Average throughput of one subcarrier prior to RS decoding, $\Gamma_{l,M}(\overline{\gamma})$, for M=4, 16, 64.

$$SER_{l,M}(\gamma) = 1 - \left(1 - \frac{4(\sqrt{M} - 1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\gamma}{M-1}}\right) + \left[\frac{2(\sqrt{M} - 1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\gamma}{M-1}}\right) \right]^2 \right) \quad (7)$$

B. Adaptive Modulation Based on Proposed Variance Aware Method

If throughput and distortion were related linearly, then maximizing average throughput would be the same as minimizing average distortion. Since in general their relationship is not linear, maximizing average throughput can be very different from minimizing average distortion if the variance of the throughput is large caused by high channel quality variability. We would therefore like to modify the objective function used in the previous subsection as it considers only the average throughput in determining the switching thresholds.

We first describe the motivation that we used to design a new cross-layer objective function for switching-threshold determination. The rate distortion function for an $N\left(0, \sigma^2\right)$ Gaussian source with a squared-error distortion is given by [12]

$$R\left(D\right) = \begin{cases} \frac{1}{2}\log\frac{\sigma^2}{D} & 0 \le D \le \sigma^2 \\ 0 & D > \sigma^2 \end{cases} \quad (9)$$

where D is the distortion and $R(D)$ is the rate. We can rewrite the above equation to express the distortion in terms of the rate as

$$D\left(R\right) = \sigma^2 2^{-2R}. \quad (10)$$

Without loss of generality, σ^2 can be set to unity. Note that R in (10) is the source rate, i.e., the number of bits used to represent one pixel. If we define the horizontal and vertical dimensions of an image to be Y and Z pixels, respectively, then (10) can be rewritten as

$$D\left(T\right) = 2^{-\frac{2}{Y \times Z}T} \quad (11)$$

where T is the number of successfully decoded source bits sequentially from the beginning of the data stream. Since T is random, assume, for simplicity, that its pdf is that of a truncated Gaussian. That is, let

$$f_T\left(T\right) = \frac{1}{1 - \phi\left(-\frac{\mu_T}{\sigma_T}\right)} \frac{1}{\sqrt{2\pi}\sigma_T} e^{-\frac{(T-\mu_T)^2}{2\sigma_T^2}} \quad (12)$$

where

$$\phi\left(x\right) \triangleq \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy. \quad (13)$$

Then, the average distortion, D_{ave}, is given by

$$\begin{aligned} D_{ave} &= \int_0^{\infty} 2^{-\frac{2}{Y \times Z}T} \frac{1}{1 - \phi\left(-\frac{\mu_T}{\sigma_T}\right)} \frac{1}{\sqrt{2\pi}\sigma_T} e^{-\frac{(T-\mu_T)^2}{2\sigma_T^2}} dT \\ &= \int_0^{\infty} e^{-\frac{2\ln 2}{Y \times Z}T} \frac{1}{1 - \phi\left(-\frac{\mu_T}{\sigma_T}\right)} \frac{1}{\sqrt{2\pi}\sigma_T} e^{-\frac{(T-\mu_T)^2}{2\sigma_T^2}} dT \\ &= \frac{1}{1 - \phi\left(-\frac{\mu_T}{\sigma_T}\right)} e^{-\frac{2\ln 2}{Y \times Z}\left(\mu_T - \frac{\ln 2}{Y \times Z}\sigma_T^2\right)} \\ &\quad \times \left(1 - \phi\left(-\frac{\mu_T - \frac{2\sigma_T^2 \ln 2}{Y \times Z}}{\sigma_T}\right)\right). \end{aligned} \quad (14)$$

If $\frac{\mu_T}{\sigma_T}$ is large, then $\phi\left(-\frac{\mu_T}{\sigma_T}\right) \ll 1$, and the above average distortion can be approximated as

$$D_{ave} \approx e^{-\frac{2\ln 2}{Y \times Z}\left(\mu_T - \frac{\ln 2}{Y \times Z}\sigma_T^2\right)}. \quad (15)$$

