## Scalable Multimedia Optimization in MIMO Systems

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Abstract—This paper studies the optimal transmission of scalable multimedia sources over multiple-input multiple-output channels. First, we derive the outage probability of a spacetime code for an arbitrary piecewise-linear diversity-multiplexing tradeoff function. We next prove that, as long as a crossover point of the outage probabilities of two space-time codes exists, then as the spectral efficiency increases, the crossover point in signalto-noise ratio monotonically increases, whereas that in outage probability monotonically decreases. Lastly, based on that, we derive a method for optimal space-time coding of a scalable multimedia sequence that contains multiple packets. It is shown that, with the proposed method, the computational complexity involved with optimal space-time coding is exponentially reduced without losing any performance.

#### I. INTRODUCTION

This paper studies the optimal transmission of scalable (or progressive) sources [1], [2] over a multiple-input multipleoutput (MIMO) system. Progressive sources have the key feature that they have steadily decreasing importance for bits later in the stream, which makes unequal target error rates and/or transmission data rates in the stream very useful. Hence, when progressive sources are transmitted over MIMO channels, and each packet of the stream can be encoded with a different space-time code, the tradeoff between the space-time codes needs to be specified in terms of their target error rates and transmission data rates.

In [3], the authors of this paper considered both vertical Bell Labs layered space-time (V-BLAST) codes, and orthogonal space-time block codes (OSTBC), and analyzed how the crossover point of the error probability curves of the two codes behaves in the high signal-to-noise (SNR) regime; it was proven that as the data rate increases, the crossover point in error probability monotonically decreases, whereas that in the SNR monotonically increases. Then, the analysis was used to determine the optimal space-time coding of progressive sources.

The work in [3] analyzed only V-BLAST with a zeroforcing (ZF) receiver and OSTBC in i.i.d. Rayleigh fading channels. On the other hand, in this paper, with different technical approaches from those used in [3], we extend the work in [3] to other space-time codes and receivers, and prove the monotonic behavior of their crossover points, which is valid in spatially correlated Rayleigh and Rician fading channels, in addition to i.i.d. Rayleigh channels. To do so, we exploit the diversity-multiplexing tradeoff (DMT) [4].

More specifically, we derive the outage probability expression of the space-time code for an arbitrary piecewise-linear DMT function, and then analyze the crossover point of the outage probabilities of two space-time codes with given DMT functions. We prove that, as long as a crossover point of the outage probabilities exists, then, as the spectral efficiency increases, the crossover point in the SNR monotonically increases, whereas that in outage probability monotonically decreases. The above analytical results hold for any type of space-time codes and receivers which retain piecewise-linear DMT characteristics. As a specific example, we analyze the crossover point for the two-layer diagonal BLAST (D-BLAST) with a group zero-forcing (GZF) receiver [5], and V-BLAST with a minimum mean-square error (MMSE) receiver, in addition to OSTBC. Based on the analysis, we derive the optimization method for the space-time coding of progressive sources with respect to D-BLAST, V-BLAST, and OSTBC.

### II. CROSSOVER POINT ANALYSIS OF THE OUTAGE PROBABILITY CURVES FOR GIVEN DMT FUNCTIONS

Consider a MIMO system with  $N_t$  transmit and  $N_r$  receive antennas communicating over a frequency flat fading channel. A space-time codeword,  $\mathbf{S} = [\mathbf{s}_1 \cdots \mathbf{s}_T]$  of size  $N_t \times T$  is transmitted over T symbol durations. The baseband model, at the kth time symbol duration  $(k = 1, \dots, T)$ , assuming perfect matched filter detection is given by  $\mathbf{y}_k = \mathbf{H}\mathbf{s}_k + \mathbf{n}_k$ , where  $\mathbf{s}_k$  is an  $N_t \times 1$  transmitted signal vector,  $\mathbf{y}_k$  is an  $N_r \times 1$  received signal vector, and  $\mathbf{n}_k$  is an  $N_r \times 1$  zero-mean complex AWGN vector with  $\mathcal{E}\left[\mathbf{n}_{k}\mathbf{n}_{l}^{H}\right] = \sigma_{n}^{2}\mathbf{I}_{N_{r}}\delta(k-l)$ , where  $(\cdot)^H$  denotes Hermitian operation. **H** denotes the  $N_r \times N_t$ channel matrix, whose entries are i.i.d.  $\sim C\mathcal{N}(0,1)$ , and it is assumed that  $\mathbf{H}$  is random, but constant over T symbol durations. Let  $\gamma_s$  denote SNR per symbol, which is defined as  $\gamma_s := \mathcal{E}\left[ |(\mathbf{s}_k)_i|^2 \right] / \sigma_n^2$  where  $(\mathbf{s}_k)_i$  is the *i*th component of  $\mathbf{s}_k$ . We assume that  $\mathbf{H}$  is known at the receiver, but not known at the transmitter.

