Optimization of Scalable Broadcast for a Large Number of Antennas

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Abstract—In this paper, for a system incorporating a large number of antennas, we address the optimal space–time coding of multimedia scalable sources, which require unequal target error rates in their bitstream. First, in terms of the number of antennas, we analyze the behavior of the crossover point of the outage probability curves for the vertical Bell Laboratories space–time (V-BLAST) architecture with a linear or a maximum-likelihood receiver, and orthogonal space–time block codes (OSTBCs). We prove that, as the number of antennas increases with the transmission data rate fixed, the crossover point in outage probability monotonically decreases. This holds for any data rate employed by the system and is valid over propagation channels such as spatially correlated Rayleigh or Rician fading channels, as well as independent and identically distributed Rayleigh channels. We next show that, over such propagation channels with a large number of antennas, those analytical results can be used to simplify the computational complexity involved with the optimal space–time coding of a sequence of scalable packets, with no performance degradation.

Index Terms—Diversity–multiplexing tradeoff (DMT), maximum-likelihood (ML) receiver, minimum mean square error (MMSE) receiver, multimedia scalable sources, multiple-input–multiple-output (MIMO) systems, orthogonal space–time block codes (OSTBC), outage probability, Rician channel, spatially correlated Rayleigh channel, vertical Bell Laboratories layered space–time (V-BLAST) architecture, zero-forcing (ZF) receiver.

I. INTRODUCTION

The cross-layer optimization of wireless multimedia communications [1]–[3] has been motivated by the increasing demand for mobile multimedia services. Multimedia scalable sources, such as embedded images or scalable video [4]–[7], have a feature that the quality of the decoded source improves when the number of successfully received bits increases. Such advances in source codecs, however, have made the source bitstreams very susceptible to the impairments of mobile fading channels.

Multiple-input–multiple-output (MIMO) channels offer large gains in terms of link reliability and data rate. MIMO systems can be embodied in different ways to provide either a diversity gain or a spectral efficiency gain. Spatial diversity schemes, such as orthogonal space–time block codes (OSTBCs) [8], [9], extract diversity gain to combat signal fading from the channels and obtain reliability. The OSTBC is an important class of the space–time block code, in that it achieves the full diversity of channels with a very simple linear receiver. Spatial multiplexing schemes use a layered approach to increase capacity [10], [11]. One popular example is the vertical Bell Laboratories layered space–time (V-BLAST) architecture, where independent data signals are transmitted over antennas to increase the data rate, but full spatial diversity is usually not achieved.

Recently, with the exploration of high-frequency microwave waveforms and the progress in very large scale integration technology, a larger number of antennas can be incorporated in a MIMO system. In this paper, we study the optimal design of such a MIMO system with a large number of antennas, for the transmission of multimedia scalable sources over spatially correlated Rayleigh or Rician fading channels. We first compare the outage probability performances of V-BLAST, using either a linear or a maximum-likelihood (ML) receiver, with OSTBC in terms of the number of antennas. The spatial correlation of MIMO channels tends to increase if more antennas are incorporated in the limited amount of space (i.e., if the antenna separation diminishes) [13], [14]. In such spatially correlated channels, OSTBC exhibits much less performance degradation than does V-BLAST [15], [16], that is, the performance of OSTBC becomes comparable with that of V-BLAST. To compare the outage probabilities of V-BLAST and OSTBC, we exploit the diversity–multiplexing tradeoff (DMT) [17], which is a standard tool in the characterization of the performance of space–time codes, in slowly varying fading channels in the high signal-to-noise ratio (SNR) regime.

Using the outage probability expression of a space–time code in [18], which is derived from the given DMT function, we analyze how the crossover point of the outage probability curves of V-BLAST and OSTBC behaves in the high-SNR regime, as a function of the number of antennas. Note that if the number of transmit or receive antennas changes, the amount of diversity or SNR gain, as well as the spatial multiplexing rate of the space–time code, does not stay the same, and it is not clear how the crossover point behaves according to the effects of those
combined factors. We prove that, as the number of antennas increases with the transmission data rate fixed, the crossover point in outage probability monotonically decreases. The results are proven for any data rate employed by the space–time codes; furthermore, the results are valid in propagation channels such as spatially correlated Rayleigh or Rician fading channels, in addition to independent and identically distributed (i.i.d.) Rayleigh fading channels. In some literature, the crossover point of the ergodic capacity curves is investigated [19], [20]. In [18], the behavior of the crossover point of the error probabilities with regard to spectral efficiency was analyzed. However, we are unaware of any work that analytically compares the error probabil-

\[ \mathbf{H} = \sqrt{\frac{K}{K+1}} \mathbf{H} + \sqrt{\frac{1}{K+1}} \mathbf{R}_s^\frac{1}{2} \mathbf{H}_w \mathbf{R}_r^\frac{1}{2}, \]  

where \( K > 0 \) is the Rician factor, and \( \mathbf{H} \) represents the channel matrix related to line-of-sight (LOS) signal components. The Frobenius norm of \( \mathbf{H} \) is normalized as \((N_rN_t)^{1/2}\), and \( \mathbf{H} \) is assumed to be known to both the transmitter and the receiver. \( \mathbf{R}_s \) is an \( N_t \times N_t \) transmit spatial correlation matrix, \( \mathbf{R}_r \) is an \( N_r \times N_r \) receive spatial correlation matrix, and \((\cdot)^{1/2}\) represents the Hermitian square root of a matrix. We use the exponential correlation model with \( (\mathbf{R}_t)_{i,j} = \rho_{i,j} \) and \( (\mathbf{R}_r)_{i,j} = \rho_{r,i,j} \), where \((\cdot)_{i,j}\) denotes the \((i,j)\)th entry of a matrix; and \( \rho_t \) and \( \rho_r \) are the transmit and receive spatial correlation coefficients between adjacent antennas, respectively. In (2), \( \mathbf{H}_w \) is an \( N_r \times N_t \) channel matrix whose entries are i.i.d. \( \sim CN(0,1) \), and \( \mathbf{H}_w \) is assumed to be known at the receiver, but not known at the transmitter (i.e., channel state information (CSI) is available only at the receiver). It is also assumed that \( \mathbf{H}_w \) is random, but constant over \( T \) symbol durations. Let \( \gamma_s \) denote the SNR per symbol, which is defined as \( \gamma_s := \mathcal{E}[(|s_k|)^2]/\sigma_n^2 \), where \((\cdot)\) denotes the \(k\)th component of a vector. Let \( N_s \) denote the number of symbols packed within a space–time codeword \( \mathbf{S} = [s_1, \ldots, s_T] \). Then, the spatial multiplexing rate of \( \mathbf{S} \) is defined as \( N_s/T \).

