

Optimal Multiplexed Hierarchical Modulation for Unequal Error Protection of Progressive Bit Streams

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Abstract—Progressive image and scalable video have gradual differences of importance in their bitstreams, which can benefit from multiple levels of unequal error protection (UEP). Though hierarchical modulation has been intensively studied as an UEP approach for digital broadcasting and multimedia transmission, methods of achieving a large number of UEP levels have rarely been studied. In this paper, we propose a multilevel UEP system using multiplexed hierarchical quadrature amplitude modulation (QAM) for progressive transmission over mobile radio channels. We suggest a specific way of multiplexing, and prove that multiple levels of UEP are achieved by the suggested method. When the BER is dominated by the minimum Euclidian distance, we derive an optimal multiplexing approach which minimizes both the average and peak powers. An asymmetric hierarchical QAM which reduces the peak-to-average power ratio (PAPR) without performance loss is also proposed. Numerical results show that the performance of progressive transmission over Rayleigh fading channels is significantly enhanced by the proposed UEP systems.

I. INTRODUCTION

When a communication system transmits messages over mobile radio channels, they are subject to errors, in part because mobile channels typically exhibit time-variant channel-quality fluctuations. For two-way communication links, these effects can be mitigated using adaptive methods. However, the adaptive schemes require a reliable feedback link from the receiver to the transmitter. Moreover, for a one-way broadcast system, those schemes are not appropriate because of the nature of broadcasting. When adaptive schemes cannot be used, the way to ensure communications is to classify the data into multiple classes with unequal error protection (UEP).

Since the theoretical and conceptual basis for UEP was initiated by Cover [1], much of the work has shown that one method of achieving UEP is based on a constellation of nonuniformly spaced signal points [2]–[5], which is called a hierarchical constellation. In this constellation, more important bits in a symbol have larger minimum Euclidian distance than less important bits. Hierarchical constellations were intensively studied for digital broadcasting systems and multimedia transmission [2][4]–[7]. Moreover, the Digital Video Broadcasting (DVB-T) standard [8], which is now commercially available, incorporated hierarchical QAM for layered video data transmission.

Progressive image and scalable video encoders [9][10], which are expected to have more prominence in the future, employ a progressive mode of transmission such that as more bits are transmitted, the source can be reconstructed with better

quality. Since these progressive transmissions have gradual differences of importance in their bitstreams, a large number of error protection levels are required. However, hierarchical modulation can achieve only a limited number of UEP levels for a given constellation size. For example, hierarchical 16 QAM provides two levels of UEP, and hierarchical 64 QAM yields at most three levels [11]. In the DVB-T standard, video data encoded by MPEG-2 consists of two different layers, and thus the use of hierarchical 16 or 64 QAM meets the required number of UEP levels. However, if scalable video is to be incorporated in a digital video broadcasting system, hierarchical 16 or 64 QAM may not meet the system needs. Most of the work about hierarchical modulation up to now has considered only two layered source coding, and to the best of our knowledge, methods of achieving an arbitrarily large number of UEP levels have not been studied.

In this paper, we propose a multilevel UEP system using multiplexed hierarchical modulation for progressive transmission over mobile radio channels. We propose a specific way of multiplexing, and prove that multiple levels of UEP are achieved by the proposed method. These results are presented in Section II. When the BER is dominated by the minimum Euclidian distance, we derive an optimal multiplexing approach which minimizes both the average and peak powers, as presented in Section III. While the suggested methods achieve multilevel UEP, the PAPR typically will be increased when constellations having distinct minimum distances are time-multiplexed. To mitigate this effect, an asymmetric hierarchical QAM constellation, which reduces the PAPR without performance loss, is proposed in Section IV. It is also shown that asymmetric hierarchical QAM can provide multilevel UEP even when multiplexed constellations need to have constant power. In Section V, the performance of the suggested UEP system for the transmission of progressive images is analyzed in terms of the expected distortion, and Section VI presents numerical results of performance analysis.

