Tradeoff Between Source Coding and Spreading in a Multicarrier DS-CDMA System Operating over Multipath Fading Channels with Narrow-Band Interference

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***Abstract*— For a fixed total bandwidth expansion factor, and for a fixed channel code rate, we consider the problem of optimal bandwidth allocation between the source coder and the spread-spectrum unit for a multicarrier direct-sequence CDMA system operating over a frequency-selective fading channel with narrow-band interference. Assuming a Gaussian source with the optimum scalar quantizer and a binary convolutional code with soft-decision decoding, we obtain both a lower and an upper bound on the end-to-end average source distortion. The optimal bandwidth allocation is then numerically computed by minimizing upper and lower bounds on the average distortion. We show that the upper bound based cost function is a convex function of the source code rate, and the optimal allocation depends on the system and the channel conditions, such as the total number of active users, the number of carriers, and the average jammer-to-signal power ratio.**

Keywords: Source coding, multicarrier CDMA, bandwidth constraint, cross-layer optimization

I. Introduction

Direct-sequence CDMA (DS-CDMA) technology is popular due to its robustness against channel fading, capability to suppress intentional/unintentional narrow-band interference (NBI) and multiple access capacity [1]. For providing wide-band multimedia services with non-availability of contiguous frequency spectrum, and for the purpose of overlaying a CDMA system on existing narrow-band systems, a multicarrier version of the traditional DS-CDMA systems can be realized by employing more than one carrier [2]. Studies have shown that, with hostile NBI, multicarrier CDMA (MC-CDMA) systems with efficient channel coding can provide improved system performance relative to their single carrier counterparts [3]- [4]. However, for a fixed total system bandwidth, transmission of high quality source information competes for the available bandwidth with channel coding and spreading. This motivates us to study the tradeoffs involved among source coding, channel coding and spreading in a multicarrier DS-CDMA system.

We now briefly review the related previous work. In [5]-[7], an information theoretic approach is taken to investigate the tradeoffs between source and channel coding. In [8] and [9], the tradeoff between coding and spreading is investigated for a spread-spectrum system. Using system level simulations, in [10], Zhao *et al.* studied the problem of optimal bandwidth allocation among source coder, channel coder, and spread-spectrum modulator for progressive transmission of images over frequency-selective fading DS-CDMA channel with MAI. Recently, in [11], an analysis was presented for the optimal bandwidth allocation on AWGN and flat Rayleigh fading CDMA channels with both block coding with hard-decision decoding and convolutional coding with soft-decision decoding.

In this paper, we study the bandwidth allocation problem between the source coding and spreading, for a fixed channel code rate, when a MC-CDMA system is used in the presence of NBI. We assume a Gaussian source with the optimum scalar quantizer and a binary convolutional code with soft-decision decoding. The NBI is a modeled as a Gaussian distributed partial-band interferer (PBI). First, using a standard Gaussian approximation for the MAI, we obtain upper and lower bounds on the pairwise error probability (PEP) with soft-decision decoding, and we use these results to bound the end-to-end average source distortion. The optimal bandwidth allocation is then numerically computed by minimizing upper and lower bounds on the average distortion. We show that the upper-bound based cost function is a convex function of the source code rate, and the optimal allocation depends on the system and the channel conditions, such as the average jammer-to-signal power ratio (JSR), the number of carriers, and total number of active users.

The rest of this paper is organized as follows. In Section II, we introduce the system and the channel model, and derive upper and lower bounds on the PEP with soft-decision decoding. Analysis of the end-to-end average distortion is presented in Section III, whereas the optimum bandwidth allocation problem is detailed in Section IV. Numerical results and discussion are provided in Section V. Finally, we conclude our work in Section VI.

II. System Model

The transmitter-receiver pair for the kth user is shown in Fig. 1. The information source is quantized by the source encoder with a rate of r_s bits per source sample, which are then mapped onto a new bit index of the same length r_s using an index assignment block, whose output bit stream is denoted by $\{b_n^{(k)}\}$. As detailed in [13], the purpose of this latter block is to permute the indices so that those with small Hamming distance between them correspond to close quantization levels.

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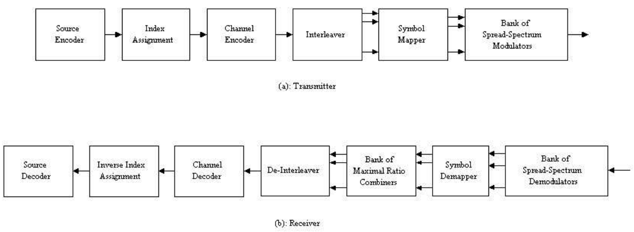

Fig. 1. Block Diagram of the transmitter-receiver pair for the desired user.

This way, the distortion caused by the most probable errors, and hence the total distortion, is kept minimum. For ease of analysis, similar to [13], [11], we assume a random index assignment with a one-to-one mapping of indices from 0 through $2^{r_s} - 1$.

Each bit $b_n^{(k)}$ is encoded by a convolutional code of rate r_c, and the resulting code symbols are interleaved. For the purpose of analysis, we assume an ideal interleaver. Each code symbol $d_n^{(k)}$ is then spread, binary phase modulated and transmitted over the M frequency bands, each of width W_1. An optional symbol mapper can be used for coding across the carriers, as studied by [3]. If T_c and W, respectively, denote the chip duration and system bandwidth of a comparable single carrier CDMA (SC-CDMA) system, then we have $W = (1+\beta)/T_c$, where $\beta \in (0,1]$ is the roll-off factor of the chip wave-shaping filter. The bandwidth available per carrier in a MC-CDMA system is then given by $W_1 = W/M = (1+\beta)/(T_c M) = (1+\beta)/T_{c_1}$, where $T_{c_1} = MT_c$ is the corresponding chip duration in MC-CDMA system.