Next, we briefly present the outage probability expression of the space–time code, which is derived in [18], for any given piecewise-linear DMT function [17]. We let \( r \) and \( d \) denote the multiplexing and diversity gains defined in [17], respectively. We consider a space–time code whose DMT characteristic function is given by

\[ d(r) = v - u_r, \quad \text{for } \alpha \leq r \leq \beta (\alpha > 0), \]  

where \( d(r) \geq 0 \), and \( u \geq 0 \) and \( v \geq 0 \) are real constants. Let \( P_{\text{out}}(\gamma_s) \) denote the outage probability for the space–time code whose DMT is given by (3). In [18], it is shown that, as \( \gamma_s \to \infty \), \( P_{\text{out}}(\gamma_s) \) can be expressed as

\[ P_{\text{out}}(\gamma_s) = k_d \left( \frac{2R}{k_r} \right)^\frac{1}{\gamma_s^{\alpha}} \leq \gamma_s \leq \left( \frac{2R}{k_r} \right)^\frac{1}{\gamma_s^{\beta}}, \]  

where \( k_d \) is an arbitrary positive constant, \( R \) is the spectral efficiency (bits/s/Hz), and \( k_r \) is an arbitrary positive constant subject to the constraint that \( 2R/k_r \) is greater than unity. Consider two space–time codes that have linear DMT characteristics as follows:

\[ d_1(r) = v_1 - u_1 r \quad \text{and} \quad d_2(r) = v_2 - u_2 r, \]

for \( \alpha \leq r \leq \beta (\alpha > 0), \)
That is, there exists a crossover in $\alpha < r < \beta$ for the two DMT functions. Let $P_{\text{out},1}(\gamma_s)$ and $P_{\text{out},2}(\gamma_s)$ denote the outage probabilities of the space–time codes whose DMT functions are given by $d_1(r)$ and $d_2(r)$, respectively. Then, from (4), as $\gamma_s \to \infty$, we have
\[
P_{\text{out},i}(\gamma_s) = k_d \left( \frac{2R}{k_r} \right)^{\frac{1}{2}} \frac{1}{\gamma_s^q} (i = 1, 2),
\]
(9)
for $(2^R/k_r)^{1/\beta} \leq \gamma_s \leq (2^R/k_r)^{1/\alpha}$. From (9), for a given spectral efficiency $R$, we find the outage probability $P_{\text{out}}$, for which $P_{\text{out},1}(\gamma_s)$ and $P_{\text{out},2}(\gamma_s)$ are identical. In [18], it is shown that the crossover point in outage probabilities $P_{\text{out}}$ is given by
\[
P_{\text{out}} = k_d \left( \frac{2R}{k_r} \right)^{\frac{1}{2}} \frac{1}{T_{\text{out}, \beta}}.
\]
Moreover, it is shown that
\[
P_{\text{out},1}(\gamma_s) < P_{\text{out},2}(\gamma_s), \quad \text{for} \quad \left( \frac{2R}{k_r} \right)^{1/\beta} \leq \gamma_s < \gamma_s^*,
\]
(10)
where $\gamma_s^*$ is a crossover point in SNR, which corresponds to $P_{\text{out}}$ in outage probability.

The DMT characteristics of V-BLAST, using either a minimum mean square error (MMSE) or a zero-forcing (ZF) linear receiver, with OSTBC, which are denoted by $d_V(r)$ and $d_O(r)$, respectively, are given by [22]
\[
d_V(r) = \left\{ \begin{array}{ll}
N_r - N_t + 1 + \frac{1}{N_t}(N_r - N_t + 1)r, & \text{for} \quad 0 \leq r \leq N_t, \\
0, & \text{for} \quad N_t < r < \infty,
\end{array} \right.
\]
(12)
\[
d_O(r) = \left\{ \begin{array}{ll}
N_r N_t - \frac{1}{N_t} N_t N_t r, & \text{for} \quad 0 \leq r \leq r_s, \\
0, & \text{for} \quad r_s < r < \infty,
\end{array} \right.
\]
(13)
where $r_s$ denotes the spatial multiplexing rate of the OSTBC. For $N_t = 2$, the Alamouti scheme achieves $r_s = 1$. On the other hand, $r_s = 3/4$ is the maximum achievable rate for $N_t = 3$ or 4 in the complex OSTBC, and $r_s = 1/2$ is the maximum rate for $N_t > 4$ [23]. To compare the preceding codes, it is assumed that $N_t \geq N_r \geq 2$.

III. BEHAVIOR OF CROSSOVER POINT FOR V-BLAST WITH A LINEAR RECEIVER AND ORTHOGONAL SPACE TIME BLOCK CODES IN TERMS OF THE NUMBER OF ANTENNAS

Using (12) and (13), we set $d_1(r) = d_V(r)$ and $d_2(r) = d_O(r)$ in (5). From $N_t \geq 2$ and $r_s \leq 1$, we have $r_s < N_t$. Thus, from (12) and (13), it is seen that, for the range of multiplexing gain, i.e., $0 < r \leq r_s$, the condition of (6) is satisfied. Furthermore, from $N_r \geq N_t \geq 2$, it can be shown that
\[
d_V(0) - d_O(0) = (N_r + 1)(1 - N_t) < 0,
\]
(14)
\[
d_V(r_s) - d_O(r_s) = (N_r - N_t + 1)(1 - r_s/N_t) > 0.
\]
(15)
Eqs. (14) and (15) satisfy the conditions of (7) and (8), respectively, when setting $\alpha = \varepsilon$ and $\beta = r_s$, where $\varepsilon > 0$ denotes an arbitrarily small positive number. As a result, (11) holds, which indicates that, in $(2^R/k_r)^{1/r_s} \leq \gamma_s < \infty$, there exists a crossover point of the outage probabilities for V-BLAST, using either an MMSE or a ZF linear receiver, with OSTBC. Furthermore, from (10), (12), and (13), it can be shown that the crossover point of the outage probabilities $P_{\text{out}}$ is given by
\[
P_{\text{out}} = k_d \left( \frac{2R}{k_r} \right)^{\frac{1}{2}} \frac{1}{T_{\text{out}, \beta}}.
\]
(16)
In the following, we will investigate the behavior of $P_{\text{out}}$ in terms of the number of antennas. If we let $N_t = N_r = n$ in (16), we have
\[
P_{\text{out}} = k_d \left( \frac{2R}{k_r} \right)^{\frac{1}{2}} \frac{1}{n^{2/\beta}}.
\]
(17)
We will prove that $P_{\text{out}}$, given by (17), is a strictly decreasing function in $n$, under the condition that $n \geq 2$. We define function $f(n)$ as
\[
f(n) = \frac{n - n^2/r_s}{n^2 - 1}.
\]
(18)
Assuming that $n$ is a real number, $df(n)/dn$ can be expressed as
\[
\frac{df(n)}{dn} = \frac{r_s n^2 + 2n - r_s}{r_s(n^2 - 1)^2}.
\]
(19)
Let $g(n) = -r_s n^2 + 2n - r_s$ be the numerator of $df(n)/dn$. Then, it can be shown that $g(n)$ is a strictly decreasing function in $n$ for $n \geq 1/r_s$. To begin, suppose that $n \geq 4$. Then, from the fact that $1/2 \leq r_s \leq 3/4$ (i.e., $4/3 \leq 1/r_s \leq 2$) for $N_t \geq 4$, we have
\[
g(n) \leq g(4) = -17r_s + 8 < 0.
\]
(20)
From (19) and (20), it follows that
\[
\frac{df(n)}{dn} < 0, \quad \text{for} \quad n \geq 4.
\]
(21)
If we substitute $n = 2, 3, \text{and } 4 \text{ along with the corresponding spatial multiplexing rates (i.e., } r_s = 1 \text{ for } N_t = 2 \text{ and } r_s = 3/4 \text{ for } N_t = 3 \text{ or } 4 \text{ into (18), we have}
\[
f(2) = -2/3, \quad f(3) = -9/8, \quad f(4) = -52/45.
\]
(22)
From (21) and the inequality of $f(2) > f(3) > f(4)$ in (22), it follows that $df(n)/dn < 0$ for $n \geq 2$. Thus, from (17), (18), and $k_d > 0$ and $2^R/k_r > 1$ given below (4), it is seen that $P_{\text{out}}$ is a strictly decreasing function in $n \geq 2$, regardless of the given spectral efficiency $R$. That is, as the number of antennas, i.e., $N_t = N_r \geq 2$, increases, the crossover point in the outage probability monotonically decreases for any fixed spectral efficiency. We note that, in the preceding analysis, the same spectral efficiency $R$ is used for both V-BLAST and OSTBC. Furthermore, $R$ stays the same when the number of antennas increases. Let $P_{\text{out},1}$ denote the crossover point when
the number of antennas $N_t = N_r = n_1$ is employed, and let $P^*_{\text{out},2}$ denote the crossover point when $N_t = N_r = n_2$ is used. Then, from the results given earlier, we have
\[ P^*_{\text{out},1} > P^*_{\text{out},2}, \quad \text{for } n_1 < n_2. \] (23)