To minimize (15), we need to maximize

$$\mu_T - \left(\frac{\ln 2}{Y \times Z}\right)\sigma_T^2. \quad (16)$$

By analogy with (16), to determine the switching thresholds, we propose to use the objective function of

$$\text{MAX} : \left\{\Gamma_{l,M}\left(\overline{\gamma}\right) - K\widetilde{\sigma}_{\Gamma_{l,M}}^2\right\} \quad (17)$$

where $\Gamma_{l,M}\left(\overline{\gamma}\right)$ is the average throughput of subcarrier l prior to RS decoding defined in (5), and $\widetilde{\sigma}_{\Gamma_{l,M}}^2$ can be shown to be given by

$$\widetilde{\sigma}_{\Gamma_{l,M}}^2 = \frac{\sigma_{\Gamma_{l,M}}^2}{N_c}. \quad (18)$$

In (18), $\sigma_{\Gamma_{l,M}}^2$ denotes the variance of the throughput of subcarrier l. When all the channels are correlated, $N_c = 1$ and $\widetilde{\sigma}_{\Gamma_{l,M}}^2$ is the largest, whereas when all the channels are independent, $N_c = L$, which makes the $\widetilde{\sigma}_{\Gamma_{l,M}}^2$ L times smaller. Note that we are not actually assuming that the throughput has a truncated Gaussian distribution. We are simply using the result of (16) to motivate the functional form of combining the mean and the variance of throughput. In (17), K is a scaling factor in units of subcarriers per bit. Note that when $K = 0$, the objective function becomes the average throughput maximization described in Section IV.A.

V. SIMULATION RESULTS AND DISCUSSION

In this section, simulations are performed on a 128×128 grey scale image, and the embedded bitstream is obtained by encoding the image using the SPIHT [13] algorithm. To evaluate image quality, we use a peak-signal-to-noise ratio (PSNR) performance metric defined as

$$\text{PSNR} = 10\log\frac{(255)^2}{D^*}\text{dB}. \quad (19)$$

In the above equation, D^* is the distortion, as shown in (3). The parameters for the simulations are as follows: $L = 16$ is the number of subcarriers, $z = 255$ is the number of symbols in one subcarrier, and there are $J = 51$ RS codewords.

We first concentrate on the case where all the subcarriers are correlated, i.e., $M_c = L = 16$, in which case the throughput variance is the largest. We compare the PSNR of the transmitted image using the average throughput maximization and using the variance-aware method. Simulations for different test images, i.e., 'Shuttle', 'Goldhill', 'Lena' and 'Tiffany', have been carried out. As can be seen from Fig. 3, these images have different degrees of complexity. However, we observed similar trends for all of these images. Hence, we only present the result for 'Shuttle' (the most complex image among these four images) in Fig. 4, and for 'Lena' in Fig. 5. With the same

number of successfully received bits, 'Shuttle' has the most distortion and the lowest PSNR among the four test images, approximately 8dB below the least complex image ('Tiffany').

As observed in Fig. 4 and Fig. 5, the variance-aware method outperforms the average throughput maximization method by as much as 4dB, which is very significant visually. Note that a dramatic drop in PSNR for average throughput maximization occurs when switching from 4-QAM to 16-QAM, and from 16-QAM to 64-QAM. This performance decline occurs when changing to a higher constellation size at a SNR value lower than the optimal threshold. The result suggests that when only the average channel state information is available, the threshold for switching to a higher constellation should be more conservative, which is what our proposed variance-aware method does.

Consider now how the optimal K value changes with different types of images. We observed that the optimum K values for these four images are $K_{opt} = 0.005$ for 'Shuttle', $K_{opt} = 0.006$ for 'Goldhill', $K_{opt} = 0.007$ for 'Lena' and $K_{opt} = 0.007$ for 'Tiffany'. Fig. 4 and 5 present the PSNR for an optimum K with the range of SNR from 10 to 42dB. To better understand the sensitivity of K, Fig. 6 is plotted. In Fig. 6, we take the average difference between the PSNR with the optimum K (as shown in Fig.4 for 'Shuttle' and Fig.5 for 'Lena') and the PSNR with a K ranging from 0 to 0.016. The range of K was chosen to be approximately twice the optimum K value. For each value of K, the average difference in PSNR is defined as

$$\Delta = \frac{\sum_{i=1}^{N} |f_i(K_{opt}) - f_i(K)|}{N} \quad (20)$$

where N is the total number of SNR points in the PSNR curve used to get the difference. Specifically, we use $N = 17$ for SNR ranging from 10dB to 42dB with 2dB intervals. The plots suggest that the test images are not very sensitive to the optimal value of K. Results from slight variations (± 0.003) in K_{opt} are still close to the optimal performance.