Mathematically, the signal at the output of the kth user's transmitter can be written as

$$\begin{aligned} S_k(t) &= \sqrt{2E_c} \sum_{n=-\infty}^{\infty} d_{\lfloor n/N \rfloor}^{(k)} c_n^{(k)} h(t - nMT_c) \times \\ & \quad \sum_{m=1}^{M} \cos(2\pi f_m t + \theta_m^{(k)}), \end{aligned} \tag{1}$$

where $\lfloor x \rfloor$ is the largest integer that is less than or equal to x, $c_n^{(k)}$ denotes the spreading sequence, f_m is the center frequency of the mth carrier, $\theta_m^{(k)}$ denotes the initial phase angle of the kth user's mth carrier, N is the number of chips-per-code symbol-per-carrier, and E_c denotes the energy-per-chip. Also, $h(t)$ denotes the chip wave-shaping filter, and we assume that $X(f) = |H(f)|^2$ satisfies the Nyquist criterion, where $H(f)$ is the Fourier transform of $h(t)$. If we denote by $\mathcal{S}_F$ the spreading factor associated with a single carrier, then we have $\mathcal{S}_F = T_s/T_c = MT_s/T_{c_1} = MN$, where T_s is the code symbol duration. With this, we can express N as $N = \mathcal{S}_F/M$.

We assume that the channel is frequency-selective over a bandwidth of W. However, the total bandwidth W is assumed to be partitioned into M disjoint frequency bands in such a way that each of the M bands experiences independent, frequency-flat fading. In [2], conditions were derived for satisfying this assumption. With this, the received signal

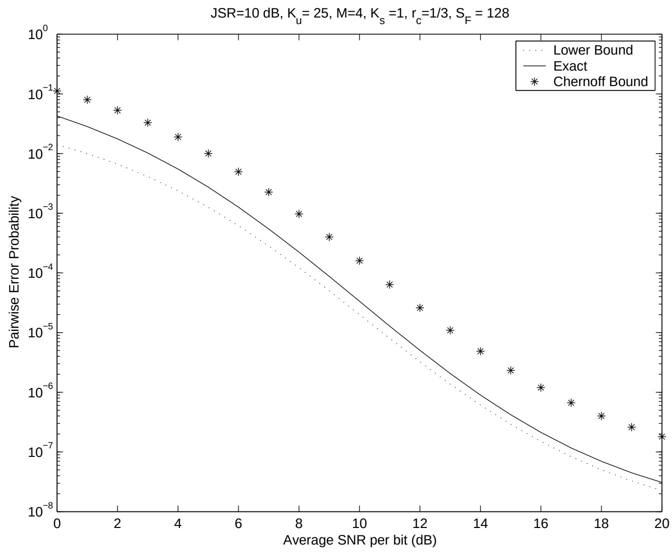

Fig. 2. Comparison of exact pairwise error probability with the one based on the Chernoff bound and the one based on a lower bound. The system parameters are the following: $K_u = 25$, JSR = 10 dB, $\rho_J = 0.25$, $r_c = 1/3$, $\mathcal{S}_F = 128$, and $M = 4$.

of the kth user can be written as

$$\begin{aligned} r(t) &= \sum_{k=1}^{K_u} \sqrt{2E_c} \sum_{n=-\infty}^{\infty} d_{\lfloor n/N \rfloor}^{(k)} c_n^{(k)} h(t - nMT_c - \tau_k) \times \\ & \quad \sum_{m=1}^{M} \alpha_m^{(k)} \cos(2\pi f_m t + \psi_{k,m}) + n_W(t) + n_J(t), \end{aligned} \tag{2}$$

where τ_k is the random time delay corresponding to the kth user, assumed to be uniformly distributed in $[0, MT_c)$, K_u is the total number of active users in the system, $\alpha_m^{(k)}$ denotes the fade amplitude, $\phi_m^{(k)}$ denotes the random phase on the mth carrier of the kth user, and $\psi_m^{(k)} = \theta_m^{(k)} + \phi_m^{(k)}$ is the resultant phase on the mth carrier. The term $n_W(t)$ denotes the additive white Gaussian noise (AWGN) with a two-sided power spectral density (PSD) of $\eta_0/2$, whereas $n_J(t)$ represents partial band Gaussian interference with a PSD of $S_J(f)$.

We assume that the fades are independent across the users, the carriers, and over time. We further assume that $\alpha_m^{(k)}$ is Rayleigh distributed with density function $f_{\alpha_m^{(k)}}(x) = 2xe^{-x^2}$, for $x \geq 0$, and $\phi_m^{(k)}$ is uniformly distributed over $(-\pi, \pi]$. The PSD of the jammer, $S_J(f)$, can be written as

$$S_J(f) = \begin{cases} \frac{\eta_J}{2} & \text{for } f_J - \frac{W_J}{2} \leq |f| \leq f_J + \frac{W_J}{2} \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where η_J is the one-sided PSD of the jammer, and W_J and f_J are the bandwidth and the center frequency of the PBI, respectively.