We next analyze how the crossover point behaves as only the number of receive antennas $N_r$ increases, for any given number of transmit antennas $N_t$ ($\leq N_r$). Substituting $N_r = m$ into (16), we have
\[ P^*_{\text{out}} = k_d \left( \frac{R}{k_r} \right) \frac{(m-N_t)(m-N_t/r_s)m}{(m+1)(m+2)(m+3)}. \] (24)

It will be shown that $P^*_{\text{out}}$, given by (24), is a strictly decreasing function in $m$, under the condition that $m \geq N_t \geq 2$. We define function $h(m)$ as
\[ h(m) = \frac{(1-N_t/r_s)}{N_t-1} \cdot \frac{(m-N_t+1)}{m+1}. \] (25)

Let $p(m) = (m-N_t+1)/(m+1)$ be the second factor of $h(m)$. If we assume that $m$ is a real number, $dp(m)/dm$ can be expressed as
\[ \frac{dp(m)}{dm} = \frac{m^2 + 2m - N_t + 1}{(m+1)^2}. \] (26)

If we let $q(m) = m^2 + 2m - N_t + 1$ be the numerator of $dp(m)/dm$, it can be shown that $q(m)$ is a monotonically increasing function in $m$ for $m \geq 2$. From this and $m \geq N_t \geq 2$, $q(m)$ satisfies
\[ q(m) \geq q(N_t) = N_t^2 + N_t + 1 > 0. \] (27)

From (26) and (27), we have
\[ \frac{dp(m)}{dm} > 0, \quad \text{for } m \geq N_t. \] (28)

Furthermore, from $N_t \geq 2$ and $1/2 \leq r_s \leq 1$, we have $(1-N_t/r_s)(N_t-1) < 0$. Hence, from (24), (25), (28), and $k_d > 0$ and $2R/k_r > 1$ given in Section II, it follows that $P^*_{\text{out}}$ is a strictly decreasing function in $m$, regardless of the given spectral efficiency $R$ and the given number of transmit antennas $N_t$.

In other words, for any fixed spectral efficiency and any fixed number of transmit antennas, as the number of receive antennas, i.e., $N_r (\geq N_t \geq 2)$, increases, the crossover point in the outage probability monotonically decreases.

Suppose that the number of transmit antennas $N_t$ is fixed. Then, we let $P^*_{\text{out},1}$ denote the crossover point when the number of receive antennas $N_r = m_1 (\geq N_t)$ is used and let $P^*_{\text{out},2}$ denote the crossover point when $N_r = m_2 (\geq N_t)$ is used. Then, from the previous results, we have
\[ P^*_{\text{out},1} > P^*_{\text{out},2}, \quad \text{for } m_1 < m_2. \] (29)

Recall that, as given by (23), when the number of antennas increases such that an $n_1 \times n_1$ system becomes an $n_2 \times n_2$ system ($n_1 < n_2$), the crossover point in the outage probability monotonically decreases. In addition, as given by (29), when the number of antennas increases such that an $n_2 \times n_2$ system becomes an $n_2 \times n_3$ system ($n_2 < n_3$), the crossover point also monotonically decreases. Based on these, the outage probabilities of V-BLAST with a linear receiver and OSTBC are qualitatively depicted in Fig. 1, where $P^*_{\text{out},1}$ denotes the crossover point for an $n_1 \times n_1$ MIMO system, and $P^*_{\text{out},2}$ is the crossover point for an $n_2 \times n_2$ MIMO system ($n_2 < n_3$). Suppose that a target outage probability $P^*_{\text{out},1}$ is smaller than $P^*_{\text{out},1}$ but greater than $P^*_{\text{out},2}$. Then, from Fig. 1, it is seen that OSTBC is preferable to V-BLAST for an $n_1 \times n_1$ system, whereas the latter is preferable to the former for an $n_2 \times n_3$ system. In Appendix A, we prove that the crossover point for V-BLAST with an ML receiver and OSTBC shows the same behavior as given by (23) and (29).

We note that DMT characteristics are not influenced by spatial correlation or LOS signal components at high SNR [24], [25]. In other words, the DMT function for spatially correlated Rayleigh or Rician fading is identical to that for i.i.d. Rayleigh fading. This is because, as described in [24], when the SNR approaches infinity, only the number of channel eigenmodes determines the performance. Spatial correlation or LOS components primarily affect the condition number of the channel matrix, and hence, the impact of such propagation on the performance is not observed at high SNR. This indicates that the analysis of the crossover points here is valid over correlated
Rayleigh or Rician channels and i.i.d. Rayleigh channels at high SNR.

IV. OPTIMAL SPACE–TIME CODING OF SCALABLE SOURCES FOR A LARGE NUMBER OF ANTENNAS

The analysis in Section III can be exploited to optimally design a MIMO system with a large number of antennas for the transmission of multimedia scalable sources. To begin, we briefly present the properties of scalable bitstreams described in [18, Sec. IV]. Scalable codecs adopt progressive transmission so that encoded data have gradual differences of importance in their bitstreams. Suppose that the system takes the bitstream from the scalable source encoder and transforms it into a sequence of \( N_{\text{pkt}} \) packets. Each of these \( N_{\text{pkt}} \) scalable packets can be encoded with different transmission data rates and powers, as well as different space–time codes, to achieve the best end-to-end performance measured by the expected distortion of the source. Such a system is depicted in Fig. 2. Note that the error probability of an earlier packet in a sequence of scalable packets should be lower than or equal to that of a later packet.

Let \( N_d \) and \( N_p \) denote the numbers of candidate transmission data rates and powers employed by a system, respectively. The numbers of possible assignments of \( N_d \) data rates and \( N_p \) powers to a sequence of \( N_{\text{pkt}} \) packets are \( N_d^{N_{\text{pkt}}} \) and \( N_p^{N_{\text{pkt}}} \), respectively, which exponentially grow as \( N_{\text{pkt}} \) increases. Furthermore, in a MIMO system, if each packet can be encoded with different space–time codes (e.g., V-BLAST or OSTBC), the assignment of space–time codes and data rates and powers to \( N_{\text{pkt}} \) packets yields a more complicated optimization problem. Note that each source, e.g., an image, has its inherent rate–distortion characteristic, from which the performance of the expected distortion is computed. For example, when a series of images is transmitted, the aforementioned optimization should be addressed in a real-time manner, considering which specific image (i.e., rate–distortion characteristic) is transmitted in the current time slot. To address this matter, for a single-input–single-output system, there have been many studies about the optimal assignment of data rates to a sequence of scalable packets [26]–[28].

