

# Superposition Coding Based Cooperative Communication with Relay Selection

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**Abstract**—We study the layered transmission of a Gaussian source over multiple relays using superposition coding. First, we analyze the outage probability and performance in terms of average throughput and distortion for decode-and-forward protocols with single layer and superposed two-layer coding. For the superposition coding approach, we consider different power allocations to the base and enhancement layers. Then, we propose a simple protocol which assigns a pre-determined number of relays to individual layers instead of repeating the superposition coded source packet at the relay. Finally, we present numerical results based on the analysis to compare the performance.

## I. INTRODUCTION

Recently, relay transmission has been considered as an attractive technology to support high data rates in mobile environments such as LTE-Advanced and WiMAX [1]. Since a cooperative relay system has an advantage over a single hop system due to the path-loss reduction in the source-relay and relay-destination paths, higher modulation schemes can be used for relay transmission. Due to this advantage, relay transmission was studied for multimedia applications, such as progressive image [2] or video [3], [4]. In this paper, we consider a block fading relay channel to transmit a Gaussian source which is successively refinable. To support the layered transmission of the source, superposition coding, which is most useful for a broadcast channel [5] [6], is considered.

In a broadcast channel, the transmitter cannot adapt to an individual node's channel gain. Instead, it may be required that all nodes can achieve some minimum performance. By assigning high priority data, which can achieve a specified quality, to the base layer of the superposition coding, most nodes can achieve at least the minimum performance, even if they have poor channel conditions. However, for nodes with good channel conditions, signals can be decoded with better performance. In [7], superposition coding is applied to relay communication to improve the throughput between the source and the destination, where the relay transmits a message which is re-encoded with a modulation appropriate for the next link. Optimal power and rate allocation in two-level superposition coding based relaying schemes are studied by Goparaju et al [8]. An information-theoretic approach for this problem is studied in [9], where the authors considered joint source-channel coding for cooperative relaying systems with superposition coding when channel state information is only known at the receiver and quasi-static channels are assumed.

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They investigated the performance in terms of the distortion exponent to quantify the decay rate of the expected distortion. In this paper, we analyze the performance of the system both with and without superposition coding in terms of the average distortion. Then, we propose a simple algorithm where the superposition coding is used for the channel from the source to the relays, which is a broadcast channel. A pre-determined number of relays are dedicated to the base layer and to the enhancement layer in the transmission from the relays to the destination, which is not a broadcast channel.

The rest of this paper is organized as follows. In Section II, the system model and the performance analysis of three different protocols based on Decode-and-Forward (DF) are presented. Numerical results are shown in Section III. Finally, in Section IV, conclusions are presented.

## II. SYSTEM MODEL AND PERFORMANCE ANALYSIS

In this paper, we consider a single source, a single destination, and multiple relays. A Gaussian source is transmitted. Assuming that the total bandwidth is divided into  $N$  equal-width, disjoint channels as shown in Fig. 1, the transmission of source and relay nodes are orthogonal in the time and frequency domains. We assume half-duplex and decode-and-forward relays, so only those relay nodes that successfully decode the source packet re-encode and forward it to the destination. Direct link (DL) between source and destination is not used. We consider a block fading channel, where the fading coefficients are only known at the corresponding receivers of the relay and destination nodes.

We assume power constraints on both the source and relay nodes. In particular, we have the individual power constraint for the relay nodes [10]. That is, if the transmit power from the source node is  $P_T$ , then we assume that the maximum transmit power per relay node is  $P_T/N$ , where  $N$  is the total number of relays. We consider the following three protocols for relays to transmit the packet:

### A. Case 1 : Repetition-based conventional single layer transmission

As presented in [2], the relay transmits the packet if the packet is decoded successfully at the relay. If not, the relay is silent. To analyze the performance, we assume that a successively refinable Gaussian source is transmitted. We derive information-theoretic expressions of the average throughput and the expected distortion as functions of the outage probability. In Case 1, the Gaussian source is transmitted in a single layer.