The receiver operation, assuming the first user is the desired user, can be briefly explained as follows. We assume that perfect carrier, code, and bit synchronization for the first user has been accomplished. The received signal of Eqn. (2) is first chip-matched filtered, using the band-pass filters $H^*(f - f_i) + H^*(f + f_i)$, $i = 1, \ldots, M$, and then low-pass filtered with $\sqrt{2}\cos(2\pi f_i t + \phi_i^{(1)})$, $i = 1, \ldots, M$. Each of these M outputs are correlated using the local pseudo-noise sequences. If z_i denotes the output of the correlator on the ith carrier, then we have

$$z_i = S_i + I_i + J_i + N_i, \tag{4}$$

where S_i is the desired signal, I_i is the signal due to the other $K_u - 1$ interfering users, J_i is the contribution due to the jammer and N_i is the output due to AWGN. From ([2], Eqn.(23), the conditional mean of z_i, conditioned upon $\alpha_i^{(1)}$ and $d_{\lfloor n/N \rfloor}^{(1)}$, can be obtained as

$$E[z_i|\alpha_i^{(1)}, d_{\lfloor n/N \rfloor}^{(1)}] = d^{(1)} N \sqrt{E_c}\alpha_i^{(1)}, \tag{5}$$

where $d^{(1)} = \pm 1$ is the corresponding transmitted code symbol.

To obtain the variance of z_i, conditioned on $\alpha_i^{(1)}$, we assume that the interference from other users, the PBI, and the AWGN are independent of each other. With this, we have

$$\begin{aligned}\text{Var}\{z_i|\alpha_i^{(1)}\} &= \sigma_i^2 \\ &= \text{Var}\{I_i|\alpha_i^{(1)}\} + \text{Var}\{J_i|\alpha_i^{(1)}\} + \text{Var}\{N_i|\alpha_i^{(1)}\} \\ &\approx NR_{I_i}(0) + NR_{J_i}(0) + N\eta_0/2,\end{aligned} \tag{6}$$

where $R_{I_i}(\tau)$ and $R_{J_i}(\tau)$ are the autocorrelation functions of the interference and jammer, respectively. In Eqn. (6), the approximation in the last line is due to ignoring the contribution of $R_{I_i}(\tau)$ and $R_{J_i}(\tau)$ when $\tau \neq 0$ (see Eqns. (25), (26) and (27) in [2]). For simplicity, we assume that the Gaussian PBI overlaps K_s carriers, where $1 \leq K_s \leq M$. Without loss of generality, now assume that the first K_s bands are affected by the jammer. Then we have [2]

$$\begin{aligned}\sigma_i^2 &= \frac{N}{2}E_c(K_u-1)(1-\frac{\beta}{4}) + N\frac{\eta_J}{2} + \frac{N\eta_0}{2},\ i = 1, \ldots, K_s, \\ &= \frac{N}{2}E_c(K_u-1)(1-\frac{\beta}{4}) + \frac{N\eta_0}{2},\ i = K_s+1, \ldots, M.\end{aligned} \tag{7}$$

We note that the total jammer power is given by $P_J = \eta_J K_s W_1 = \eta_J W(K_s/M)$. By defining JSR $= P_J/(E_c/T_c)$ as the jammer-to-signal power ratio, we can solve for η_J as $\eta_J = \text{JSR} \times \frac{E_c}{\rho_J(1+\beta)}$, where $\rho_J = K_s/M$ is the fraction of the carriers affected by the jammer.

For each code symbol, the M outputs, $z_m, m = 1, \ldots, M$, are processed using the maximal ratio combiner (MRC) to result in an output Z. Since each z_m is affected by the fade $\alpha_m^{(1)}$ and has a noise variance of σ_m^2, the MRC weights should be proportional to $\alpha_m^{(1)}/\sigma_m^2$ to yield maximum signal-to-noise ratio (SNR). Assuming perfect knowledge of $\{\alpha_m^{(1)}\}$ and $\{\sigma_m^2\}$ at the receiver, the output of the MRC, Z, can then be expressed as

$$Z = \sum_{m=1}^{M}\frac{\alpha_m^{(1)}}{\sigma_m^2}z_m = d^{(1)}N\sqrt{E_c}\sum_{m=1}^{M}(\frac{\alpha_m^{(1)}}{\sigma_m})^2 + \xi, \tag{8}$$

where ξ is zero-mean Gaussian with variance $\sigma_\xi^2 = \sum_{m=1}^{M}(\alpha_m^{(1)}/\sigma_m)^2$. The instantaneous SNR random variable, γ, at the output of MRC is given by

$$\gamma = \frac{(E[Z|\alpha_1^{(1)}, \ldots, \alpha_M^{(1)}])^2}{2\text{Var}\{Z|\alpha_1^{(1)}, \ldots, \alpha_M^{(1)}\}} = \sum_{m=1}^{M}[\alpha_m^{(1)}]^2\overline{\gamma}_m, \tag{9}$$

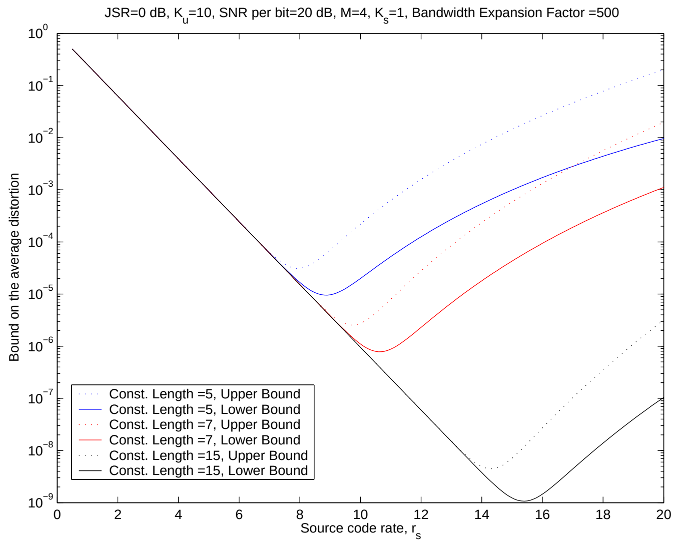

Fig. 3. Lower and upper bounds on the average distortion on Rayleigh fading channels with $M = 4$ carriers. The channel code rate is fixed to $r_c = 1/2$ and the bandwidth expansion factor is set to 500. Binary convolutional codes with various constraint lengths are used with an optimum distance spectrum. The other system parameters are: JSR = 0 dB, $\rho_J = 0.25$, $K_u = 10$ and $\gamma_b = 20$ dB.