On the other hand, for a MIMO system with a large number of antennas, we exploit the analysis in the previous section for the optimal assignment of space–time codes to a sequence of scalable packets. Suppose that the assignment of space–time codes in an \( n_1 \times n_1 \) MIMO system has been optimized, with the result that the \( k \)th packet (\( 1 \leq k \leq N_{\text{pkt}} \)), which has a data rate \( R_k \), has been encoded with V-BLAST. Then, our analysis indicates that, in an \( n_2 \times n_2 \) MIMO system (\( n_1 < n_2 \leq n_3 \)), the aforementioned \( k \)th packet, which has data rate \( R_k \), should also be encoded with V-BLAST rather than with OSTBC. This is because we have proven that, for a fixed data rate (i.e., spectral efficiency times signal bandwidth), as the number of antennas increases, the crossover point in the outage probability monotonically decreases. In other words, if V-BLAST is preferable for a packet in an \( n_1 \times n_1 \) system, then another packet with the same data rate in an \( n_2 \times n_2 \) system (\( n_1 < n_2 \leq n_3 \)) should be also encoded with V-BLAST, as long as the target error rates of the two packets are the same (see Fig. 1). Note that the target error rates of scalable packets depend on their application source (e.g., an embedded image with a specific rate–distortion characteristic) and their positions in the scalable bitstream, and that we evaluate the same application source, which is packetized in the same way, for both \( n_1 \times n_1 \) and \( n_2 \times n_3 \) systems.

Based on that, we further suppose that the \( k_1 \)th, ..., \( k_{N_{\text{ch}}} \)th packets (\( 1 \leq k_1 < \cdots < k_{N_{\text{ch}}} \leq N_{\text{pkt}} \)) have been encoded with V-BLAST as a result of the optimization for an \( n_1 \times n_1 \) system. Then, in an \( n_2 \times n_3 \) system, the aforementioned \( N_{\text{ch}} \) packets should be also encoded with V-BLAST, and only the remaining \( N_{\text{pkt}} - N_{\text{ch}} \) packets need to be optimized for their space–time coding.

For the optimal assignment of space–time codes to \( N_{\text{pkt}} \) packets, the expected distortion of the source should be evaluated for each possible assignment. Note that each evaluation includes the decoding of the packets encoded with V-BLAST. An MMSE or a ZF linear receiver for V-BLAST incorporates
an inversion operation of a channel matrix $H$, and the amount of computation required for the inversion is in general $O(n^3)$, where $n$ is the size of the channel matrix in an $n \times n$ MIMO system. Thus, the computational complexity involved with the decoding of V-BLAST exponentially grows as the number of antennas increases, which indicates that the V-BLAST receiver will be highly complex for a large number of antennas. Note that the complexity of V-BLAST decoding for an $n_1 \times n_1$ system becomes insignificant compared with that for an $n_2 \times n_3$ system when $n_1$ is chosen such that it is sufficiently smaller than $n_2$. Based on the preceding statements, we propose an optimization strategy for the space–time coding of scalable sources in a MIMO system with a large number of antennas, as follows.

Step 1) Optimize the assignment of the space–time codes (i.e., OSTBC or V-BLAST) to a sequence of $N_{\text{pkt}}$ scalable packets for an $n_1 \times n_1$ system, which is referred to as a pilot system.

Step 2) If the $k_1, \ldots, k_{N_{\text{vb}}}$th packets ($1 \leq k_1 < \cdots < k_{N_{\text{vb}}} \leq N_{\text{pkt}}$) have been encoded with V-BLAST in Step 1, then assign V-BLAST to the same $N_{\text{vb}}$ packets in an $n_2 \times n_3$ system ($n_1 < n_2 \leq n_3$), which is referred to as a target system.

Step 3) Optimize the assignment of the space–time codes to the remaining $N_{\text{pkt}} - N_{\text{vb}}$ packets for a target system.

In Steps 1 and 3, we employ the same scalable source that is packetized identically. In addition, we use the same system parameters such as the sets of transmission data rates and powers assigned to a sequence of $N_{\text{pkt}}$ packets.

Note that, in [18], it is assumed that a constant transmission power is used for a sequence of $N_{\text{pkt}}$ scalable packets. Thus, with constant transmission power, the earlier packet requires a transmission data rate that is less than or equal to that of the later packet, to provide the unequal error protection of scalable packets. On the other hand, the work in this paper considers a more general case where unequal transmission powers are allowed for a sequence of scalable packets for the purpose of unequal error protection. As a result, the earlier packet does not necessarily have a data rate that is less than or equal to that of the later packet. Without any such assumptions, the number of possible assignments of two space–time codes (i.e., OSTBC and V-BLAST) to a sequence of $N_{\text{pkt}}$ packets is $2^{N_{\text{pkt}}}$. It can be shown that, by exploiting the optimization results in a pilot system, the number of possible assignments of two space–time codes to $N_{\text{pkt}}$ scalable packets is reduced from $2^{N_{\text{pkt}}}$ to $2^{N_{\text{pkt}} - N_{\text{vb}}}$.

Finally, in the following, we present the message operation between the transmitter and the receiver required for the space–time coding of scalable packets, as depicted in Fig. 3.

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1The use of Gaussian elimination is assumed [29].

2Note that this differs from $N_{\text{pkt}} + 1$, which is derived in [18] based on the assumption that the earlier packet has a data rate that is less than or equal to that of the later packet.

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V. NUMERICAL EVALUATION

First, we numerically evaluate the outage probabilities of V-BLAST with an MMSE receiver and OSTBC for various numbers of antennas.
To begin, we consider spatially correlated Rayleigh fading channels. The outage probabilities are evaluated, as an example, for spatial correlation coefficients of $\rho_t = \rho_r = 0.3$ and a spectral efficiency of $R = 12$ bits/s/Hz. The results are shown in Fig. 4, where the high-SNR approximate outage probabilities are derived from (3), (4), (12), and (13). Note that the analyses in Sections II and III are valid for any $k_r > 0$, subject to the constraint that $2^R / k_r > 1$ and $k_d > 0$ in the high-SNR approximate outage probabilities as given by (4) [18]. Let $P_{out,V}(\gamma_s)$ and $P_{out,O}(\gamma_s)$ denote the outage probabilities of V-BLAST with an MMSE receiver and OSTBC, respectively. In Fig. 4, the exact outage probabilities are obtained by numerically evaluating [30, Eqs. (6) and (9)] and [31, Eq. (20)] for V-BLAST with an MMSE receiver and OSTBC, respectively. Note that, in those equations, mutual information is normalized by the time duration of a space–time codeword (i.e., $T$ as defined below (1)) for the computation of the outage probabilities. The curves in Fig. 4 are repeated in Fig. 5, which show that, when the number of transmit and receive antennas increases, as stated in the last paragraph in Section III, the crossover point of the outage probability in the correlated Rayleigh fading channels behaves as predicted by the analysis given by (23) (see Fig. 1).