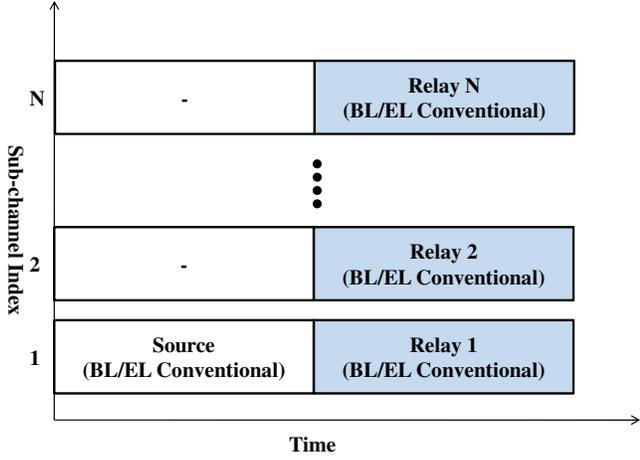


Fig. 1: Case 1 : Repetition-based conventional single layer transmission.

We denote by  $\alpha_{sr}^j$  the fading amplitude on the path from the source to the  $j$ th relay node, and by  $\alpha_{rd}^j$  the fading amplitude on the path from the  $j$ th relay node to the destination. We assume that  $\alpha_{sr}^j$  and  $\alpha_{rd}^j$ , for  $j = 1, \dots, N$ , are independent and Rayleigh distributed, with second moments  $E[(\alpha_{sr}^j)^2] = \Omega_{sr}^j$  and  $E[(\alpha_{rd}^j)^2] = \Omega_{rd}^j$ . Then, the mutual information (MI), conditioned on the channel gain, and the outage probability for the channel from the source to the  $j$ th relay with given rate  $R$  can be represented as [2] [12]

$$\text{MI}(j) = \log_2 \left( 1 + \frac{P_T}{\sigma_N^2} (\alpha_{sr}^j)^2 \right) \quad (1)$$

$$\begin{aligned} \pi_{r_j}(R) &= \text{Prob}(\text{MI}(j) < R) \\ &= \text{Prob} \left( \log_2 \left( 1 + \frac{P_T}{\sigma_N^2} (\alpha_{sr}^j)^2 \right) < R \right) \\ &= \text{Prob} \left( \frac{P_T}{\sigma_N^2} (\alpha_{sr}^j)^2 < R \right) \\ &= \gamma_{\text{inc}} \left( \frac{2^R - 1}{\frac{P_T}{\sigma_N^2} \Omega_{sr}^j}, 1 \right), \end{aligned} \quad (2)$$

respectively, where  $\gamma_{\text{inc}}$  is the incomplete Gamma function, which is defined as [11]

$$\gamma_{\text{inc}} \triangleq \frac{1}{\int_{\mu=0}^{\infty} e^{-\mu} \mu^{n-1} d\mu} \int_{\mu=0}^x e^{-\mu} \mu^{n-1} d\mu. \quad (3)$$

Note that MI is normalized by  $W$ , which is the bandwidth of a subchannel, throughout this paper. The single-sided power spectral density of the additive Gaussian noise is denoted by  $N_0$ , so that the noise power in a bandwidth  $W$  is  $\sigma_N^2 = N_0 W$ . We assume there are  $N$  relays and that  $P_T$ , which is the total transmit power from the source, is equally allocated to the relays. Then, the relays which decode the packet successfully will transmit the packet to the destination. At the destination, all packets are maximal-ratio combined and then decoded. Let  $\Phi_k$  denote the possible set of relay nodes which successfully decode the source packet, where  $k = 1, \dots, 2^N$ . The probability

of the set  $\Phi_k$  is

$$\text{Prob}(\Phi_k) = \prod_{i \in \Phi_k} (1 - \pi_{r_i}(R)) \prod_{j \notin \Phi_k} \pi_{r_j}(R), \quad (4)$$

where independence of the decoding errors at the relays, due to the spatial independence of the channel fading, is assumed. At the destination, the MI and the outage probability, conditioned on  $\Phi_k$ , are