Next, we derive the outage probability expression of the space-time code for any given piecewise-linear DMT function. Let r and d denote the multiplexing and diversity gains defined in [4], respectively. That is,

$$r = \lim_{\gamma_s \to \infty} \frac{R(\gamma_s)}{\log_2 \gamma_s} \quad \text{and} \quad d = -\lim_{\gamma_s \to \infty} \frac{\log_2 P_{out}(\gamma_s)}{\log_2 \gamma_s} \quad (1)$$

where  $R(\gamma_s)$  is the spectral efficiency (bits/s/Hz), and  $P_{out}(\gamma_s)$  is the outage probability. By L'Hopital's rule, the multiplexing gain, r, in (1) can be expressed as

$$r = \lim_{\gamma_s \to \infty} \frac{\partial R(\gamma_s) / \partial \gamma_s}{\partial \log_2 \gamma_s / \partial \gamma_s} = \lim_{\gamma_s \to \infty} \ln 2 \cdot \gamma_s \frac{\partial R(\gamma_s)}{\partial \gamma_s}$$
(2)

From (2), it can be shown that

$$\lim_{\gamma_s \to \infty} R(\gamma_s) = \lim_{\gamma_s \to \infty} r \log_2 \gamma_s + c_r$$
(3)

where  $c_r$  is an arbitrary real constant. Let  $k_r = 2^{c_r} > 0$ . Then, as  $\gamma_s \to \infty$  (i.e., high SNR),  $R(\gamma_s)$  can be expressed as

$$R(\gamma_s) = \log_2(k_r \, \gamma_s^r). \tag{4}$$

In a similar way, from (1), it can be shown that, as  $\gamma_s \to \infty$ ,  $P_{out}(\gamma_s)$  is expressed as

$$P_{out}(\gamma_s) = k_d \, \gamma_s^{-d} \tag{5}$$

where  $k_d = 2^{c_d} > 0$ , and  $c_d$  is an arbitrary real constant.

Consider a space-time code whose DMT characteristic function, defined in [4], is given by

$$d^{\dagger}(r) = v - ur, \quad \text{for } \alpha \le r \le \beta \ (\alpha > 0)$$
 (6)

where  $d^{\dagger}(r) \geq 0$ , and where  $u \geq 0$  and  $v \geq 0$  are real constants. Let  $P_{out}^{\dagger}(\gamma_s)$  denote the outage probability for the space-time code whose DMT is given by (6). From (5) and (6), as  $\gamma_s \to \infty$ ,  $P_{out}^{\dagger}(\gamma_s)$  can be expressed as

$$P_{out}^{\dagger}(\gamma_s) = k_d \gamma_s^{-d^{\dagger}(r)} = k_d \frac{\gamma_s^{ur}}{\gamma_s^v}$$
(7)

Eq. (4) can be rewritten as

$$\gamma_s^r = \frac{2^{R(\gamma_s)}}{k_r} > 1 \tag{8}$$

where the inequality follows from  $\gamma_s >> 1$  and r > 0. Substituting (8) into (7), as  $\gamma_s \to \infty$ ,  $P_{out}^{\dagger}(\gamma_s)$  can be rewritten as

$$P_{out}^{\dagger}(\gamma_s) = k_d \left(\frac{2^{R(\gamma_s)}}{k_r}\right)^u \frac{1}{\gamma_s^v} \tag{9}$$

for  $(2^{R(\gamma_s)}/k_r)^{1/\beta} \leq \gamma_s \leq (2^{R(\gamma_s)}/k_r)^{1/\alpha}$ . The range of  $\gamma_s$  is derived as follows: Since r > 0, (8) can be rewritten as  $\gamma_s = (2^{R(\gamma_s)}/k_r)^{1/r}$ . Thus, from the inequality in (8) and  $\alpha \leq r \leq \beta$  in (6), we have the range in (9). We focus on the situation where the spectral efficiency,  $R(\gamma_s)$ , does not change as  $\gamma_s$  increases. Thus, from here onwards, we denote the spectral efficiency,  $R(\gamma_s)$ , simply by R. In the following subsections, for given piecewise-linear DMT functions of the space-time codes, the crossover points of their outage probability curves are analyzed.