As another example, Fig. 6 depicts the exact outage probabilities for spatial correlation coefficients of $\rho_t = \rho_r = 0.7$ and a spectral efficiency of $R = 12$ bits/s/Hz. It is shown that the gap between the high-SNR approximate outage probability and the exact one is small. In Fig. 4, the exact outage probabilities are obtained by numerically evaluating [30, Eqs. (6) and (9)] and [31, Eq. (20)] for V-BLAST with an MMSE receiver and OSTBC, respectively. Note that, in those equations, mutual information is normalized by the time duration of a space–time codeword (i.e., $T$ as defined below (1)) for the computation of the outage probabilities. The curves in Fig. 4 are repeated in Fig. 5, which show that, when the number of transmit and receive antennas increases, as stated in the last paragraph in Section III, the crossover point of the outage probabilities in the correlated Rayleigh fading channels behaves as predicted by the analysis given by (23) (see Fig. 1).
Crossover points in Fig. 6 exhibit the same behavior as those in Fig. 5. If we focus on an outage probability of $10^{-3}$, in Fig. 6, OSTBC outperforms V-BLAST for $N_t = N_r = 2$ or $4$, whereas the latter outperforms the former for $N_t = N_r = 6$, $8$, $10$, or $12$. Note that this preference is a function of the target outage probability of the application. For example, if the target is $10^{-6}$, OSTBC outperforms V-BLAST for $N_t = N_r = 2$, $4$, $6$, $8$, or $10$, and the latter outperforms the former only for $N_t = N_r = 12$. In addition, from Figs. 5 and 6, it is observed that the outage probability performance of V-BLAST tends to be more sensitive to the spatial correlation of MIMO channels than that of OSTBC. This is because, as stated in [32], spatial correlation leads to a disparity in the distribution of the spatial eigenmodes, which reduces the spatial multiplexing capability; spatial correlation in general leads to much less performance degradation for spatial diversity schemes than for spatial multiplexing [15], [19].

We next consider Rician fading channels. The outage probabilities are evaluated, as an example, for the Rician factor of $K = 2$ and a spectral efficiency of $R = 12$ bits/s/Hz. The numerical results are depicted in Fig. 7, where the high-SNR approximate and exact outage probabilities are derived in the same way as that for Fig. 4. The curves in Fig. 7 are repeated in Fig. 8, which indicate that the crossover points also exhibit the behavior as given by (23). Fig. 9 depicts the exact outage probabilities, as another example, for the Rician factor of $K = 5$ and a spectral efficiency of $R = 12$ bits/s/Hz. It is also shown that the crossover points in Fig. 9 behave as given by (23). Furthermore, in Figs. 8 and 9, it is shown that OSTBC works better in a channel with a stronger LOS, whereas V-BLAST fails to work well in the LOS case [15], [16].

The outage probabilities in i.i.d. Rayleigh fading channels for a spectral efficiency of $R = 4$ bits/s/Hz are shown in Figs. 10 and 11. It is again observed that the crossover points behave as given by (23). Fig. 12 depicts the outage probabilities for the number of antennas taken to be larger than those in Fig. 6, for spatial correlation coefficients of $\rho_t = \rho_r = 0.7$ and a spectral efficiency of $R = 12$ bits/s/Hz. It is shown that the crossover points also behave as predicted by (23).

Next, we evaluate the behavior of the crossover points for the case when the number of receive antennas increases, but the number of transmit antennas remains fixed. Figs. 13 and 14 depict the results of such a case for correlated Rayleigh fading channels having $\rho_t = \rho_r = 0.7$ and for the Rician fading channels with $K = 5$, when a spectral efficiency of $R = 12$ bits/s/Hz is used, respectively. When only the number of receive antennas increases, as described in the last paragraph in Section III, the crossover point of the outage probabilities in the correlated Rayleigh or Rician channels behaves as predicted by the analysis given by (29).

In Section IV, we presented the optimal space–time coding for the transmission of scalable sources. In the following,
we will compare the performance of the optimal space–time coding and the suboptimal ones for scalable transmission. We evaluate the end-to-end performance measured by the expected distortion of the image for 8 × 8 MIMO systems using the source coder SPIHT [33] as an example and provide results for the standard 8 bpp 512 × 512 Lena image with a transmission rate of 0.5 bpp.

To begin, we summarize the evaluation of the expected distortion as stated in [18, Sec. V]. The system takes a compressed scalable bitstream from the source encoder and transforms it into a sequence of \( N_{\text{pkt}} \) packets with error detection and correction capability. Then, as shown in Fig. 2, the packets are encoded by the space–time codes. At the receiver, if a received packet is correctly decoded, the next packet is considered by the source decoder. Otherwise, the decoding is terminated, and the source is reconstructed from only the correctly decoded packets. We assume a slow-fading channel such that channel coefficients are nearly constant over an image, which consists of a sequence of \( N_{\text{pkt}} \) scalable packets, and the channel estimation at the receiver is perfect. Let \( P_i(\gamma_{s,i}) \) denote the conditional probability of a decoding error of the \( i \)th packet \((1 \leq i \leq N_{\text{pkt}})\), conditioned on \( \gamma_{s,i} \) where \( \gamma_{s,i} \) is the instantaneous postprocessing SNR per symbol for the \( i \)th packet. Then, the probability that no decoding errors occur in the first \( n \) packets with an error in the next one, i.e., \( P_{c,n} \), is given by

\[
P_{c,n} = P_{n+1}(\gamma_{s,n+1}) \prod_{i=1}^{n} (1 - P_i(\gamma_{s,i})) , \quad 1 \leq n \leq N_{\text{pkt}} - 1.
\]

\( P_{c,0} = P_1(\gamma_{s,1}) \) is the conditional probability of an error in the first packet, and \( P_{c,N_{\text{pkt}}} = \prod_{i=1}^{N_{\text{pkt}}} (1 - P_i(\gamma_{s,i})) \) is the conditional probability that all \( N_{\text{pkt}} \) packets are correctly decoded. Let \( d_n \) denote the distortion of the source using the first \( n \) packets for the source decoder \((0 \leq n \leq N_{\text{pkt}})\). Then, \( d_n \) can be expressed as \( d_n = D(\sum_{i=1}^{n} r_i) \), where \( r_i \) is the number of source bits in the \( i \)th packet, \( D(x) \) denotes the operational distortion-rate function of the source, and \( d_0 = D(0) \) refers to the distortion when the decoder reconstructs the source without any of the received information. The expected distortion of the source, i.e., \( E[D] \), can be expressed as (31), shown at the bottom of the next page, where \( p(\gamma_{s,i}) \) is the probability density function of the instantaneous postprocessing SNR for the \( i \)th packet, i.e., \( \gamma_{s,i} \). Note that \( p(\gamma_{s,i}) \) is a function of the average SNR per symbol \( \gamma_s \), which is defined below (2), as well as the transmission data rate, power, and space–time code assigned to the \( i \)th packet. Thus, \( E[D] \) is also a function of those system parameters.