where $\overline{\gamma}_m = N^2E_c/(2\sigma_m^2)$ is the average signal-to-interference-plus-noise ratio (SINR) on the mth carrier. Using Eqn. (7) we can simplify $\overline{\gamma}_m$ as

$$\begin{aligned}\overline{\gamma}_m &= \frac{1}{M}\frac{r_c\gamma_b}{1+\frac{r_c}{\mathcal{S}_F}\gamma_b(K_u-1)(1-\beta/4)+\frac{r_c}{\mathcal{S}_F}\gamma_b\frac{\text{JSR}}{\rho_J(1+\beta)}},\ m = 1, \ldots, K_s \\ &= \frac{1}{M}\frac{r_c\gamma_b}{1+\frac{r_c}{\mathcal{S}_F}\gamma_b(K_u-1)(1-\beta/4)},\ m = K_s+1, \ldots, M.\end{aligned} \tag{10}$$

For convenience, we define $\mathbf{a} = r_c\gamma_b/M$, $\Delta_0 = \gamma_b(K_u - 1)(1-\beta/4) + \gamma_b\frac{\text{JSR}}{\rho_J(1+\beta)}$ and $\Delta_1 = \gamma_b(K_u-1)(1-\beta/4)$. Clearly, $\Delta_0 \geq \Delta_1$ and $\overline{\gamma}_m$ can be expressed in terms of $\mathbf{a}$, Δ_0, Δ_1, r_c and $\mathcal{S}_F$ as

$$\begin{aligned}\overline{\gamma}_m &= \frac{\mathbf{a}}{1+r_c\Delta_0/\mathcal{S}_F},\ m = 1, \ldots, K_s, \\ &= \frac{\mathbf{a}}{1+r_c\Delta_1/\mathcal{S}_F},\ m = K_s+1, \ldots, M.\end{aligned} \tag{11}$$

A. Frame Error Rate with Convolutional Coding

The MRC outputs, $\{Z_n\}$, corresponding to a given coded frame are passed to the deinterleaver and then to the Viterbi decoder for soft-decision decoding. It is well known that at high SNR regions, the key performance metric with channel coding is the pairwise error probability (PEP) between two codewords [14]. The PEP between two codewords $\mathbf{x} = (x_1, \ldots, x_{N_F})$ and $\mathbf{y} = (y_1, \ldots, y_{N_F})$ which differ in d positions, when $\mathbf{x}$ is the transmitted codeword, is given by

$$\begin{aligned}P_2(d|\underline{\alpha}) &= \text{Prob}\left(\sum_{n=1}^{N_F}(Z_n - x_n)^2 > \sum_{n=1}^{N_F}(Z_n - y_n)^2\right) \\ &= \text{Prob}\left(\sum_{n=1}^{N_F}\xi_n(x_n - y_n) < -\sum_{n=1}^{N_F}N\sqrt{E_c}x_n \times \right. \\ &\quad \left. \sum_{m=1}^{M}(\alpha_{m,n}^{(1)}/\sigma_m)^2(x_n - y_n)\right),\end{aligned} \tag{12}$$

where in the last step of Eqn. (12) we have used Eqn. (8), N_F is the coded frame length of the terminated convolutional code, and the additional subscript n in $\alpha_{m,n}^{(1)}$ shows the time index of the code symbol. Without loss of generality, we assume that the codewords $\mathbf{x}$ and $\mathbf{y}$ differ in the first d positions. Then, using the Chernoff bound, $Q(x) \leq 1/2\exp(-x^2/2)$ for $x \geq 0$, we can upper bound Eqn. (12) as

$$P_2(d|\underline{\alpha}) \leq \frac{1}{2}\exp\left(-\sum_{n=1}^{d}\sum_{m=1}^{M}(\alpha_{m,n}^{(1)})^2\overline{\gamma}_m\right). \quad (13)$$

Upon taking the expectation of Eqn. (13) over the distribution of $\{\alpha_{m,n}^{(1)}\}$, we obtain

$$\overline{P}_2(d)E\left\{P_2(d|\underline{\alpha})\right\} \leq \frac{1}{2}\prod_{m=1}^{M}\left(\frac{1}{1+\overline{\gamma}_m}\right)^d < \frac{1}{2}\prod_{m=1}^{M}\overline{\gamma}_m^{-d}$$
$$= \frac{1}{2\mathbf{a}^{Md}}(1+r_c\Delta_0/\mathcal{S}_F)^{K_s d}(1+r_c\Delta_1/\mathcal{S}_F)^{(M-K_s)d}, \quad (14)$$

where Eqn. (11) was used in Eqn. (13). An exact expression for the frame error rate (FER) for a convolutional code with soft-decision decoding is difficult to derive, which motivates us to employ the union bound, as it is sufficiently tight at high SNRs. A tight upper bound on the FER of a convolutional code with block lengths larger than the constraint length is obtained in [11], using which we obtain the FER for our system as