#### A. When There Exists a Crossover in the DMT Functions

Consider two space-time codes which have linear DMT characteristics as follows:

$$d_1(r) = v_1 - u_1 r \quad \text{and} \quad d_2(r) = v_2 - u_2 r,$$
  
for  $\alpha \le r \le \beta \quad (\alpha > 0)$  (10)

where

$$u_i > 0$$
 and  $v_i > 0$   $(i = 1, 2),$  (11)

$$v_1 - u_1 \alpha < v_2 - u_2 \alpha$$
 and  $v_1 - u_1 \beta > v_2 - u_2 \beta$  (12)

That is, there exists a crossover in  $\alpha < r < \beta$  for the two DMT functions. Let  $P_{out,1}(\gamma_s)$  and  $P_{out,2}(\gamma_s)$  denote the outage

probabilities of the space-time codes whose DMT functions are given by  $d_1(r)$  and  $d_2(r)$ , respectively. Then, from (9), as  $\gamma_s \to \infty$ , we have

$$P_{out,i}(\gamma_s) = k_d \left(\frac{2^R}{k_r}\right)^{u_i} \frac{1}{\gamma_s^{v_i}} \quad (i = 1, 2)$$
(13)

for  $(2^R/k_r)^{1/\beta} \leq \gamma_s \leq (2^R/k_r)^{1/\alpha}$ . From (13), for a given spectral efficiency, R, we find the SNR,  $\gamma_s^*$ , for which  $P_{out,1}(\gamma_s)$  and  $P_{out,2}(\gamma_s)$  are identical. It can be readily shown that, for  $v_1 \neq v_2$ ,  $\gamma_s^*$  is given by

$$\gamma_s^* = \left(\frac{2^R}{k_r}\right)^{\frac{u_2 - u_1}{v_2 - v_1}} \tag{14}$$

In the following, we will show that  $\gamma_s^*$  exists within the range of SNR given below (13), and that  $v_1 \neq v_2$  (or, more precisely,  $v_2 > v_1$ ) for (14).

- i) From (12), we obtain  $(\beta \alpha)u_2 > (\beta \alpha)u_1$ , or, equivalently,  $u_2 > u_1$ . It can also be shown that  $v_2 > v_1$ .
- ii) From  $u_2 > u_1$  and  $v_2 > v_1$ , (12) can be rewritten as  $1/\alpha > (u_2 - u_1)/(v_2 - v_1)$  and  $1/\beta < (u_2 - u_1)/(v_2 - v_1)$ . From this and the inequality in (8), we have  $(2^R/k_r)^{1/\beta} \le \gamma_s^* \le (2^R/k_r)^{1/\alpha}$ .

Further, from  $u_2 > u_1$  and  $v_2 > v_1$ , it follows that  $\gamma_s^*$ , given by (14), is a strictly increasing function in R, for any  $k_r > 0$  given below (3). In other words, as the spectral efficiency increases, the crossover point of the outage probability curves in SNR monotonically increases.

If we substitute  $\gamma_s^*$ , given by (14), into (13), it can be shown that the corresponding outage probability,  $P_{out}^*$ , is given by

$$P_{out}^* = k_d \left(\frac{2^R}{k_r}\right)^{\frac{u_1 v_2 - u_2 v_1}{v_2 - v_1}}$$
(15)

We will prove that  $P_{out}^*$  is a strictly decreasing function in R. From  $u_2 > u_1$  and  $v_2 > v_1$ , (12) can be rewritten as  $\beta > (v_2 - v_1)/(u_2 - u_1)$ . From  $d_2(\beta) = v_2 - u_2\beta \ge 0$ , which is assumed in (6) and (10), and  $u_2 > 0$ , given by (11), it is seen that  $\beta \le v_2/u_2$ . Thus, we have  $(v_2 - v_1)/(u_2 - u_1) < v_2/u_2$ . From this,  $u_2 > 0$ , and  $u_2 > u_1$ , it can be shown that  $u_2v_1 > u_1v_2$ . In addition, from  $v_2 > v_1$ , we have

$$\frac{u_1v_2 - u_2v_1}{v_2 - v_1} < 0. \tag{16}$$

Eqs. (15) and (16) show that  $P_{out}^*$  is a strictly decreasing function in R, regardless of what the constants  $k_r > 0$  and  $k_d > 0$ , given below (3) and (5), respectively, are. That is, as the spectral efficiency increases, the crossover point in the outage probability monotonically decreases.