Let \( D_i \), \( P_i \), and \( C_i \) denote the transmission data rate, power, and space–time code assigned to the \( i \)th packet, respectively. For given sets of \( \{D_1, \ldots, D_{N_{\text{pkt}}}\} \) and \( \{P_1, \ldots, P_{N_{\text{pkt}}}\} \) assigned to a sequence of \( N_{\text{pkt}} \) packets, one can find the optimal set of space–time codes \( C_{\text{opt}} = [C_1, \ldots, C_{N_{\text{pkt}}}]_{\text{opt}} \), which

![Fig. 8. Exact and high-SNR approximate outage probabilities for \( R = 12 \) bits/s/Hz in the Rician fading channels with \( K = 2 \). Solid curves denote the outage probabilities of OSTBC, and dotted curves denote those of V-BLAST with an MMSE receiver. The crossover points of outage probabilities are marked with circles. (a) High-SNR approximate outage probabilities. (b) Exact outage probabilities.](image)

![Fig. 9. Exact outage probabilities for \( R = 12 \) bits/s/Hz in the Rician fading channels with \( K = 5 \). Solid curves denote the outage probabilities of OSTBC, and dotted curves denote those of V-BLAST with an MMSE receiver. The crossover points of outage probabilities are marked with circles.](image)
minimizes the expected distortion over a range of average SNRs using the weighted cost function as follows:

\[
E[D] = \int_0^\infty \cdots \int_0^\infty \left\{ D(0)P_1(\gamma_{s,1}) + \sum_{n=1}^{N_{\text{pkt}}-1} \left( D \left( \sum_{i=1}^n r_i \right) P_{n+1}(\gamma_{s,n+1}) \prod_{i=1}^n (1 - P_i(\gamma_{s,i})) \right) \right. \\
+ D \left. \left( \sum_{i=1}^{N_{\text{pkt}}} r_i \right) \prod_{i=1}^{N_{\text{pkt}}} (1 - P_i(\gamma_{s,i})) \right\} p(\gamma_{s,1}), \ldots, p(\gamma_{s,N_{\text{pkt}}}) \, d\gamma_{s,1}, \ldots, d\gamma_{s,N_{\text{pkt}}}. 
\]  

where \(w(\gamma_s)\) in \([0, 1]\) is the weight function. For example, \(w(\gamma_s)\) can be chosen such that

\[
w(\gamma_s) = \begin{cases} 
1, & \text{for } \gamma_{s,1} \leq \gamma_s \leq \gamma_{s,2}, \\
0, & \text{otherwise}. 
\end{cases}
\]
In broadcast systems, the weight function in (33) indicates that SNRs of multiple receivers are uniformly distributed in the range of $\gamma_{s,1} \leq \gamma_s \leq \gamma_{s,2}$. Eq. (32) indicates that the \{C_1, \ldots, C_N_{pkt}\} are chosen such that the total sum of the expected distortion of the receivers distributed in the range of $\gamma_{s,1} \leq \gamma_s \leq \gamma_{s,2}$ is minimized. Note that the amount of computation involved in (32) exponentially grows as $N_{pkt}$ increases. Alternatively, as presented in Section IV, we may choose the set of codes $\{C_1, \ldots, C_{N_{pkt}}\}$, with the constraint that V-BLAST should be assigned to the $i$th packet (i.e., OSTBC is excluded) if the packet has been encoded with V-BLAST in a pilot system as a result of the optimization.

To compare the image quality, we use the PSNR, which is defined as $10 \log_{10}(255^2/E[D])$ (dB). The PSNR performance is evaluated as follows: We compute (32) using the expected distortion $E[D]$, given by (31), and the weight function $w(\gamma_s)$, given by (33). Next, with the optimal set of codes $C_{opt} = \{C_1, \ldots, C_{N_{pkt}}\}_{opt}$ obtained from (32), we evaluate the PSNR over a range of SNRs given by (33).

The performance is evaluated, as an example, when a sequence of 12 scalable packets is transmitted (i.e., $N_{pkt} = 12$) in $8 \times 8$ MIMO systems, and we assume that the transmission data rates and powers are assigned to scalable packets in a manner that $R = [12 10 8 7 6 5 4.5 4 3.5 3 2.5 2]$ (bits/s/Hz) and $P = [0 - 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 10 - 11]$ (dB), where the $i$th component of $R$ is the spectral efficiency employed by the $i$th packet, and the $i$th component of $P$ is the ratio of the transmission power of the $i$th packet to that of the first packet. With this specific setup, the optimal set of space–time codes, which is computed from (32), is given by $C_1 = \cdots = C_4 = $ V-BLAST, $C_5 = C_6 = $ OSTBC, $C_7 = C_8 = C_9 = $ V-BLAST, and $C_{10} = C_{11} = C_{12} = $ OSTBC in the correlated Rayleigh fading channel of $\rho_t = \rho_r = 0.7$.

Fig. 15 shows the PSNR of such an optimal set of space–time codes, in addition to showing the PSNRs of other suboptimal sets of codes, such as the sets at the 75th and 50th percentiles among the sets of codes, and the worst set of codes, which has the poorest performance. Fig. 15 also shows the PSNR corresponding to the expected distortion that is averaged over all the possible sets of space–time codes. From this example, it is seen that the PSNR performance of the scalable source is sensitive to the way space–time codes are assigned to scalable packets in the bitstream. In particular, in the case of unequal target error rates in the bitstream, Fig. 15 also depicts the PSNR performance when (32) is computed with the constraint presented in Section IV. A $2 \times 2$ MIMO system has been employed as a pilot system. With the same setup used for an $8 \times 8$ target system, the optimal set of codes in a $2 \times 2$ pilot system,
which is computed from (32), is given by $C_1 = C_2 = C_3 = \text{V-BLAST}, C_4 = C_5 = C_6 = \text{OSTBC}, C_7 = C_8 = \text{V-BLAST},$ and $C_9 = \cdots = C_{12} = \text{OSTBC}$ (i.e., $N_{\text{vb}} = 5$). Recall that, with the constraint in Section IV, the number of possible sets of space–time codes in a target system is reduced from $2^{N_{\text{rktx}}}$ to $2^{N_{\text{rktx}} - N_{\text{vb}}}$ (i.e., from 4096 to 128 in this example).

We note that the same optimal set of space–time codes has been obtained when (32) is computed with and without the constraint. That is, without losing any PSNR performance, the computational complexity involved with the optimization can be reduced by exploiting the monotonic behavior of the crossover point, as shown in Fig. 1. It is further seen that the PSNR, which corresponds to the expected distortion averaged over all the possible sets of codes, becomes much better when the constraint in Section IV is introduced, which shows that, on average, the constraint is a good strategy for the space–time coding of scalable sources.

Fig. 16 shows the PSNR performances in the Rician fading channel of $K = 5$, where we use the same system parameters as those for the correlated Rayleigh channel whose results are shown in Fig. 15. We also note that, in Fig. 16, the same optimal set of space–time codes has been obtained when (32) is computed with and without the constraint. Furthermore, like the case in Fig. 15, the PSNR corresponding to the expected distortion averaged over all the possible sets of space–time codes becomes better when the constraint in Section IV is used. This indicates that the constraint is also a good optimization strategy in Rician fading channels.