$$P_B \leq \sum_d t(d)\overline{P}_2(d)$$
$$< \sum_d \frac{t(d)}{2\mathbf{a}^{Md}}(1+r_c\Delta_0/\mathcal{S}_F)^{K_s d}(1+r_c\Delta_1/\mathcal{S}_F)^{(M-K_s)d}, \quad (15)$$

where $t(d)$ is a function of the weight spectrum [14] of the underlying convolutional code. We are also interested in a lower bound on the FER, which can be obtained by taking only the dominant term of Eqn. (15). However, the Chernoff upper bound on the PEP of Eqn. (14) is no longer useful. A lower bound on the FER can be obtained as

$$P_B \geq t(d_{free})C(m,d_{free})\prod_{m=1}^{M}(1+\overline{\gamma}_m)^{-d_{free}}, \quad (16)$$

where d_{free} is the free distance of the code, $C(m,d) = \frac{1}{2\pi}\beta\left(Md+\frac{1}{2},\frac{1}{2}\right)$, and $\beta(p,q)$ is the standard beta integral [18].

III. End-to-End Average Distortion

We assume that the information source is Gaussian-distributed with independent and identically distributed source samples, each with unit variance. If r_s denotes the number of bits-per-source sample, then the average source distortion with minimum mean square error scalar quantization on a noise-free channel is given by $D(r_s) = \epsilon 2^{-2r_s}$, where ϵ depends on the quantizer [15]. Note that each coded frame of length N_F contains $N_F r_c/r_s$ source samples. Then, the average distortion per source sample can be written as ([16], Eqn. (10))

$$\mathcal{D}(r_s,r_c,\mathcal{S}_F) = (1-P_B(r_c,\mathcal{S}_F))\epsilon 2^{-2r_s} + P_B(r_c,\mathcal{S}_F)$$
$$\leq \epsilon 2^{-2r_s} + P_B(r_c,\mathcal{S}_F)$$
$$\leq \epsilon 2^{-2r_s} + \sum_d \frac{t(d)}{2\mathbf{a}^{Md}}(1+r_c\Delta_0/\mathcal{S}_F)^{K_s d}(1+r_c\Delta_1/\mathcal{S}_F)^{(M-K_s)d}$$
$$\triangleq D_u(r_s,r_c,\mathcal{S}_F), \quad (17)$$

where the above upper bound is quite accurate in the high SNR region.

A lower bound on the end-to-end average distortion can be obtained by first lower bounding the frame error rate, $P_B(r_c,\mathcal{S}_F)$ by the term with minimum free distance d_{free} as $P_B(r_c,\mathcal{S}_F) \geq t(d_{free})P_2(d_{free})$. With this a lower bound on the average distortion can be obtained as

$$D(r_s,r_c,\mathcal{S}_F) \geq \epsilon 2^{-2r_s} + t(d_{free})P_2(d_{free})(1-\epsilon 2^{-2r_s})$$
$$= \epsilon 2^{-2r_s} + (1-\epsilon 2^{-2r_s})t(d_{free})C(m,d_{free})\prod_{m=1}^{M}(1+\overline{\gamma}_m)^{-d_{free}}$$
$$\triangleq D_l(r_s,r_c,\mathcal{S}_F), \quad (18)$$

where in the second step of Eqn. (18) we have used Eqn. (16).

In what follows, we consider both the upper bound and lower bounds on the average distortion of Eqns. (17) and (18), respectively, as our objective functions.

IV. Optimum Bandwidth Allocation

If we denote by $\mathcal{U}$ the number of source samples-per-second available to the source coder, then the chip rate at the output of the spread-spectrum modulator is given by $\mathcal{U}r_s\frac{1}{r_c}\mathcal{S}_F$, which is limited to $W/(1+\beta)$, where W is the spread-spectrum bandwidth and β is the excess fractional bandwidth due to Nyquist chip wave-shaping filtering. That is, the variables r_s, r_c and $\mathcal{S}_F$ are related by $r_s\mathcal{S}_F/r_c \leq C_0$, where $C_0 = W/((1+\beta)\mathcal{U})$. The distortion function is given by $D(r_s,r_c,\mathcal{S}_F)$, which can also be written as $D(r_s,r_c,C_0r_c/r_s)$. We notice that by fixing the channel code rate, r_c, the distortion can be expressed only as a function of the source rate r_s, together with the bandwidth constraint C_0. In this section, we minimize the objective functions, $D_u(r_s,r_c,\mathcal{S}_F)$ of Eqn. (17) and $D_l(r_s,r_c,\mathcal{S}_F)$ of Eqn. (18), as a function of the source code rate r_s.