II. MULTILEVEL UEP BASED ON MULTIPLEXING HIERARCHICAL QAM CONSTELLATIONS

A. Hierarchical 16 QAM Constellation

Fig. 1 shows a hierarchical 16 QAM constellation with Gray coded bit mapping [8]. The two most significant bits (MSBs), i_1 and q_1 , determine one of the four clusters, and

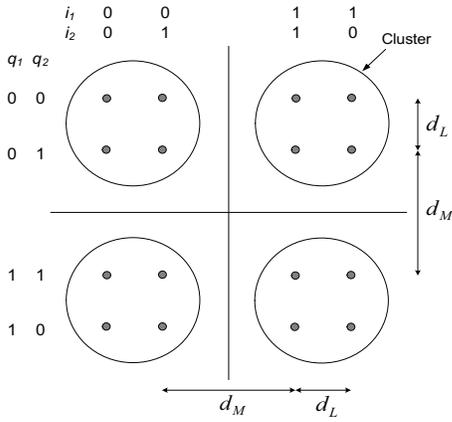


Fig. 1. Hierarchical 16 QAM constellation.

their minimum Euclidian distance is d_M . The two least significant bits (LSBs), i_2 and q_2 , determine which of the four signal points within the cluster is chosen, and their minimum Euclidian distance is d_L . The distance ratio $\alpha = d_M/d_L (> 1)$ determines how much more the MSBs are protected against errors than are the LSBs. Since hierarchical 16 QAM has one embedded QPSK subconstellation consisting of four clusters and provides two levels of UEP, it is denoted by 4/16 QAM.

We consider multiplexing N hierarchical 16 QAM constellations. The average power per symbol of all the multiplexed constellations, S_{avg} , is given by

$$S_{avg} = \frac{1}{N} \sum_{i=1}^N S_{avg,i} \quad (1)$$

where $S_{avg,i}$ is the average power per symbol of constellation i . $S_{avg,i}$ is given by

$$S_{avg,i} = \left(\frac{d_{M,i}}{2}\right)^2 + \left(\frac{d_{M,i}}{2} + d_{L,i}\right)^2 = \frac{d_{M,i}^2}{2} + d_{M,i}d_{L,i} + d_{L,i}^2 \quad (2)$$

where $d_{M,i}$ and $d_{L,i}$ are minimum distances for the MSBs and LSBs of constellation i , respectively. The BERs of the MSBs and LSBs of hierarchical 16 QAM constellation i , denoted by $P_{M,i}$ and $P_{L,i}$, respectively, are given by [11]

$$P_{M,i} = \frac{1}{2}Q\left(\frac{d_{M,i}}{2}\sqrt{\frac{2\gamma_s}{S_{avg}}}\right) + \frac{1}{2}Q\left(\left(\frac{d_{M,i}}{2} + d_{L,i}\right)\sqrt{\frac{2\gamma_s}{S_{avg}}}\right)$$

$$P_{L,i} = Q\left(\frac{d_{L,i}}{2}\sqrt{\frac{2\gamma_s}{S_{avg}}}\right) + \frac{1}{2}Q\left(\left(d_{M,i} + \frac{d_{L,i}}{2}\right)\sqrt{\frac{2\gamma_s}{S_{avg}}}\right) - \frac{1}{2}Q\left(\left(d_{M,i} + \frac{3d_{L,i}}{2}\right)\sqrt{\frac{2\gamma_s}{S_{avg}}}\right) \quad (3)$$

$$P_{M_n,i}^{app} = \begin{cases} \sum_{p=0}^{2^{K-n}-1} \frac{1}{2^{K-n}} Q\left(\left(\frac{d_{M_n,i}}{2} + \sum_{q=n+1}^K \lfloor \frac{p+2^{K-q}}{2^{K-q+1}} \rfloor d_{M_q,i}\right)\sqrt{\frac{2\gamma_s}{S_{avg}}}\right) & \text{for } 2 \leq n \leq K-1 \\ Q\left(\frac{d_{M_{K-1},i}}{2}\sqrt{\frac{2\gamma_s}{S_{avg}}}\right) + \frac{1}{2}Q\left(d_{M_{K-1},i} + \frac{d_{M_{K,i}}}{2}\sqrt{\frac{2\gamma_s}{S_{avg}}}\right) & \text{for } n = K \end{cases} \quad (6)$$

where γ_s is the signal-to-noise ratio (SNR) per symbol, and $Q(x) = 1/\sqrt{2\pi} \int_x^\infty e^{-y^2/2} dy$. The following theorem states