Moreover, from (13) and  $v_2 > v_1$ , it can be shown that

$$P_{out,1}(\gamma_s) < P_{out,2}(\gamma_s) \quad \text{for } \left(\frac{2^R}{k_r}\right)^{\frac{1}{\beta}} \le \gamma_s < \gamma_s^*$$
$$P_{out,1}(\gamma_s) > P_{out,2}(\gamma_s) \quad \text{for } \gamma_s^* < \gamma_s \le \left(\frac{2^R}{k_r}\right)^{\frac{1}{\alpha}} \quad (17)$$

Let  $P_{out,f}^*$  and  $\gamma_{s,f}^*$  denote the crossover point when a spectral efficiency  $R = R_f$  is used, and let  $P_{out,g}^*$  and  $\gamma_{s,g}^*$  denote

(11)

the crossover point when  $R = R_g$  is employed. Suppose that  $R_f < R_g$ . Since  $\gamma_s^*$  and  $P_{out}^*$  are strictly increasing and decreasing functions in R, respectively, we have

$$\gamma_{s,f}^* < \gamma_{s,g}^* \quad \text{and} \quad P_{out,f}^* > P_{out,g}^* \quad \text{for } R_f < R_g.$$
 (18)

Based on (17) and (18), the outage probabilities of the two space-time codes, for the same given spectral efficiency, are qualitatively depicted in Fig. 1. Suppose that a target outage probability,  $P_{out,T}$ , is smaller than  $P_{out,f}^*$  but greater than  $P_{out,g}^*$ . Then, from Fig. 1, it is seen that, for a spectral efficiency  $R_f$ , the space-time code with the DMT of  $d_2(r)$ given by (10) is preferable to that with the DMT of  $d_1(r)$ . For a spectral efficiency  $R_g$ , however, the latter is preferable to the former. Note that the analyses in this subsection are valid for any  $k_d > 0$  and  $k_r > 0$ .

# B. When the DMT Functions Coincide Only at the Lowest Multiplexing Gain

We next consider the case when the DMT functions coincide only at the smallest multiplexing gain in  $\alpha \leq r \leq \beta$ . Consider two space-time codes which have linear DMT characteristics given by (10) and (11), with

$$v_1 - u_1 \alpha = v_2 - u_2 \alpha$$
 and  $v_1 - u_1 \beta < v_2 - u_2 \beta$  (19)

From (19), in a similar way to Subsection A, it can be shown that  $\gamma_s^*$ , given by (14), exists in the range of SNR given below (13), such that  $(2^R/k_r)^{1/\beta} \leq \gamma_s \leq (2^R/k_r)^{1/\alpha} (= \gamma_s^*)$ .

Moreover, it can be shown that  $\gamma_s^*$  is a strictly increasing function in R, for any  $k_r > 0$ . It can also be proven that  $P_{out}^*$ , given by (15), is a strictly decreasing function in R, for any  $k_d > 0$  and  $k_r > 0$ .

Further, from (13), it can be shown that

$$P_{out,1}(\gamma_s) > P_{out,2}(\gamma_s) \text{ for } \left(\frac{2^R}{k_r}\right)^{\frac{1}{\beta}} \le \gamma_s < \left(\frac{2^R}{k_r}\right)^{\frac{1}{\alpha}}$$
 (20)

From (20), it is seen that, except at the highest SNR,  $\gamma_s = (2^R/k_r)^{1/\alpha}$ , the space-time code with the DMT given by

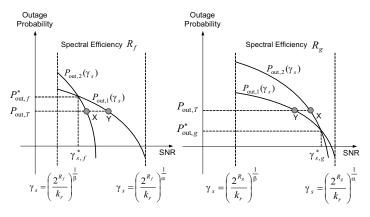


Fig. 1. Outage probabilities of the two space-time codes for the same given spectral efficiency. For  $R_f < R_g$ , we have  $\gamma^*_{s,f} < \gamma^*_{s,g}$  and  $P^*_{out,f} > P^*_{out,g}$ .

 $d_2(r)$  is always preferable to the space-time code with the DMT given by  $d_1(r)$ , for any spectral efficiency R and target outage probability  $P_{out,T}$ . Note that this differs from the result of Subsection A which is stated below (18). However, as described below (19), the crossover point,  $\gamma_s^*$  and  $P_{out}^*$ , retains the same properties as those of Subsection A, as given by (18).

# C. When the DMT Functions Coincide Only at the Highest Multiplexing Gain

For this case, we have similar results to Subsection B, which are not presented here due to limited space.