A. Upper Bound Based Optimal Allocation

With r_c fixed, we substitute $\mathcal{S}_F = C_0r_c/r_s$ in Eqn. (17) and rewrite Eqn. (17) as a function of only r_s as follows:

$$D_u(r_s) = \epsilon 2^{-2r_s} +$$
$$\sum_d \frac{t(d)}{2\mathbf{a}^{Md}}(1+\frac{\Delta_0}{C_0}r_s)^{dK_s}(1+\frac{\Delta_1}{C_0}r_s)^{(M-K_s)d}. \quad (19)$$

The first derivative with respect to r_s can be obtained as

$$\frac{d}{dr_s}D_u(r_s) = (-2\epsilon\ln 2)2^{-2r_s} + \sum_d \frac{t(d)}{2\mathbf{a}^{Md}}\left\{\frac{K_s d\Delta_0}{C_0}\times\right.$$
$$\left(1+\frac{\Delta_0}{C_0}r_s\right)^{K_s d-1}\left(1+\frac{\Delta_1}{C_0}r_s\right)^{(M-K_s)d} + \frac{(M-K_s)d\Delta_1}{C_0}\times$$
$$\left.\left(1+\frac{\Delta_0}{C_0}r_s\right)^{K_s d}\left(1+\frac{\Delta_1}{C_0}r_s\right)^{(M-K_s)d-1}\right\} \quad (20)$$

By taking the derivative of Eqn. (20) we arrive at

$$\frac{d^2}{dr_s^2}D_u(r_s) = 4\epsilon(\ln 2)^2 2^{-2r_s} + \sum_d \frac{t(d)}{2\mathbf{a}^{Md}}(1+\Delta_0 r_s/C_0)^{K_s d}(1+\Delta_1 r_s/C_0)^{(M-K_s)d} \times \left\{ \left(\frac{K_s d\Delta_0}{C_0 + r_s\Delta_0} + \frac{(M-K_s)d\Delta_1}{C_0 + r_s\Delta_1} \right)^2 - \frac{K_s d\Delta_0^2}{(C_0+\Delta_0 r_s)^2} - \frac{(M-K_s)d\Delta_1^2}{(C_0+\Delta_1 r_s)^2} \right\}. \quad (21)$$

Since $1 \leq K_s \leq M$, both the terms within $(\cdot)^2$ of Eqn. (21) are positive. Using $(x+y)^2 \geq x^2 + y^2$ for $x \geq 0$ and $y \geq 0$, we can simplify Eqn. (21) as

$$\frac{d^2}{dr_s^2}D_u(r_s) \geq 4\epsilon(\ln 2)^2 2^{-2r_s} + \sum_d \frac{t(d)}{2\mathbf{a}^{Md}}(1+\Delta_0 r_s/C_0)^{K_s d}(1+\Delta_1 r_s/C_0)^{(M-K_s)d} \times \left\{ \frac{K_s d(K_s d-1)\Delta_0^2}{(C_0+\Delta_0 r_s)^2} + \frac{(M-K_s)d((M-K_s)d-1)\Delta_1^2}{(C_0+\Delta_1 r_s)^2} \right\} \quad (22)$$

By noting that the expression inside $\{\cdot\}$ of Eqn. (22) is non-negative, we conclude that $D_u(r_s)$ is a convex function of r_s. The optimal 2-tuple is then given by $(r_s^*, r_c, C_0 r_c/r_s^*)$, where r_s^* uniquely solves $\frac{d}{dr_s}D_u(r_s) = 0$.

B. Lower Bound Based Optimal Allocation

By fixing r_c, and, as before, substituting $\mathcal{S}_F = C_0 r_c / r_s$ we express $D_{lower}(\cdot,\cdot,\cdot)$ of Eqn. (18) as a function of only r_s. For convenience, let us define

$$f(r_s) \triangleq t(d_{free})C(m, d_{free}) \left(1 + \frac{\mathbf{a}}{1+\frac{\Delta_0 r_s}{C_0}}\right)^{-K_s d_{free}} \times \left(1 + \frac{\mathbf{a}}{1+\frac{\Delta_1 r_s}{C_0}}\right)^{-(M-K_s)d_{free}} \quad (23)$$

so that Eqn. (18) can be expressed as

$$D_l(r_s) = \epsilon 2^{-2r_s} + (1-\epsilon 2^{-2r_s})f(r_s). \quad (24)$$

Notice that since $f(r_s)$ is a lower bound on the frame error rate, we have $0 \leq f(r_s) \leq 1$. The first derivative of Eqn. (24) with respect to r_s can then be computed as

$$\frac{d}{dr_s}D_l(r_s) = -\epsilon(2\ln 2)2^{-2r_s} + f(r_s)\epsilon(2\ln 2)2^{-2r_s} + (1-\epsilon 2^{-2r_s})\frac{d}{dr_s}f(r_s), \quad (25)$$

where, after some simplification, we can show that

$$\frac{d}{dr_s}f(r_s) = f(r_s) \times \left[\frac{K_s d_{free}\mathbf{a}\Delta_0 C_0}{(C_0 + r_s\Delta_0)((1+\mathbf{a})C_0 + r_s\Delta_0)} + \frac{(M-K_s)d_{free}\mathbf{a}\Delta_1 C_0}{(C_0 + r_s\Delta_1)((1+\mathbf{a})C_0 + r_s\Delta_1)} \right], \quad (26)$$

which is positive for all r_s. Upon setting $d/dr_s D_l(r_s) = 0$ and solving for r_s, we arrive at the following implicit equation:

$$r_s^* = \frac{1}{2}\log_2 \left(\frac{\epsilon \times \left[\frac{d}{dr_s}f(r_s)\big|_{r_s=r_s^*} + (2\ln 2)\left(1 - f(r_s)\big|_{r_s=r_s^*}\right) \right]}{\frac{d}{dr_s}f(r_s)\big|_{r_s=r_s^*}} \right). \quad (27)$$

Since $\frac{d}{dr_s}f(r_s)|_{r_s=r_s^*} > 0$, the argument of the logarithm in Eqn. (27) is always positive.