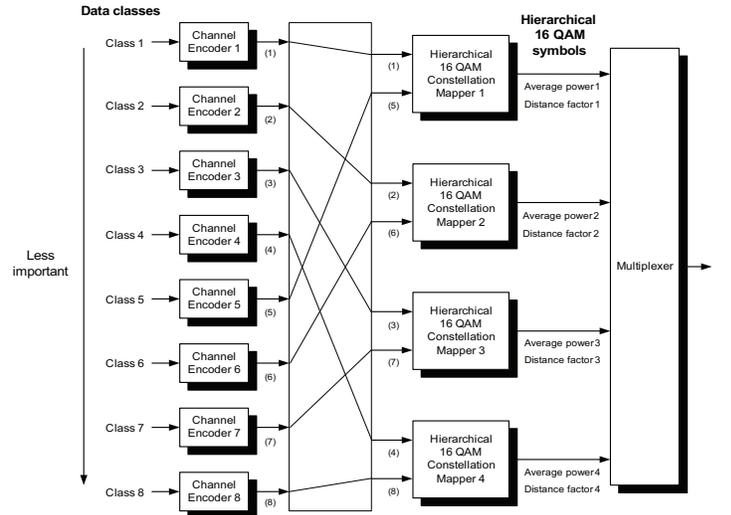


Fig. 2. The multilevel UEP system using multiplexed hierarchical 16 QAM constellations based on Theorem 1.

that $2N$ levels of UEP can be achieved by multiplexing N hierarchical 16 QAM constellations.

Theorem 1: For N hierarchical 16 QAM constellations, $P_{M,i}$ and $P_{L,i}$, given by (3), satisfy

$$P_{M,1} < P_{M,2} < \dots < P_{M,N} < P_{L,1} < P_{L,2} < \dots < P_{L,N} \quad (4)$$

for all SNR if

$$d_{M,1} > d_{M,2} > \dots > d_{M,N} > d_{L,1} > d_{L,2} > \dots > d_{L,N}. \quad (5)$$

Proof: The proof of this theorem as well as the proofs of all other results are not included here due to space limitations, but they can be found in [12]. \square

Fig. 2 depicts the multilevel UEP system using multiplexed hierarchical 16 QAM constellations based on Theorem 1 for eight data classes ($N = 4$).

B. Hierarchical 2^{2K} ($K \geq 3$) QAM Constellation

Next, we consider multiplexing N hierarchical 2^{2K} ($K \geq 3$) QAM constellations. Let $d_{M_n,i}$ ($1 \leq n \leq K$) denote the minimum distance for the n th MSBs of constellation i ($1 \leq i \leq N$).

Theorem 2: If the SNR of interest for the n th MSBs ($2 \leq n \leq K$) is sufficiently large so that the probability of the noise exceeding the Euclidian distance of $d_{M_{n-1},i} + \frac{1}{2}d_{M_n,i}$ is insignificant compared to that of the noise exceeding $\frac{1}{2}d_{M_n,i}$ (note that the distance ratio of the hierarchical constellation, $d_{M_{n-1},i}/d_{M_n,i}$ is greater than unity), the BER of the n th MSBs ($2 \leq n \leq K$), $P_{M_n,i}$, becomes

where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x . Note that for the MSBs (i.e., $n = 1$), the top line of (6) is the exact BER expression when n is set to unity (i.e., $P_{M_1,i}^{app} = P_{M_1,i}$).

Theorem 3: For N hierarchical 2^{2K} QAM constellations, $P_{M_n,i}^{app}$, given by (6), satisfy

$$P_{M_1,1}^{app} < \dots < P_{M_1,N}^{app} < P_{M_2,1}^{app} < \dots < P_{M_2,N}^{app} < \dots < P_{M_K,1}^{app} < \dots < P_{M_K,N}^{app}$$

if

$$d_{M_1,1} > \dots > d_{M_1,N} > d_{M_2,1} > \dots > d_{M_2,N} > \dots > d_{M_K,1} > \dots > d_{M_K,N} \quad (7)$$

Theorem 3 tells us that, by multiplexing N hierarchical 2^{2K} ($K \geq 3$) QAM constellations having minimum distances satisfying (7), KN levels of UEP are achieved under the assumption that the SNR of interest for the n th MSBs ($2 \leq n \leq K$) is reasonably large so that the condition of Theorem 2 is satisfied.

III. OPTIMAL MULTIPLEXING OF HIERARCHICAL QAM CONSTELLATIONS FOR HIGH SNR

In this section, we define high SNR as an SNR which is sufficiently large so that the BER is dominated by the error function term having the minimum Euclidian distance.