### III. CROSSOVER POINT ANALYSIS OF THE OUTAGE PROBABILITIES FOR D-BLAST, V-BLAST AND OSTBC

Based on the analysis in Section II, we analyze the behavior of the crossover point of the outage probabilities for specific space-time codes. As an example, we consider two-layer D-BLAST with a GZF receiver [5], V-BLAST with an MMSE receiver, and OSTBC. GZF (group decoding) is a recent decoding method studied in the literature. From here onwards, D-BLAST and V-BLAST are assumed to be with those specific receivers. Let  $d_D(r)$ ,  $d_V(r)$  and  $d_O(r)$  denote the DMT characteristics of D-BLAST, V-BLAST, and OSTBC, respectively. Then, we have  $d_D(r) = N_r N_t - N_t + 1 - N_r (N_t + 1)r/2$  for  $0 \le r \le 2/(N_t+1)$ ; and  $d_D(r) = (N_r-1)\{N_t-(N_t+1)r/2\}$ for  $2/(N_t + 1) \leq r \leq 2N_t/(N_t + 1)$  [5]. In addition,  $d_V(r) = (N_r - N_t + 1)(1 - r/N_t)$  for  $0 \le r \le N_t$ , and  $d_O(r) = N_r N_t (1 - r/r_s)$  for  $0 \le r \le r_s$ . In the range of r, which is not specified above, we have  $d_D(r)=d_V(r)=d_O(r)=0$ . To compare the above codes, it is assumed that  $N_r \ge N_t \ge 0$ .

#### A. Two-Layer D-BLAST and V-BLAST

The range of multiplexing gain given above can be divided into  $0 < r \le 2/(N_t + 1)$ ,  $2/(N_t + 1) \le r \le 2N_t/(N_t + 1)$ ,  $2N_t/(N_t + 1) \le r \le N_t$ , and  $N_t \le r < \infty$ , such that the DMT functions of both D-BLAST and V-BLAST are linear over each range. We analyze the crossover point of the outage probabilities for each of the ranges. Let  $P_{out,D}(\gamma_s)$  and  $P_{out,V}(\gamma_s)$  denote the outage probabilities of D-BLAST and V-BLAST, respectively. Then, based on the analysis in Section II, it can be shown that

$$P_{out,D}(\gamma_s) = P_{out,V}(\gamma_s) \quad \text{for } 1 < \gamma_s \le \left(\frac{2^R}{k_r}\right)^{\frac{1}{N_t}}$$
$$P_{out,D}(\gamma_s) > P_{out,V}(\gamma_s) \quad \text{for } \left(\frac{2^R}{k_r}\right)^{\frac{1}{N_t}} < \gamma_s < \gamma_s^*$$
$$P_{out,D}(\gamma_s) < P_{out,V}(\gamma_s) \quad \text{for } \gamma_s^* < \gamma_s < \infty$$
(21)

where  $\gamma_s^*$  exists in the range of  $(2^R/k_r)^{(N_t+1)/2N_t} < \gamma_s^* < (2^R/k_r)^{(N_t+1)/2}$ , and it exhibits monotonic behavior, as given by (18):

$$\gamma_{s,f}^* < \gamma_{s,g}^*$$
 and  $P_{out,f}^* > P_{out,g}^*$  for  $R_f < R_g$ . (22)

Suppose that a target outage probability,  $P_{out,T}$ , is smaller than  $P_{out,f}^*$  but greater than  $P_{out,g}^*$ . Then, from (21) and (22), it follows that D-BLAST is preferable to V-BLAST for a spectral efficiency  $R_f$ , but V-BLAST is preferable for  $R_g$  (see Fig. 1).

#### B. Two-Layer D-BLAST and OSTBC

Let  $P_{out,O}(\gamma_s)$  denote the outage probability of OSTBC. Based on the results of Section II, it can be shown that

$$P_{out,D}(\gamma_s) = P_{out,O}(\gamma_s) \quad \text{for } 1 < \gamma_s \le \left(\frac{2^R}{k_r}\right)^{\frac{N_t+1}{2N_t}}$$
$$P_{out,D}(\gamma_s) < P_{out,O}(\gamma_s) \quad \text{for } \left(\frac{2^R}{k_r}\right)^{\frac{N_t+1}{2N_t}} < \gamma_s < \gamma_s^*$$
$$P_{out,D}(\gamma_s) > P_{out,O}(\gamma_s) \quad \text{for } \gamma_s^* < \gamma_s < \infty \qquad (23)$$

where  $\gamma_s^*$  exists in  $\gamma_s^* > (2^R/k_r)^{(N_t+1)/2}$ , and it exhibits monotonic behavior, as given by (22). Thus, the same argument as given below (22) can be made for this case.