The previous plots (Fig. 4 through Fig. 6) are for cases where all the 16 subcarriers are identically correlated; the subsequent three figures examine the performance of the 'Lena' image with eight correlated subcarriers (Fig. 7), four correlated subcarriers (Fig. 8) and all independent subcarriers (Fig. 9). The optimum K for 'Lena' obtained from the case of fully correlated subcarriers, $K_{opt} = 0.007$, is used for the simulations of the cases of eight correlated subcarriers, four correlated subcarriers and all independent subcarriers. Again, the performance of the variance-aware method is superior to the average throughput maximization method. However, we observe that the gain decreases with reduced number of correlated channels. This is because the throughput variance decreases with fewer correlated channels.

In conclusion, this paper identifies the throughput variance as an important factor for determining the constellation size switching threshold. Large throughput variance caused the av-erage throughput maximization results to diverge from average distortion minimization ones. This paper provides an objective function which considers both the average and the variance of the throughput in determining the switching threshold. For adaptive modulation in progressive image transmission with multiple description coding, simulation results suggest that our variance-aware method increases the system performance significantly.

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(a)　　　　　　(b)

(c)　　　　　　(d)

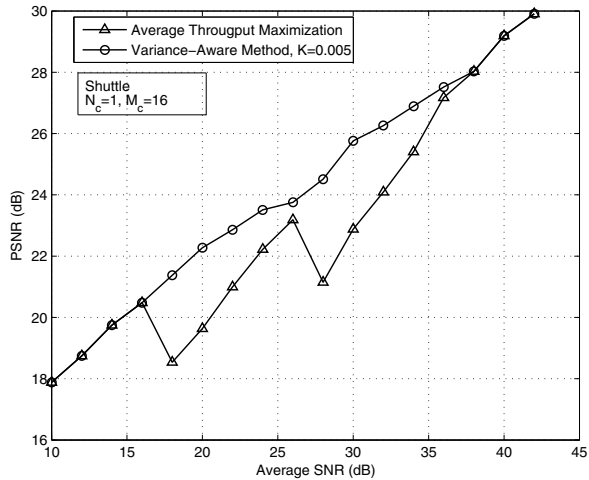Fig. 3. Test images (a) Shuttle, (b) Goldhill, (c) Lena, and (d) Tiffany.

Fig. 4.   PSNR plot of 'Shuttle' image for average throughput maximization method and variance-aware method, $N_c = 1$, $M_c = 16$.
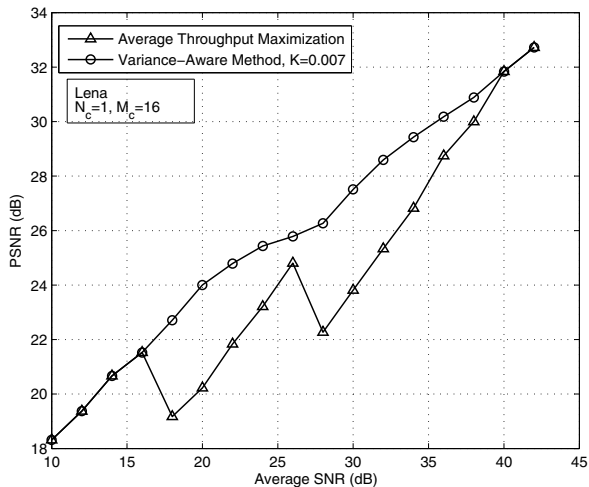