A. Hierarchical $2^{2J}/2^{2K}$ ($K > J \geq 1$) QAM Constellation

We first analyze a hierarchical 4/16 QAM (or 16 QAM) constellation, as a simple example of the hierarchical $2^{2J}/2^{2K}$ QAM constellations [11]. For high SNR, from (3), the BERs of a hierarchical 16 QAM constellation i ($1 \leq i \leq N$) are given by

$$P_{M,i} \approx \frac{1}{2} Q \left(\frac{d_{M,i}}{2} \sqrt{\frac{2\gamma_s}{S_{avg}}} \right) \text{ and } P_{L,i} \approx Q \left(\frac{d_{L,i}}{2} \sqrt{\frac{2\gamma_s}{S_{avg}}} \right). \quad (8)$$

Theorem 4: Suppose that there are N multiplexed hierarchical 16 QAM constellations, and the minimum distances satisfying (5) are given. Also, suppose the given minimum distances can be permuted such that $d_{M,1}, \dots, d_{M,N}$ for the MSBs can be arbitrarily combined with $d_{L,1}, \dots, d_{L,N}$ for the LSBs. After the distances are permuted, the resultant minimum distances for the MSBs and LSBs of constellation i , denoted by $\tilde{d}_{M,i}$ and $\tilde{d}_{L,i}$, respectively, can be expressed as

$$\tilde{d}_{M,i} = d_{M,i} \text{ and } \tilde{d}_{L,\pi(i)} = d_{L,i} \quad (9)$$

where $\pi(i)$ is the index of the constellation to which $d_{L,i}$ is permuted. Then, with the permuted distances given by (9), we have

$$P_{M,1} < \dots < P_{M,N} < P_{L,\pi(1)} < \dots < P_{L,\pi(N)} \quad (10)$$

In contrast to Theorem 1, Theorem 4 tells us that $2N$ levels of UEP are achieved for high SNR even after the minimum distances satisfying (5) are arbitrarily permuted.

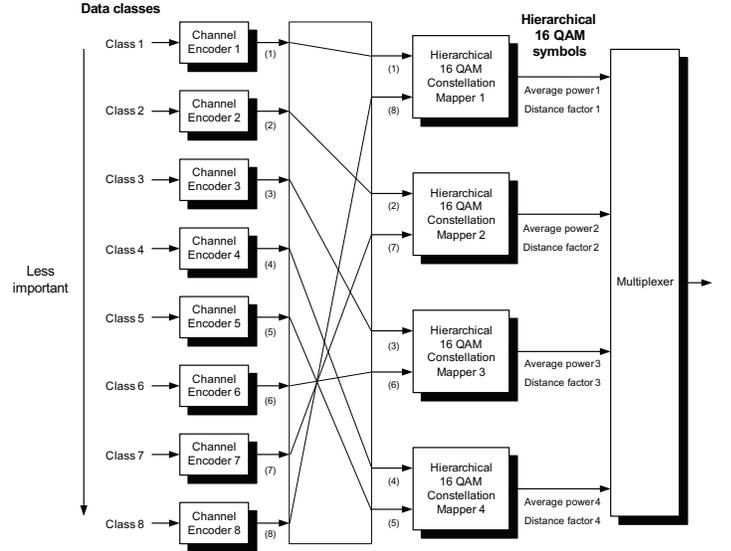


Fig. 3. The multilevel UEP system using multiplexed hierarchical 16 QAM constellations based on Theorems 6 and 7.

Corollary 5: From Theorem 4, regardless of how the minimum distances are permuted, the BERs given by (10) stay the same.

Theorem 6: After the distances are permuted as described in Theorem 4, the average power, S_{avg} , given by

$$S_{avg} = \frac{1}{N} \sum_{i=1}^N \left(\frac{\tilde{d}_{M,i}^2}{2} + \tilde{d}_{M,i} \tilde{d}_{L,i} + \tilde{d}_{L,i}^2 \right) \quad (11)$$

is minimized if and only if distances are permuted such that $d_{M,i}$ is combined with $d_{L,N+1-i}$ in the same constellation. That is,

$$\tilde{d}_{M,i} = d_{M,i} \text{ and } \tilde{d}_{L,i} = d_{L,N+1-i} \quad (1 \leq i \leq N). \quad (12)$$

Corollary 5 and Theorem 6 indicate that the average power is minimized by permuting distances according to (12), while the BERs are unchanged for high SNR.