### IV. OPTIMAL SPACE-TIME CODING OF A SEQUENCE OF NUMEROUS PROGRESSIVE PACKETS

We exploit the analysis in Section III to address the optimization of progressive transmission in MIMO systems. Progressive encoders produce data with gradual differences of importance in their bitstreams. Suppose that the bitstream from a progressive source encoder is transformed into a sequence of  $N_P$  packets. Each of those packets can be encoded with different transmission data rates, as well as different spacetime codes, so as to yield the best end-to-end performance. The error probability of an earlier packet needs to be less than or equal to that of a later packet, due to the gradually decreasing importance in the bitstream. Thus, given the same transmission power, the earlier packet requires a data rate which is less than or equal to that of the later packet.

Let  $N_D$  denote the number of candidate data rates employed by a system. The number of possible assignments of  $N_D$ data rates to  $N_P$  packets would exponentially grow as  $N_P$ increases. Further, in a MIMO system, if each packet can be encoded with different space-time codes (e.g., D-BLAST, V-BLAST, or OSTBC), the assignment of space-time codes as well as data rates to  $N_P$  packets yields a more complicated optimization problem. Note that each source, such as an image, has its inherent rate-distortion characteristic, from which the performance of the expected distortion is computed. Hence, for example, when a series of images is transmitted, the above optimization should be addressed in an image-by-image manner (i.e., in a real time manner), considering which specific image (i.e., rate-distortion characteristic) is transmitted in the current time slot. To address this matter, for a SISO system, there have been many studies about the optimal assignment of data rates for a sequence of progressive packets. For a MIMO system, we exploit the results in the previous sections to optimize the assignment of space-time codes for progressive packets.

First, we focus on D-BLAST and V-BLAST. Suppose that we can employ one of the two codes for each packet, and that the *k*th packet in a sequence of  $N_P$  packets is encoded with V-BLAST. Then, our analysis tells us that the k + 1st, k + 2nd, ...,  $N_P$ th packets also should be encoded with V-BLAST rather than with D-BLAST. This is because in Section III we have shown that, when V-BLAST is preferable for a packet with a data rate (i.e., spectral efficiency times signal bandwidth) of  $R_f$ , a packet with a data rate of  $R_g$  (>  $R_f$ ) also should be encoded with V-BLAST, as long as the target error rate of the latter is the same as or higher than that of the former (see Fig. 1). That is, in a sequence of  $N_P$  progressive packets, the last *i* consecutive packets should be encoded with V-BLAST, and the other  $N_P - i$  packets are encoded with D-BLAST ( $0 \le i \le N_P$ ).

Next, suppose that either D-BLAST or OSTBC can be employed for each packet, and that the kth packet is encoded with OSTBC. Then, our analysis indicates that the 1st, 2nd,  $\ldots$ , k-1st packets also should be encoded with OSTBC rather than with D-BLAST. Hence, the earliest *i* consecutive packets should be encoded with OSTBC, whereas the other  $N_P - i$ packets are encoded with D-BLAST ( $0 \le i \le N_P$ ).

From the statements above, the optimization strategy can be derived as follows: Suppose that the system can employ D-BLAST, V-BLAST or OSTBC for each progressive packet. Then, the earliest *i* consecutive packets should be encoded with OSTBC, the last *j* consecutive packets should be encoded with V-BLAST, and the remaining  $N_P - i - j$  packets are encoded with D-BLAST ( $0 \le i, j \le N_P$  and  $0 \le i+j \le N_P$ ). Note that the strategy is based on the properties of progressive sources that are involved with unequal target error rates and spectral efficiencies in the bitstream. As a result, the number of possible assignments of the three space-time codes to a sequence of  $N_P$  packets is reduced from  $3^{N_P}$  to  $\binom{N_P+1}{2}$ . That is, the computational complexity involved with the optimization can be exponentially simplified.

### V. NUMERICAL EVALUATION

First, we evaluate the outage probabilities of D-BLAST, V-BLAST, and OSTBC for various spectral efficiencies and numbers of transmit and receive antennas. As an example, the results for  $2 \times 3$  and  $2 \times 4$  MIMO systems are depiced in Fig. 2, where solid curves denote the exact outage probabilities, and dashed curves denote the high SNR approximate outage probabilities which are derived from (9). Both Fig. 2 (a) and 2 (b) show that as spectral efficiency increases, the exact crossover points as well as the approximate ones behave in a manner predicted by the analysis as given by (22).