V. Results and Discussion

In this section, we present some numerical results based on the analysis presented in Sections II-IV. First, Fig. 2 shows the tightness of the lower and the upper bounds on the PEP on Rayleigh fading channel with $M = 4$ carriers. Also shown is the true PEP, which is numerically evaluated. We conclude from Fig. 2 that the bounds are sufficiently tight. In particular, for SNR-per-bit, γ_b, values less than 15 dB, the upper bound is within 2 dB of the true PEP.

The lower and upper bounds on the average end-to-end source distortion, as derived in Section III, are plotted in Fig. 3. For a fixed spread bandwidth and channel code rate, the average distortion is plotted as a function of the source code rate. A family of such curves is obtained for varying levels of channel code complexity, as measured by its constraint length. We notice from Fig. 3 that $i)$ there exists a source code rate at which the distortion can be minimized, $ii)$ the minimum source code rate shifts to the right for increasing values of the channel code complexity, since a stronger channel code enables the spread-spectrum modulator to use a small value of the spread factor, $iii)$the lower and the upper bounds coincide at all r_s that are below the r_s at which the distortion is minimized, after which the bounds differ by an order of magnitude. This difference in the lower and the upper bounds can be explained as follows: Notice that for a fixed channel code rate with moderate constraint lengths, increasing source code rate limits the available spread factor. This results in increasing variances for both the MAI and the PBI, which makes the Chernoff based union upper bound ineffective and is not comparable with the dominant term-based lower bound.

In Table I, we present the optimum source code rate, the optimum spreading factor, and the resulting average distortion for various channel code rates. For all the channel codes, the complexity of the encoder is fixed at a constraint length of 6. Both lower and upper bounds on the distortion are considered with the constraint on the bandwidth expansion factor set to 500 (i.e., $r_s \frac{1}{r_c}\mathcal{S}_F = 500$). The number of users is fixed at $K_u = 5$. From Table I, we observe that both the lower and the upper bounds result in approximately the same optimum bandwidth allocation. Table I also indicates that with decreasing channel code rates, it is beneficial to allocate more bandwidth to the source coder than to the spread-spectrum modulator. This can be explained as follows: For a given constraint length, a low rate channel code provides higher free distance, and hence large diversity order, which, together with the M-fold diversity provided by the MRC, helps to reduce the burden on the spread-spectrum modulator in combating the interference.

The effect of increasing system load (i.e., the number of users, K_u) on the bandwidth allocation is also investigated. The results are summarized in Table II, which corresponds to $M = 4$ carriers, $\gamma_b = 20$ dB and JSR = 0 dB. The bandwidth constraint is set to 500. From Table II, for a given channel code rate, it is seen that the source code rate has to be decreased as the number of users increases to allow sufficient processing gain to suppress the additional MAI. Also, for a given number of users, the best performance is seen to be achieved at the lowest channel code rate (with the exception of the lower bound result for $K_u = 25$ and 50 users, where the use of a rate $1/3$ code yielded slightly better performance than the use of a rate $1/4$ code).

Finally, we compare the performance of a single-carrier CDMA system employing a RAKE receiver against the performance of a MC-CDMA system in terms of the optimum bandwidth allocation. We assume that the number of multipaths resolved by the SC-CDMA system is the same as the number of carriers in an MC-CDMA system. Furthermore, we assume that the multipath fading in SC-CDMA is Rayleigh distributed with uniform intensity profile. The two systems are compared by varying the JSR, and the results are tabulated in Table III. It is evident, from Table III, that the single-carrier version performs worse than the MC-CDMA system for increasing values of the JSR. Also, as the JSR increases, the source coding rate is reduced in the SC-CDMA system and more bandwidth is allocated to spreading in order to combat the jammer. However, as we notice from the resulting minimum distortion in Table III, even with increasing spread factor the SC-CDMA cannot reach the performance of the MC-CDMA. This is due to the fact that no additional signal processing is employed in SC-CDMA, apart from a simple RAKE processing, whereas the MC-CDMA system has effectively nullified the effect of the jammer using the MRC receiver, in a simple way, by attenuating all the carriers that are affected by the PBI. We also note that, by incorporating a notch-filter, although at a higher complexity, to mitigate the effects of the jammer, the tradeoff performance of SC-CDMA can be expected to improve.

VI. Conclusions

For a fixed total bandwidth expansion factor, and for a fixed channel code rate, we studied the problem of optimal bandwidth allocation between the source coder and the spread-spectrum unit for an MC-CDMA system operating over a frequency-selective fading channel with NBI. By assuming a Gaussian source with the optimum scalar quantizer and a binary convolutional code with soft-decision decoding, we obtained both a lower and an upper bound on the end-to-end average source distortion. The optimal bandwidth allocation was then numerically obtained by minimizing upper and lower bounds on the average distortion. We have shown that the upper bound-based cost function is a convex function of the source code rate, and the optimal allocation depends on the system and the channel conditions, such as the total number of active users, the number of carriers, and the average jammer-to-signal power ratio.

r_c	Lower Bound			Upper Bound		
	r_s^*	$\mathcal{S}_F$	$D(r_s^*)$	r_s^*	$\mathcal{S}_F$	$D(r_s^*)$
$\frac{1}{2}$	9.15	27	$1.07 \cdot 10^{-5}$	8.40	29	$2.68 \cdot 10^{-5}$
$\frac{1}{3}$	10.48	15	$1.57 \cdot 10^{-6}$	9.69	17	$4.23 \cdot 10^{-6}$
$\frac{1}{4}$	10.65	11	$1.16 \cdot 10^{-6}$	10.18	12	$2.11 \cdot 10^{-6}$

TABLE I

Optimum source code rate, spreading factor, and the minimum distortion, for a fixed channel code rate, based on both upper and lower bounds on the end-to-end average distortion. Number of sub-carriers $M = 4$. The JSR is 10 dB with the jammer completely overlapping one sub-carrier (i.e., $K_s = 1$). $K_u = 5$, $\gamma_b = 10$ dB and frame length = 500 bits.