Next, we consider the peak signal power of the multiplexed hierarchical constellations. If we assume that all the constellations are time-multiplexed, the peak power of all the multiplexed hierarchical constellations, S_{peak} , is given by

$$S_{peak} = \max \left[\left\{ S_{peak,i} \mid 1 \leq i \leq N \right\} \right] \quad (13)$$

where $\max[X]$ denotes the maximum element of the set X , and $S_{peak,i}$ is the peak power of a hierarchical constellation i given by

$$S_{peak,i} = 2 \left(\frac{d_{M,i}}{2} + d_{L,i} \right)^2 = \frac{d_{M,i}^2}{2} + 2d_{M,i}d_{L,i} + 2d_{L,i}^2. \quad (14)$$

Theorem 7: After the distances are permuted as described in Theorem 4, the peak power, S_{peak} , given by

$$S_{peak} = \max \left[\left\{ \frac{\tilde{d}_{M,i}^2}{2} + 2\tilde{d}_{M,i}\tilde{d}_{L,i} + 2\tilde{d}_{L,i}^2 \mid 1 \leq i \leq N \right\} \right] \quad (15)$$

is minimized if the distances are permuted according to (12) of Theorem 6. \square

Theorems 6 and 7 tell us that the permutation of the distances that minimizes the average power also, coincidentally, minimizes the peak power. Fig. 3 depicts the multilevel UEP system using multiplexed hierarchical 16 QAM constellations based on Theorems 6 and 7 for eight data classes ($N = 4$).

It can be shown that the results for hierarchical 4/16 QAM (or 16 QAM) can be generalized to hierarchical $2^{2J}/2^{2K}$ ($K > J \geq 1$) QAM.

IV. ASYMMETRIC HIERARCHICAL QAM CONSTELLATION

We propose asymmetric hierarchical QAM which reduces the PAPR of time-multiplexed hierarchical constellations without performance loss. From here onwards, we refer to conventional hierarchical QAM, which has been presented in Sections II and III, as symmetric hierarchical QAM, in order to distinguish it from asymmetric hierarchical QAM.

A. Hierarchical 2^{2K} ($K \geq 2$) QAM Constellation

For an asymmetric hierarchical 2^{2K} QAM, the minimum Euclidian distances for the inphase and quadrature components are different from each other. We present asymmetric hierarchical 16 QAM, depicted in Fig. 4, as a simple example. The MSB i_1 for the inphase component determines the first cluster, and its minimum distance is $d_M^{A,I}$. The MSB q_1 for the quadrature component determines the second cluster within the first cluster that i_1 determined, and its minimum distance is $d_M^{A,Q}$. The LSB i_2 for the inphase component determines the third cluster, and its minimum distance is $d_L^{A,I}$, and the LSB q_2 for the quadrature component determines the specific signal point within the third cluster, and has minimum distance $d_L^{A,Q}$. Asymmetric hierarchical 16 QAM has three embedded subconstellations, and it provides four levels of UEP when $d_M^{A,I} > d_M^{A,Q} > d_L^{A,I} > d_L^{A,Q}$.

In order to provide $2N$ levels of UEP, we consider multiplexing $N/2$ (N is assumed to be even) asymmetric hierarchical 16 QAM constellations.

Theorem 8: Suppose that there are N multiplexed symmetric hierarchical 16 QAM constellations whose minimum distances are given by $d_{M,1}, \dots, d_{M,N}$ and $d_{L,1}, \dots, d_{L,N}$. Also, suppose that there are $N/2$ asymmetric hierarchical 16 QAM constellations, and the minimum distances for the inphase and quadrature components of asymmetric hierarchical constellation i are the same as those of two distinct symmetric hierarchical constellations $x(i)$ and $y(i)$, respectively ($1 \leq i \leq N/2$). In other words, for $1 \leq i \leq N/2$,

$$\begin{aligned} d_{M,i}^{A,I} &= d_{M,x(i)} \text{ and } d_{L,i}^{A,I} = d_{L,x(i)} \\ d_{M,i}^{A,Q} &= d_{M,y(i)} \text{ and } d_{L,i}^{A,Q} = d_{L,y(i)} \end{aligned} \quad (16)$$

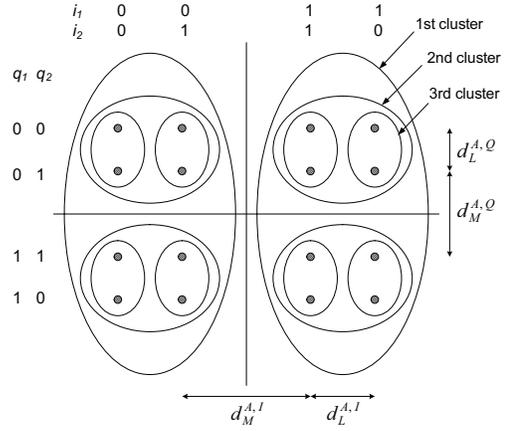


Fig. 4. Asymmetric hierarchical 16 QAM constellation .