Next, we compare the performances of the optimal spacetime coding and the suboptimal ones for the transmission of progressive sources. We evaluate the performances using the source coder SPIHT [2], for the 8 bits per pixel (bpp)  $512 \times 512$ Lena image with a transmission rate of 0.5 bpp. The endto-end performance is measured by the expected distortion of the image. To begin, we briefly present the evaluation of the expected distortion of the image: The system takes the compressed bitstream from the progressive source encoder, and transforms it into a sequence of  $N_P$  packets. Then, each packet is encoded by a space-time code. At the receiver, if a received packet is correctly decoded, the next packet is considered by the source decoder. Otherwise, the decoding is stopped, and the source is reconstructed from only the

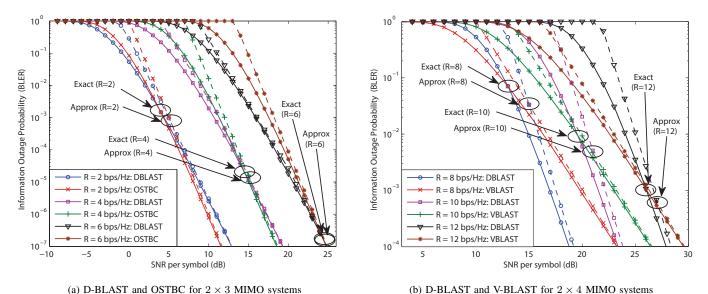


Fig. 2. Exact and high SNR approximate outage probabilities in i.i.d. Rayleigh fading channels. The crossover points are marked with circles.

correctly decoded packets. We assume a slow fading channel such that channel coefficients are nearly constant over an image which consists of a sequence of  $N_P$  packets.

Let  $P_i(\dot{\gamma}_s)$  denote the conditional probability of a decoding error of the *i*th packet  $(1 \leq i \leq N_P)$ , where  $\dot{\gamma}_s$  is the instantaneous SNR per symbol, conditioned on fading channel state. Then, the conditional probability that no decoding errors occur in the first n packets with an error in the next one,  $P_{c,n}$ , is given by  $P_{c,n} = P_{n+1}(\dot{\gamma}_s) \prod_{i=1}^n (1 - P_i(\dot{\gamma}_s))$   $(1 \le n \le N_P - 1)$ . Note that  $P_{c,0} = P_1(\dot{\gamma}_s)$  is the probability of an error in the first packet, and  $P_{c,N_P} = \prod_{i=1}^{N_P} (1 - P_i(\dot{\gamma}_s))$  is the probability that all  $N_P$  packets are correctly decoded. Let  $d_n$ denote the distortion of the source using the first n packets for the source decoder ( $0 \le n \le N_P$ ). Then,  $d_n$  can be expressed as  $d_n = D(\sum_{i=1}^n r_i)$ , where D(x) denotes the operational distortion-rate function of the source, and  $r_i$  is the number of source bits in the *i*th packet  $(1 \le i \le N_P)$ . Then, the expected distortion, denoted by E[D], can be evaluated from  $P_{c,n}$ ,  $d_n$ , and the probability distribution of the instantaneous SNR,  $\dot{\gamma}_s$ (refer to [6, Sec. V] for more details of the evaluation). Note that E[D] is a function of the SNR as well as the spectral efficiency and the space-time code that are assigned to each of  $N_P$  progressive packets.

Let  $C_i$  denote the space-time code assigned to the *i*th packet. One can find the optimal set of space-time codes  $\mathbf{C}_{\text{opt}} = [C_1, \ldots, C_{N_P}]_{\text{opt}}$  which minimizes the expected distortion over a range of SNRs using the weighted cost function as follows:

$$\underset{C_1,\ldots,C_{N_P}}{\operatorname{arg\,min}} \; \frac{\int_0^\infty \omega(\gamma_s) E[D] d\gamma_s}{\int_0^\infty \omega(\gamma_s) d\gamma_s} \tag{24}$$

where  $w(\gamma_s) \in [0,1]$  is the weight function. For example,  $w(\gamma_s)$  can be chosen such that  $w(\gamma_s) = 1$  for  $\gamma_{s,a} \leq \gamma_s \leq \gamma_{s,b}$ , and  $w(\gamma_s) = 0$  otherwise. In broadcast or multicast systems, that weight function indicates that SNRs of multiple receivers are uniformly distributed in  $\gamma_{s,a} \leq \gamma_s \leq \gamma_{s,b}$ . Eq. (24) indicates that a set of codes,  $C_1, \ldots, C_{N_P}$ , is chosen such that the total sum of the expected distortion of the receivers distributed in  $\gamma_{s,a} \leq \gamma_s \leq \gamma_{s,b}$  is minimized. Note that the amount of computation involved in (24) exponentially grows as  $N_P$  increases. Alternatively, as presented in Section IV, we may choose a set of codes with the constraint that the earliest *i* consecutive packets are encoded with OSTBC, the last *j* consecutive packets are encoded with D-BLAST, and the remaining  $N_P - i - j$  packets are encoded with D-BLAST.