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r_c	K_u	Lower Bound			Upper Bound		
		r_s^*	$\mathcal{S}_F$	$D(r_s^*)$	r_s^*	$\mathcal{S}_F$	$D(r_s^*)$
$\frac{1}{2}$	5	10.30	24	$2.146 \cdot 10^{-6}$	9.66	25	$4.776 \cdot 10^{-6}$
	10	7.93	31	$4.536 \cdot 10^{-5}$	7.20	34	$1.104 \cdot 10^{-4}$
	25	5.26	47	$1.481 \cdot 10^{-3}$	4.43	56	$3.909 \cdot 10^{-3}$
	50	3.65	68	$1.203 \cdot 10^{-2}$	2.79	89	$3.142 \cdot 10^{-2}$
$\frac{1}{3}$	5	11.75	14	$2.658 \cdot 10^{-7}$	11.06	15	$6.410 \cdot 10^{-7}$
	10	8.93	18	$1.081 \cdot 10^{-5}$	8.15	20	$2.836 \cdot 10^{-5}$
	25	5.87	28	$6.197 \cdot 10^{-4}$	5.00	33	$1.754 \cdot 10^{-3}$
	50	4.08	40	$6.598 \cdot 10^{-3}$	3.18	52	$1.851 \cdot 10^{-2}$
$\frac{1}{4}$	5	11.98	10	$1.805 \cdot 10^{-7}$	11.56	10	$3.095 \cdot 10^{-7}$
	10	8.96	13	$9.805 \cdot 10^{-6}$	8.49	14	$1.751 \cdot 10^{-5}$
	25	5.75	21	$6.939 \cdot 10^{-4}$	5.21	23	$1.312 \cdot 10^{-3}$
	50	3.90	32	$7.935 \cdot 10^{-3}$	3.33	37	$1.516 \cdot 10^{-2}$

TABLE II

OPTIMUM SOURCE CODE RATE, SPREADING FACTOR, AND THE MINIMUM DISTORTION, FOR A FIXED CHANNEL CODE RATE, BASED ON BOTH UPPER AND LOWER BOUNDS ON THE END-TO-END AVERAGE DISTORTION. THE OTHER SYSTEM PARAMETERS ARE AS FOLLOWS: SNR-PER-BIT, $\gamma_b = 20$ DB, JSR = 0 DB, $\rho_J = 0.25$. THE BANDWIDTH CONSTRAINT, C_0, IS SET TO 500. THE CONSTRAINT LENGTH OF THE CHANNEL CODE IS FIXED TO 6.

r_c	JSR	MC-CDMA			SC-CDMA		
	(dB)	r_s^*	$\mathcal{S}_F$	$D(r_s^*)$	r_s^*	$\mathcal{S}_F$	$D(r_s^*)$
$\frac{1}{2}$	0	9.66	25	$4.776 \cdot 10^{-6}$	9.85	25	$3.710 \cdot 10^{-6}$
	10	8.40	29	$2.668 \cdot 10^{-5}$	7.90	31	$4.458 \cdot 10^{-5}$
	20	7.72	32	$7.530 \cdot 10^{-5}$	3.14	79	0.02012
	25	7.65	32	$8.540 \cdot 10^{-5}$	1.31	190	0.1995
$\frac{1}{3}$	0	11.06	15	$6.410 \cdot 10^{-7}$	11.27	14	$4.778 \cdot 10^{-7}$
	10	9.69	17	$4.233 \cdot 10^{-6}$	8.97	18	$9.735 \cdot 10^{-6}$
	20	9.04	18	$1.126 \cdot 10^{-5}$	3.57	46	0.0112
	25	8.97	18	$1.258 \cdot 10^{-5}$	1.52	108	0.1502
$\frac{1}{4}$	0	11.56	10	$3.095 \cdot 10^{-7}$	11.79	10	$2.276 \cdot 10^{-7}$
	10	10.18	12	$2.114 \cdot 10^{-6}$	9.35	13	$5.673 \cdot 10^{-6}$
	20	9.55	13	$5.401 \cdot 10^{-6}$	3.73	33	0.009006
	25	9.49	13	$5.988 \cdot 10^{-6}$	1.62	77	0.13402

TABLE III

COMPARISON BETWEEN SC-CDMA WITH A RAKE RECEIVER AND MC-CDMA FOR THE SAME SYSTEM BANDWIDTH AND CHANNEL CODE RATE. THE NUMBER OF CARRIERS IN MC-CDMA SYSTEM IS SET TO 4 WHEREAS AN EQUAL NUMBER OF MULTIPATH COMPONENTS, WITH I.I.D RAYLEIGH FADING, ARE ASSUMED TO BE RESOLVED BY THE SINGLE-CARRIER CDMA SYSTEM. AN UPPER BOUND ON THE AVERAGE DISTORTION IS MINIMIZED AND THE RESULTING OPTIMAL ALLOCATION IS TABULATED. THE OTHER SYSTEM PARAMETERS ARE AS FOLLOWS: SNR-PER-BIT, $\gamma_b = 10$ DB, $\rho_J = 0.25$ AND $K_u = 5$. THE BANDWIDTH CONSTRAINT, C_0, IS SET TO 500. THE CONSTRAINT LENGTH OF THE CHANNEL CODE IS FIXED TO 6.