where $d_{M,i}^{A,I}$, $d_{L,i}^{A,I}$, $d_{M,i}^{A,Q}$, and $d_{L,i}^{A,Q}$ are the minimum distances for the inphase MSB and LSB, and quadrature MSB and LSB, respectively, and $x(i)$ and $y(i)$ satisfy

$$\begin{aligned} x(i), y(i) &\in \{1, \dots, N\}, \\ \{x(i), y(i) \mid 1 \leq i \leq N/2\} &= \{1, \dots, N\}. \end{aligned} \quad (17)$$

With the minimum distances given by (16), the average power and BERs of $N/2$ multiplexed asymmetric hierarchical 16 QAM constellations are the same as those of N multiplexed symmetric hierarchical 16 QAM constellations, regardless of the choice of $x(i)$ and $y(i)$ satisfying (17). \square

Theorem 9: Suppose that the minimum distances of the N multiplexed symmetric hierarchical 16 QAM constellations satisfy (5) of Theorem 1. Then, with the minimum distances given by (16), the peak power of all $N/2$ multiplexed asymmetric hierarchical 16 QAM constellations is less than that of all N multiplexed symmetric hierarchical 16 QAM, S_{peak} , given by (13) and (14), regardless of the choice of $x(i)$ and $y(i)$ satisfying (17). \square

Theorems 8 and 9 tell us that when asymmetric hierarchical 16 QAM is used instead of symmetric hierarchical 16 QAM, the PAPR is reduced without performance loss. Note that this result holds for all SNR.

We now consider the case where it is desirable for the multiplexed hierarchical QAM constellations to have the same average power (i.e., constant power), either due to the limited capability of a power amplifier, or for cochannel interference control.

Theorem 10: Suppose that $N/2$ multiplexed asymmetric hierarchical 16 QAM constellations are required to have constant power, and their minimum distances are given by (16). If $x(i)$ and $y(i)$ are chosen as $x(i) = i$ and $y(i) = N + 1 - i$ ($1 \leq i \leq N/2$), $2N$ levels of UEP still can be achieved. \square

It can be shown that the results for asymmetric hierarchical 16 QAM can be generalized to asymmetric hierarchical 2^{2K} ($K \geq 2$) QAM.

V. THE PERFORMANCE OF THE PROPOSED UEP SYSTEM FOR PROGRESSIVE BITSTREAM TRANSMISSION

We analyze the performance of the proposed UEP system for progressive image source transmission over Rayleigh fading channels. We first consider the UEP system depicted in Fig. 2. The system takes successive blocks (data classes) of the compressed progressive bitstream, and transforms them into a sequence of channel codewords of fixed length l_c with error detection and correction capability. Then, the coded classes are mapped to the multiplexed symmetric hierarchical 16 QAM constellations, whose minimum distances satisfy (5) of Theorem 1 to achieve $2N$ levels of UEP. At the receiver, if a received class is correctly decoded, then the next class is considered by the decoder. Otherwise, the decoding is stopped.

Let r_i be an error correction code rate for class i ($1 \leq i \leq 2N$), and $\underline{d}_i = (d_{M,c(i)}, d_{L,c(i)})$ be a pair of minimum distances of some specific constellation $c(i)$ ($1 \leq c(i) \leq N$) to which class i is mapped. Let $p(r_i, \underline{d}_i, \gamma_s)$ denote the probability of a decoding error of class i . Then, the probability that no decoding errors occur in the first i classes with an error in the next one, $P_{c,i}$ ($1 \leq i \leq 2N - 1$), is given by

$$P_{c,i} = p(r_{i+1}, \underline{d}_{i+1}, \gamma_s) \prod_{j=1}^i (1 - p(r_j, \underline{d}_j, \gamma_s)) \quad (18)$$

Note that $P_{c,0} = p(\gamma_1, \underline{d}_1, \gamma_s)$ is the probability of an error in the first class, and $P_{c,2N} = \prod_{j=1}^{2N} (1 - p(r_j, \underline{d}_j, \gamma_s))$ is the probability that all $2N$ classes are correctly decoded. The end-to-end performance can be measured by the expected distortion, $E[D]$, given by

$$E[D] = \sum_{i=0}^{2N} P_{c,i} D_i \quad (19)$$

where D_i is the reconstruction error using the first i classes ($1 \leq i \leq 2N$), and D_0 is a constant. For the case of an uncoded system, D_i is given by $D_i = V(il_c)$, where $V(x)$ denotes the operational rate-distortion function of the source coder. Also, for the uncoded system, $p(r_i, \underline{d}_i, \gamma_s)$ can be obtained analytically:

$$p(r_i, \underline{d}_i, \gamma_s) = p(\underline{d}_i, \gamma_s) = 1 - \{1 - P_i(\underline{d}_i, \gamma_s)\}^{l_c} \quad (20)$$

where P_i , a function of \underline{d}_i and γ_s , is the BER of data class i .