To compare the image quality, we use the PSNR, defined as  $255^2/E[D]$ . We evaluate the PSNR performance as follows. We first compute (24) using the expected distortion, E[D], obtained from both  $d_n$  and  $P_c(n)$ , and the weight function,  $w(\gamma_s)$ , given below (24). Next, with the optimal set of codes,  $C_{\rm opt}$ , obtained from (24), we evaluate the PSNR over a range of SNRs,  $\gamma_{s,a} \leq \gamma_s \leq \gamma_{s,b}$ . The performance is numerically evaluated for the case when a sequence of  $N_P = 11$  progressive packets is transmitted in  $2 \times 2$  MIMO systems as an example, and we assume that the spectral efficiencies are assigned in a way such that  $\mathbf{R}$ = [2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 7.0, 8.0] (bits/s/Hz), where the *i*th component is the spectral efficiency assigned to the *i*th packet. For this specific setup, the optimal set of spacetime codes computed from (24) is given by  $C_1 = \text{OSTBC}$ ,  $C_2 = \cdots = C_9 = \text{D-BLAST}$ , and  $C_{10} = C_{11} = \text{V-BLAST}$ . Fig. 3 shows the PSNR of such an optimal set of space-time codes, in addition to showing the PSNRs of other suboptimal sets of codes, such as the sets at the 75th and 50th percentiles among the sets of codes (note that the number of possible sets is  $3^{N_P}$ ), and the worst set of codes which shows the poorest performance. Fig. 3 also shows the PSNR corresponding to the expected distortion that is averaged over all possible sets of codes. From this example, it is seen that PSNR performance of the progressive source is sensitive to the way space-time codes are assigned to a sequence of packets, in part due to the unequal target error rates and spectral efficiencies of the bitstream.

Fig. 3 also shows the PSNR performance when (24) is computed with the constraint proposed in Section IV. In this case, the number of possible sets of space-time codes is reduced from  $3^{N_P}$  to  $\binom{N_P+1}{2}$ . We note that the same optimal set of codes has been obtained when (24) is computed with and without the constraint. That is, without losing any PSNR, the computational complexity involved with the optimization can be exponentially reduced by exploiting the monotonic behavior of the crossover point, as shown in Fig. 1. It is further seen that the PSNR performance which corresponds to the expected distortion averaged over all possible sets of codes becomes better when the constraint in Section IV is introduced, which shows that, on the average, the proposed constraint is a good strategy for the space-time coding of progressive sources.

Lastly, we consider spatially correlated Rayleigh and Rician fading channels, instead of the i.i.d. Rayleigh channels described in Section II. Note that DMT characteristics, with multiplexing and diversity gains defined in (1) at high SNR, are not influenced by spatial correlation or line-of-sight (LOS) signal components [7], [8]. This is because, as stated in [7], when the SNR approaches infinity, only the number of channel eigenmodes determines the performance, i.e., the relative strength of eigenmodes, which is primarily affected by spatial correlation or LOS components, does not affect high SNR behavior. From this, it follows that the analysis in Sections II and III is also valid over correlated Rayleigh or Rician channels at high SNR. Our numerical evaluations show that the crossover points in such propagation channels behave in the same way as do those for the i.i.d. Rayleigh channels, though they are not depicted here for limited space.

### VI. CONCLUSIONS

When a sequence of multimedia scalable (or progressive) packets is transmitted over MIMO channels, due to the differences of importance in the bitstream, the tradeoff between the space-time codes needs to be specified in terms of their target error rates and spectral efficiencies. To address the matter, by exploiting DMT functions, we analyzed the crossover point of the outage probabilities of the space-time codes. The work in this paper extended [3] to more general cases, in that the above results can be applied to any space-time codes, receivers, and propagation channels with given DMT functions.

As a specific example, we considered D-BLAST with a GZF receiver, V-BLAST with an MMSE receiver, and OS-TBC, and proved the monotonic behavior of their crossover points, which holds in spatially correlated Rayleigh and Rician channels, as well as in i.i.d. Rayleigh channels. Based on that, we proposed an optimization method for the space-time coding of a sequence of numerous progressive packets. The numerical evaluation showed that, with the proposed method, the computational complexity involved with the optimization

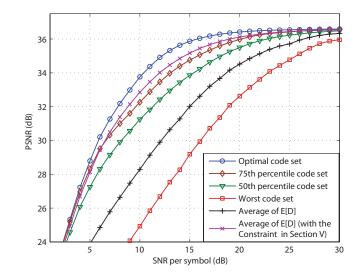


Fig. 3. PSNR for  $2 \times 2$  MIMO systems in i.i.d. Rayleigh channels.

is exponentially reduced without any PSNR degradation, compared to an exhaustive search.

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