We note that, in addition to embedded images, our analysis in Sections III and IV can be also applied to scalable video. In scalable video, the base layer is more important than the enhancement layer. If we split the base layer into multiple packets, those packets often have a similar level of importance. However, the enhancement layer, for example, with medium-grain scalability, can usually be split into multiple packets with successively decreasing importance. Hence, for real-time scalable video, we can apply our analytical results to the sequence of high-importance base layer packets and successively less important enhancement layer packets.

Finally, we note that, when selecting the optimal space–time code in an $8 \times 8$ MIMO system, in addition to the OSTBC that uses all the eight transmit antennas with the spatial multiplexing rate of $r_x = 1/2$, we also consider the OSTBC that uses only two or four transmit antennas with $r_x = 1$ and $3/4$, respectively. That is, for an $8 \times 8$ MIMO system, the OSTBC scheme that uses either a $2 \times 8$ or a $4 \times 8$ antenna configuration is also considered. The motivation for considering such an OSTBC scheme is as follows: The OSTBCs can be classified into three groups, according to their maximum achievable spatial multiplexing rates: 1) $r_x = 1/2$ for $N_t \geq 5$; 2) $r_x = 3/4$ for $N_t = 3$ or 4; and 3) $r_x = 1$ for $N_t = 2$. In Appendix B, we show that, for two OSTBCs, which belong to distinct groups earlier, there exists a crossover point of their outage probabilities; furthermore, the crossover point in outage probability is a strictly decreasing function in spectral efficiency. This indicates that, as an example, for a $3 \times N_r$ MIMO system, it is possible that the OSTBC that uses only two transmit antennas with $r_x = 1$ (i.e., OSTBC that uses a $2 \times N_r$ configuration) outperforms the OSTBC that uses all the three transmit antennas with $r_x = 3/4$ (i.e., OSTBC using a $3 \times N_r$ configuration). Note that which OSTBC scheme performs better depends on the target error rate of the application and the spectral efficiency employed by the system. For this reason, in $8 \times 8$ MIMO systems, we consider the OSTBC scheme that uses only two or four transmit antennas and the one that uses all the eight transmit antennas as a candidate space–time code.

VI. Conclusion

Because of increasing use of higher carrier frequencies and the advances in the computational capability of hardware, a MIMO system is capable of employing a large number of antennas. When a series of scalable sources is transmitted over MIMO channels, space–time coding should be optimized for each individual rate–distortion characteristic. For a large

![Fig. 15. PSNR performances of the optimal set of space–time codes and suboptimal ones for the transmission of the embedded 512 × 512 Lena image for 8 × 8 MIMO systems in spatially correlated Rayleigh fading channels with $\rho_t = \rho_r = 0.7$.](image1)

![Fig. 16. PSNR performances of the optimal set of space–time codes and suboptimal ones for the transmission of the embedded 512 × 512 Lena image for 8 × 8 MIMO systems in Rician fading channels with $K = 5$.](image2)
number of antennas, however, the optimization is complex. To address this matter, we suggested an efficient method to optimize the space–time coding of scalable sources. To begin, we analyzed the crossover point of the outage probabilities of V-BLAST, using either a linear or an ML receiver, with OSTBC, in terms of the number of antennas. The results showed that, as the number of antennas increases, the crossover point in outage probability monotonically decreases, which were proven for arbitrary spectral efficiency employed by the system. These results are conceptually depicted in Fig. 1, and we are unaware of any other literature that either states or proves them. The results are valid in spatially correlated Rayleigh or Rician fading channels, as well as in i.i.d Rayleigh fading channels.

Based on the analysis, we suggested a method to optimally assign V-BLAST or OSTBC to a sequence of scalable packets over spatially correlated Rayleigh or Rician channels with a large number of antennas. It was shown, that, without any PSNR degradation, the computational complexity involved with optimal space–time coding is exponentially reduced by the use of the suggested method. Furthermore, the PSNR performance averaged over all the possible sets of space–time codes becomes better when our method is used, which indicates that, on the average, it is a good strategy for the space–time coding of multimedia scalable sources. The technical approach in this paper, which was used to analyze the tradeoff between space–time codes in terms of their target error rates and the number of antennas, may be used for other space–time codes such as quasi-OSTBC or Golden codes as future work.

APPENDIX A
CROSSOVER POINT FOR V-BLAST WITH A MAXIMUM LIKELIHOOD RECEIVER AND ORTHOGONAL SPACE TIME BLOCK CODES

The DMT characteristic of V-BLAST with an ML receiver, i.e., \( d_{V,ML}(r) \), is given by [22]

\[
d_{V,ML}(r) = \begin{cases} N_r - N_t r / N_t, & \text{for } 0 \leq r \leq N_t, \\ 0, & \text{for } N_t \leq r < \infty. \end{cases}
\]

Using (13) and (34), we set \( d_1(r) = d_{V,ML}(r) \) and \( d_2(r) = d_O(r) \) in (5). Then, from (13), (34), and \( N_t > r_s \), it follows that, for the range of \( 0 < r \leq r_s \), the condition of (6) is satisfied. Moreover, from \( N_r \geq r \geq 2 \), we have

\[
\begin{align*}
d_{V,ML}(0) - d_O(0) &= N_r(1 - N_t) < 0, \\
d_{V,ML}(r_s) - d_O(r_s) &= N_r(1 - r_s / N_t) > 0.
\end{align*}
\]

Eqs. (35) and (36) meet the conditions of (7) and (8), respectively, when we set \( \alpha = \varepsilon = \beta = R_x \), where \( \varepsilon > 0 \) is an arbitrarily small positive number. Hence, (11) holds, and this tells us that there exists a crossover point of the outage probabilities for V-BLAST with an ML receiver and OSTBC in the range of \( (2^R / k_r)^{1 / \gamma_s} \leq \gamma_s < \infty \). In addition, from (10), (13), and (34), \( P_{out}^* \) is given by

\[
P_{out}^* = k_d \left( \frac{2^R}{k_r} \right)^{-\frac{N-r_s / r_s}{N_t-1}}.
\]

Letting \( N_t = N_r = n \) in (37), we have

\[
P_{out}^* = k_d \left( \frac{2^R}{k_r} \right)^{-\frac{n-r_s / r_s}{n-1}}.
\]

Let \( s(n) = (n - n^2 / r_s) / (n - 1) \) be the exponent of \( P_{out}^* \) given by (38). If we assume that \( n \) is a real number, \( ds(n) / dn \) can be expressed as

\[
\frac{ds(n)}{dn} = -n^2 + 2n - r_s / r_s(n - 1)^2.
\]

Let \( w(n) = -n^2 + 2n - r_s \) be the numerator of \( ds(n) / dn \). It can be shown that \( w(n) \) is a strictly decreasing function in \( n \) for \( n \geq 2 \). From this and \( n (= N_t = N_r) \geq 2 \), \( w(n) \) satisfies

\[
w(n) \leq w(2) = -r_s < 0.
\]

Eqs. (39) and (40) show that

\[
\frac{ds(n)}{dn} < 0, \quad \text{for } n \geq 2.
\]

From (38), (41), and \( k_d > 0 \) and \( 2^R / k_r > 1 \) given in Section II, it is seen that \( P_{out}^* \) is a strictly decreasing function in \( n (\geq 2) \), regardless of a given spectral efficiency \( R \). In other words, for any fixed spectral efficiency, as the number of antennas, i.e., \( N_r = N_t (\geq 2) \), increases, the crossover point in the outage probability monotonically decreases. Thus, the same argument given by (23) can be also made for V-BLAST with an ML receiver and OSTBC.