Note that for a given SNR of γ_s , $E[D]$ is the conditional expected distortion. In situations when exact SNR information is not available at the transmitter, one can find the minimum distances, $\underline{d}_1, \dots, \underline{d}_{2N}$ (or $d_{M,1}, \dots, d_{M,N}$ and $d_{L,1}, \dots, d_{L,N}$), which minimize the expected distortion over a range of expected SNRs using the weighted cost function

$$\arg \max_{\underline{d}_1, \dots, \underline{d}_{2N}} \frac{\int_0^\infty \omega(\gamma_s) E[D] d\gamma_s}{\int_0^\infty \omega(\gamma_s) d\gamma_s} \quad (21)$$

where $\omega(\gamma_s)$ in $[0, 1]$ is the weight function. For example, $\omega(\gamma_s)$ can be given by

$$\omega(\gamma_s) = \begin{cases} 1, & \text{for } \gamma_s^a \leq \gamma_s \leq \gamma_s^b \\ 0, & \text{otherwise.} \end{cases} \quad (22)$$

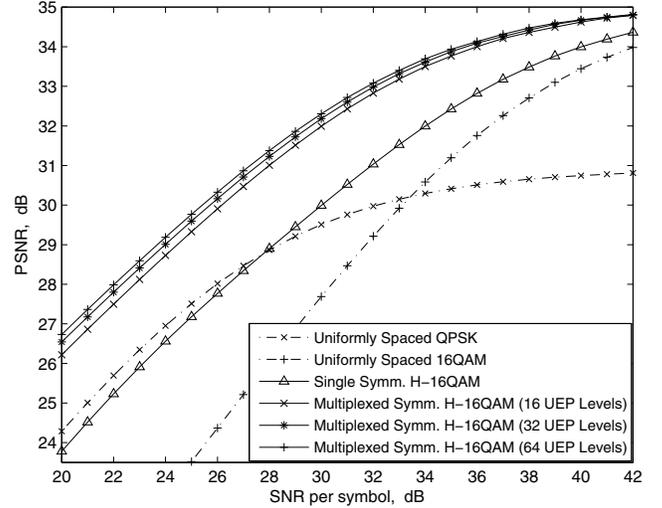


Fig. 5. PSNR performance of UEP system using multiplexed symmetric hierarchical 16 QAM (H-16QAM denotes hierarchical 16 QAM).

VI. NUMERICAL RESULTS

We evaluate the performance of the suggested UEP system for the progressive source coder SPIHT [9] as an example. We provide the results for the standard 8 bits per pixel (bpp) 512×512 Lena image with a transmission rate of 0.375 bpp. We define a frame as a group of constellation symbols to which one image bitstream is mapped. It is assumed that the transmitted signal experiences block Rayleigh fading in which channel coefficients are nearly constant over a frame. Lastly, to compare the image quality, we use peak-signal-to-noise ratio (PSNR) defined as $255^2/E[D]$, where $E[D]$ is given by (19).

We present the PSNR performance for the uncoded case by numerically evaluating (18) – (22) as follows: We first compute (21) using the expected distortion, $E[D]$, derived from (18) – (20) for the block Rayleigh fading channel, and the weight function, $\omega(\gamma_s)$, given by (22). Next, with $\underline{d}_1, \dots, \underline{d}_{2N}$ (or $d_{M,1}, \dots, d_{M,N}$ and $d_{L,1}, \dots, d_{L,N}$) obtained from (21), we evaluate PSNR over a range of expected SNRs given by (22).