$$\text{MI}_d(\Phi_k) = \log_2 \left( 1 + \sum_{l \in \Phi_k} \frac{P_T/N}{\sigma_N^2} (\alpha_{rd}^l)^2 \right), \quad (5)$$

$$\pi_d(R, \Phi_k) = \sum_{l \in \Phi_k} \lambda(\Phi_k) \gamma_{\text{inc}} \left( \frac{2^R - 1}{\frac{P_T/N}{\sigma_N^2} \Omega_{rd}^l}, 1 \right), \quad (6)$$

respectively, where  $\lambda(\Phi_k)$  is defined as [2]

$$\lambda(\Phi_k) = \prod_{m \in \Phi_k, m \neq l} \frac{\Omega_{rd}^l}{\Omega_{rd}^l - \Omega_{rd}^m}. \quad (7)$$

For simplicity, we assume channels from the relays to the destination have identical statistics. Then,

$$\pi_d(R, \Phi_k) = \gamma_{\text{inc}} \left( \frac{2^R - 1}{\frac{P_T/N}{\sigma_N^2} \Omega_{rd}}, |\Phi_k| \right), \quad (8)$$

where  $\Omega_{rd}$  represents the mean channel gain from the relays to the destination, and  $|\Phi_k|$  is the number of relays in the set  $\Phi_k$ . Finally, on averaging (6) over all possible decoding sets, the outage probability is

$$\pi_d(R) = \sum_{\Phi_k} \text{Prob}(\Phi_k) \pi_d(R, \Phi_k). \quad (9)$$

From (9), the average throughput at the destination can be computed as

$$E[T(R)] = 0 \cdot \pi_d(R) + R(1 - \pi_d(R)). \quad (10)$$

To obtain the expected distortion, we use the well-known rate-distortion function for the unit variance Gaussian source presented in [6] as

$$D(R) = 2^{-2R}. \quad (11)$$

Based on this function, the expected distortion can be computed as

$$E[D(R)] = 2^0 \cdot \pi_d(R) + 2^{-2R}(1 - \pi_d(R)). \quad (12)$$

## B. Case 2 : Repetition-based superposition coding

In this case, the base and enhancement layers are multiplexed using superposition coding, where the base layer (BL) and enhancement layer (EL) are generated by partitioning the successively refinable Gaussian source bitstream and the EL cannot be decoded without the BL. Since the priority of the BL is higher than that of the EL, more power can be assigned to the BL. At the relay, the BL is decoded first. To do that, the EL is considered as interference. If the BL is decoded successfully, then it is subtracted from the received signal to decode the EL. If a relay decodes both layers successfully, then both layers are superposed and transmitted. However if a relay decodes only the BL, then it will transmit the BL only. Since the EL cannot

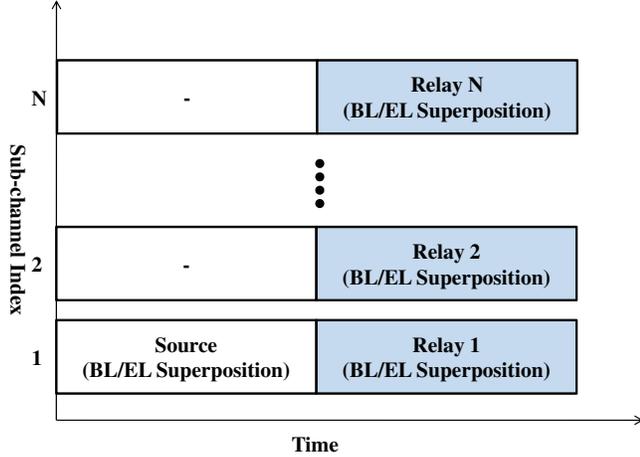


Fig. 2: Case 2 : Repetition-based superposition coding.