In addition, from \( N_t \geq 2 \) and \( 1 / 2 \leq r_s \leq 1 \), it follows that \( P_{out}^* \), given by (37), is a strictly decreasing function in \( N_r (\geq N_t \geq 2) \), regardless of a given spectral efficiency \( R \), as well as a given number of transmit antennas \( N_t \). That is, for any fixed spectral efficiency and any fixed number of transmit antennas, as only the number of receive antennas increases, the crossover point in the outage probability monotonically decreases. Hence, the results given by (29) also hold for V-BLAST with an ML receiver and OSTBC.

APPENDIX B
CROSSOVER POINT FOR ORTHOGONAL SPACE TIME BLOCK CODES WITH DISTINCT SPATIAL MULTIPLEXING RATES

Let \( d_O^p(r) \), \( d_O^\theta(r) \), and \( d_O^\phi(r) \) denote the DMT characteristics of the OSTBCs with spatial multiplexing rates \( r_s = 1 / 2, 3 / 4 \), and 1, respectively. Then, from (13), we have

\[
d_O^p(r) = \begin{cases} N_r N_t^p - 2N_r N_t^p r, & \text{for } 0 \leq r \leq \frac{1}{2}, \\ 0, & \text{for } \frac{1}{2} \leq r < \infty, \end{cases}
\]

\[
d_O^\theta(r) = \begin{cases} N_r N_t^\theta - \frac{4}{3}N_r N_t^\theta r, & \text{for } 0 \leq r \leq \frac{3}{4}, \\ 0, & \text{for } \frac{3}{4} \leq r < \infty, \end{cases}
\]

\[
d_O^\phi(r) = \begin{cases} 2N_r - 2N_r r, & \text{for } 0 \leq r \leq 1, \\ 0, & \text{for } 1 \leq r < \infty, \end{cases}
\]

where \( N_t^p \geq 5 \), and \( N_t^\theta = 3 \) or 4. In the following, we analyze the crossover point of the outage probabilities of the OSTBCs, which have different spatial multiplexing rates.
1) The OSTBCs with $r_s = 1/2$ and $r_s = 3/4$: From (42) and (43), it follows that, for the range of $0 < r \leq 1/2$, the condition of (6) is met if we set $d_1(r) = d_0^y(r)$ and $d_2(r) = d_0^o(r)$ in (5). In addition, since $N_t^r \geq 5$ and $N_t^y \geq 3$ or 4, it can be shown that

\[ d_0^o(0) - d_0^o(0) = N_r(N_t^r - N_t^y) < 0, \tag{45} \]
\[ d_0^y \left( \frac{1}{2} \right) - d_0^o \left( \frac{1}{2} \right) = \frac{1}{3} N_r N_t^y > 0. \tag{46} \]

Eqs. (45) and (46) meet the conditions of (7) and (8), respectively, when we set $\alpha = \varepsilon$ and $\beta = 1/2$, where $\varepsilon > 0$ is an arbitrarily small positive number. Hence, from (11), we have

\[ P^y_{out,O}(\gamma_s) < P^z_{out,O}(\gamma_s), \quad \text{for } \left( \frac{2R}{k_r} \right)^2 < \gamma_s < \gamma_s^*, \]
\[ P^z_{out,O}(\gamma_s) > P^y_{out,O}(\gamma_s), \quad \text{for } \gamma_s^* < \gamma_s < \infty, \tag{47} \]

where $P^z_{out,O}(\gamma_s)$ and $P^y_{out,O}(\gamma_s)$ denote the outage probabilities of the OSTBCs with $r_s = 1/2$ and $3/4$, respectively. That is, there exists a crossover point of the outage probabilities of the OSTBCs with $r_s = 1/2$ and $3/4$, and this holds for any number of receive antennas $N_r$ and any spectral efficiency $R$. In addition, from (10), (42), and (43), it can be shown that the crossover point of the outage probabilities is given by $P^x_{out} = k_d(2R/k_r) - 2N_r N_t^y/(3N_t^y - N_t^x)$. For $N_t^x \geq 5$ and $N_t^y = 3$ or 4, we have $N_r N_t^y N_t^x/(N_t^x - N_t^y) > 0$. From this and $k_d > 0$, $3R/k_r > 1$ given in Section II, it is seen that $P^x_{out}$ is a strictly decreasing function in $R$.

2) The OSTBCs with $r_s = 1/2$ and $r_s = 1$: From (42) and (44), it is seen that, for the range of multiplexing gain, i.e., $0 < r \leq 1/2$, the condition of (6) is satisfied if we set $d_1(r) = d_0^y(r)$ and $d_2(r) = d_0^o(r)$ in (5). In addition, since $N_t^r \geq 5$, it can be shown that

\[ d_0^o(0) - d_0^o(0) = N_r(2 - N_t^r) < 0, \tag{48} \]
\[ d_0^y \left( \frac{1}{2} \right) - d_0^o \left( \frac{1}{2} \right) = N_r > 0. \tag{49} \]

Eqs. (48) and (49) satisfy the conditions of (7) and (8), respectively, when setting $\alpha = \varepsilon$ and $\beta = 1/2$, where $\varepsilon > 0$ denotes an arbitrarily small positive number. Thus, from (11), we have

\[ P^y_{out,O}(\gamma_s) < P^z_{out,O}(\gamma_s), \quad \text{for } \left( \frac{2R}{k_r} \right)^2 < \gamma_s < \gamma_s^*, \]
\[ P^z_{out,O}(\gamma_s) > P^y_{out,O}(\gamma_s), \quad \text{for } \gamma_s^* < \gamma_s < \infty, \tag{50} \]

where $P^z_{out,O}(\gamma_s)$ is the outage probability of the OSTBC with $r_s = 1$. This indicates that there exists a crossover point of the outage probabilities of the OSTBCs with $r_s = 1/2$ and 1, which is valid for any number of receive antennas $N_r$ and any spectral efficiency $R$. Furthermore, from (10), (42), and (44), it can be shown that the crossover point of the outage probabilities is given by $P^x_{out} = k_d(2R/k_r) - 2N_r N_t^y/(N_t^x - 2)$. Since $N_t^x N_r/(N_t^x - 2) > 0$ for $N_t^x \geq 5$ and from $2R/k_r > 1$ given in Section II, it follows that $P^x_{out}$ is a strictly decreasing function in $R$.

3) OSTBCs with $r_s = 3/4$ and $r_s = 1$: In a similar manner, it can be shown that

\[ P^x_{out,O}(\gamma_s) < P^y_{out,O}(\gamma_s), \quad \text{for } \left( \frac{2R}{k_r} \right)^2 < \gamma_s < \gamma_s^*, \]
\[ P^y_{out,O}(\gamma_s) > P^x_{out,O}(\gamma_s), \quad \text{for } \gamma_s^* < \gamma_s < \infty. \tag{51} \]

In addition, the crossover point of the outage probabilities $P^x_{out} = k_d(2R/k_r) - 2N_r N_t^y/(3N_t^y - 2)$ is a strictly decreasing function in $R$.

REFERENCES


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