Fig. 5 shows the PSNR performance of the multiplexed symmetric hierarchical 16 QAM as well as that for single symmetric hierarchical 16 QAM. The PSNR of single symmetric hierarchical 16 QAM is evaluated in the same way as that for multiplexed symmetric hierarchical 16 QAM. From Fig. 5, it is seen that multiplexed symmetric hierarchical 16 QAM improves the performance more than does single symmetric hierarchical 16 QAM. It is also seen that 32 multiplexed symmetric hierarchical 16 QAM constellations, which provide 64 levels of UEP, have almost saturated performance in this evaluation. Note that the performance of $N/2$ multiplexed asymmetric hierarchical 16 QAM is the same as that of N multiplexed symmetric hierarchical 16 QAM ($N = 8, 16, 32$), as stated by Theorem 8, though the former is not depicted here. Table I shows the PAPRs of the multiplexed symmetric or

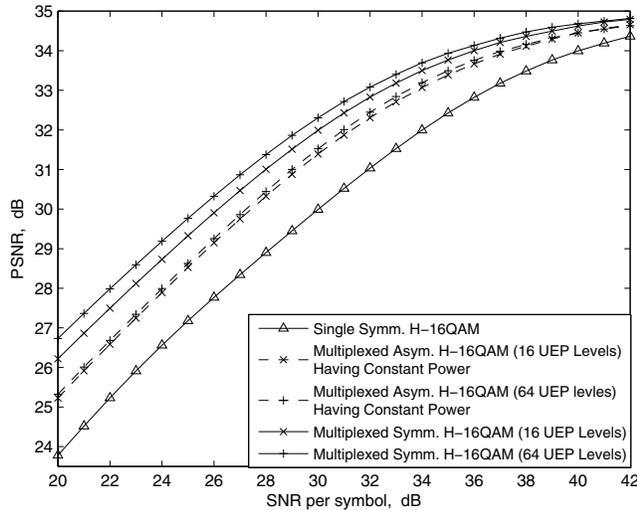


Fig. 6. PSNR performance of UEP system using multiplexed asymmetric hierarchical 16 QAM having constant power (H-16QAM denotes hierarchical 16 QAM).

asymmetric hierarchical 16 QAM. For reference, the PAPRs of single symmetric hierarchical 16 QAM and uniformly spaced 16 QAM are listed in Table II. From Tables I and II, it is seen that when symmetric hierarchical 16 QAM constellations are time-multiplexed, they have larger PAPR than does uniformly spaced 16 QAM. Table I also shows that the PAPR is reduced when an asymmetric hierarchical constellation is used, as stated in Theorem 9.

Fig. 6 shows the PSNR performance of the multiplexed asymmetric hierarchical 16 QAM constellations having constant power. It is seen that the performance is degraded when constellations are required to have constant power. However, the PAPR problem is completely solved.

VII. CONCLUSIONS

Progressive image and scalable video encoders employ progressive transmission, which benefits from multiple levels of UEP. Though hierarchical modulation has been intensively

TABLE I
PAPR OF MULTIPLEXED HIERARCHICAL 16 QAM

| Number of UEP Levels | 4 | 16 | 64 |
|--|------|------|------|
| Multiplexed Symm.H-16QAM | 3.31 | 6.87 | 9.43 |
| Multiplexed Asym. H-16QAM | 1.11 | 4.18 | 6.60 |
| Multiplexed Asym. H-16QAM Having Constant power | 1.11 | 1.43 | 1.46 |

TABLE II
PAPR OF UNIFORMLY SPACED 16 QAM AND SINGLE SYMMETRIC HIERARCHICAL 16 QAM

| | PAPR (dB) |
|--------------------------------------|-----------|
| Uniformly Spaced 16 QAM | 2.55 |
| Single Symmetric Hierarchical 16 QAM | 0.90 |

studied for digital broadcasting and multimedia transmission, methods of achieving an arbitrarily large number of UEP levels, to the best of our knowledge, have not been studied.

In this paper, we proposed a multilevel UEP system using multiplexed hierarchical modulation for progressive transmission over mobile radio channels. We suggested a specific way of multiplexing N hierarchical 2^{2K} QAM constellations ($K \geq 2$) and proved that the suggested method achieves KN levels of UEP. When the BER is dominated by the error function term having the minimum Euclidian distance, we derived an optimal multiplexing approach which minimizes both the average and peak powers for hierarchical $2^{2J}/2^{2K}$ QAM ($K > J \geq 1$) constellations (typical examples are 4/16 QAM and 4/64 QAM, which are employed in the DVB-T standard). We designed an asymmetric hierarchical QAM constellation, which reduces the PAPR without performance loss. The asymmetric hierarchical QAM also can be used to provide multilevel UEP even when multiplexed constellations are required to have constant power. Numerical results showed that the proposed multilevel UEP system based on multiplexed modulation significantly enhances the performance for progressive transmission over Rayleigh fading channels.

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