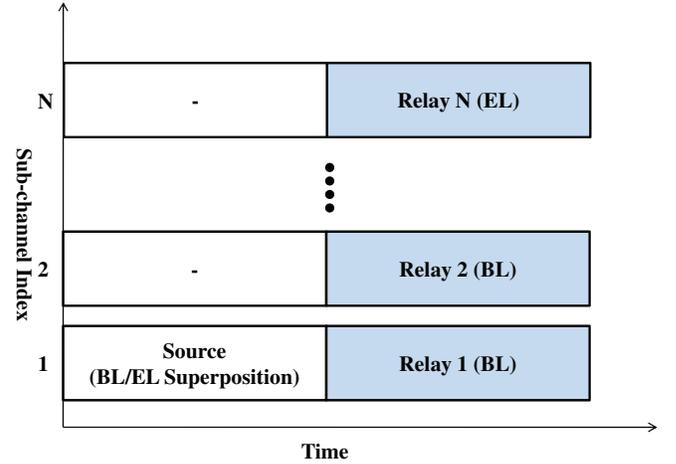


Fig. 3: Case 3 : Relay dedication-based superposition coding.

be used to reconstruct the source without the BL, we ignore the case that only the EL is available. Relays with good channel conditions will likely decode both the BL and the EL, while other relays, with poor channel condition, may decode nothing, or the BL only. Therefore, at the destination, maximal-ratio combining (MRC) cannot be used over all received signals.

Except for the cases where all relays decode both the BL and the EL, or the BL only, we have the following decoding procedure at the destination :

- 1) If there are any relays transmitting only the BL, they will be used to decode the BL at the destination. If there is no relay transmitting only the BL, decoding is done for the BL by considering the EL as interference.
- 2) The EL is decoded after subtracting the BL from the received signal.

For Case 2, the mutual information and corresponding outage probability need to be derived for the BL and EL separately. Detailed analysis for Case 2 is presented in [15].

### C. Case 3 : Relay dedication-based superposition coding

The decoding procedure of this protocol at the relays is exactly the same as that of Case 2. However, in this case, we assume all relays can communicate with each other to know whether other relays decode the BL and EL successfully. Based on that, we dedicate specific relays to the BL or the EL, instead of transmitting the superposed signal.

For Case 3, the outage probability from the source to the relays is exactly the same as for Case 2. However, from the relays to the destination, the outage is dependent on the number of relays dedicated to a specific layer. Assume at most  $M$  relays are assigned to the BL. If there are more than  $M$  relays which decode both the BL and the EL, then only  $M$  relays transmit the BL and the others transmit the EL. However, if there are less than  $M$  relays which decode the BL or both the BL and the EL, then all relays transmit only the BL. Then, the following decoding procedure is applied to Case 3, where  $y_k$  and  $z_k$  represent the number of relays which successfully decode only the BL, or both the BL and the EL, respectively :

- 1) If  $(y_k + z_k) \leq M$ , all relays transmit only the BL.
- 2) If  $(y_k + z_k) > M$ , but  $y_k \leq M$ , then the  $y_k$  relays which decode only the BL and the  $(M - y_k)$  relays which decode both layers are dedicated to transmit only the BL. The remaining  $(z_k - (M - y_k))$  relays which have both BL and EL transmit only the EL.
- 3) If  $y_k > M$ , then the  $z_k$  relays which have both layers transmit only the EL. The other  $y_k$  relays transmit only the BL.

A detailed analysis of the performance for Case 3 can be found in [15].

## III. NUMERICAL RESULTS

In this section, we present numerical results. To generate the results, we consider either two or four relays located midway between the source and the destination. All relays have the same mean channel gain to the destination. In Fig. 4, the expected throughputs for Cases 1, 2, and 3 are presented using (10) and the analysis presented in [15]. With two relays, we assign a single relay to the BL and the other to the EL to implement Case 3. As shown in the figure, Case 3 outperforms the other protocols in terms of the expected throughput. In particular, Cases 1 and 2 have almost the same throughput, where both of them are based on the DF protocol in the relays. The expected distortions for the three different protocols are shown in Fig. 5. This shows that layered transmission with superposition coding can reduce the expected distortion, compared to single layer transmission with a conventional DF protocol. Compared with the goal of maximizing the expected throughput, when we aim to minimize the expected distortion for Case 2 and 3, we need to assign higher order diversity to the BL to reduce the expected distortion.