Fig. 5.   PSNR plot of 'Lena' image for average throughput maximization method and variance-aware method, $N_c = 1$, $M_c = 16$.
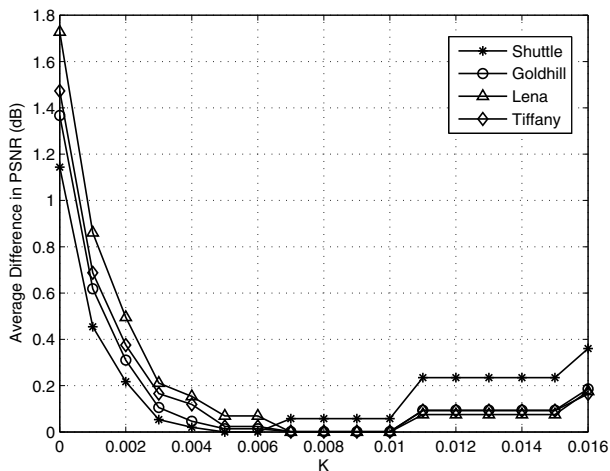

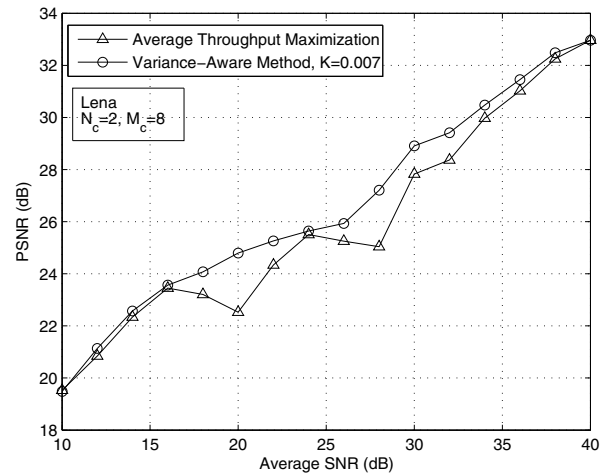Fig. 6.   Average difference in PSNR (dB), Δ versus different value of K.

Fig. 7.   PSNR plot of 'Lena' image for average throughput maximization method and variance-aware method with 8 subcarriers correlated, $N_c = 2$, $M_c = 8$.
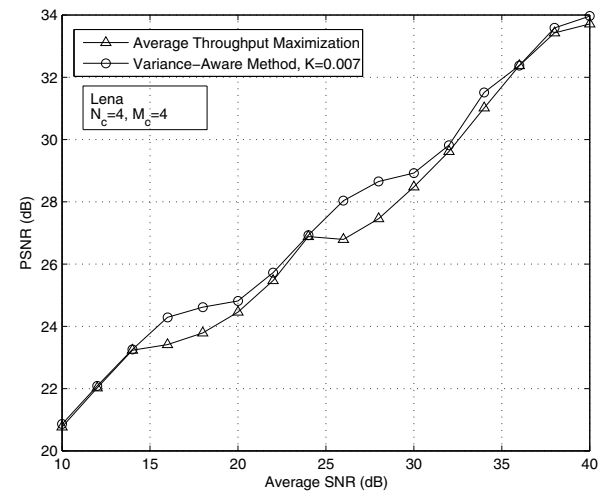

Fig. 8.   PSNR plot of 'Lena' image for average throughput maximization method and variance-aware method with 4 subcarriers correlated, $N_c = 4$, $M_c = 4$.
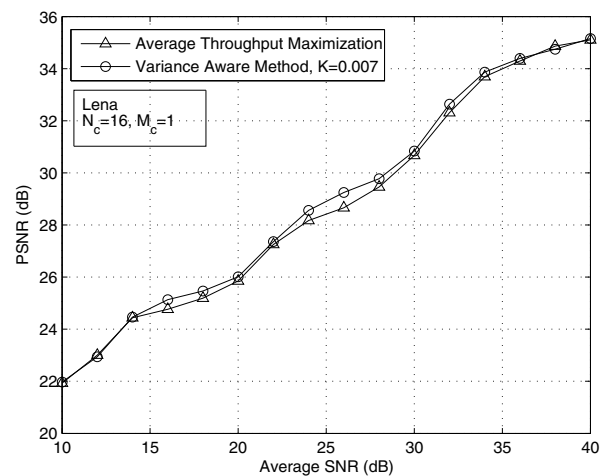

Fig. 9.   PSNR plot of 'Lena' image for average throughput maximization method and variance-aware method with all subcarriers independant, $N_c = 16$, $M_c = 1$.