In Fig. 6, the expected throughputs for Cases 1, 2, and 3 are shown, where four relays and a single receive antenna at the destination are considered. For Case 3, we consider  $M = 1, 2, 3$ . That is, one, two, or three relays are assigned to the BL. As shown, in terms of the expected throughput, Case 3 with  $M = 2$  achieves the best performance. Note that, from Fig. 4, Case 2 does not provide much improvement over Case

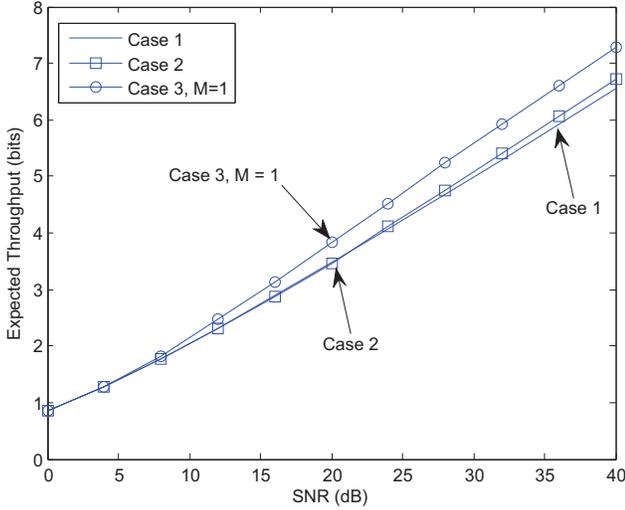


Fig. 4: Expected throughput with 2 relays and 1 receive antenna at the destination.

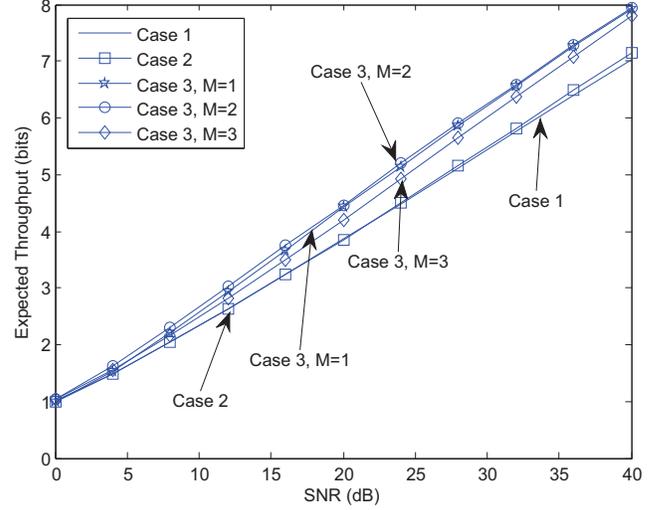


Fig. 6: Expected throughput with 4 relays and 1 receive antenna at the destination.

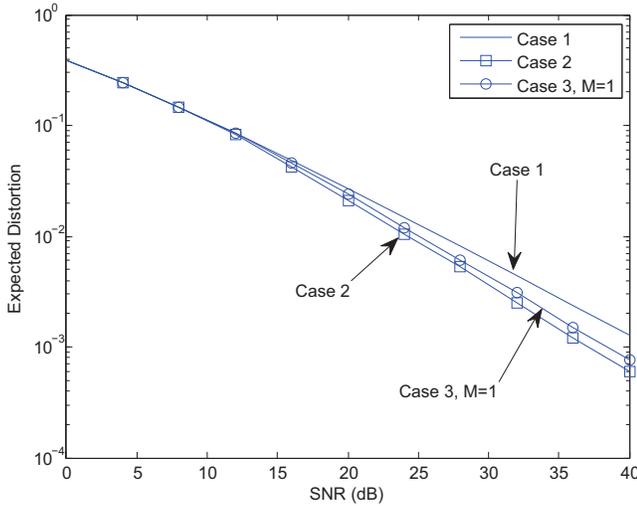


Fig. 5: Expected distortion with 2 relays and 1 receive antenna at the destination.

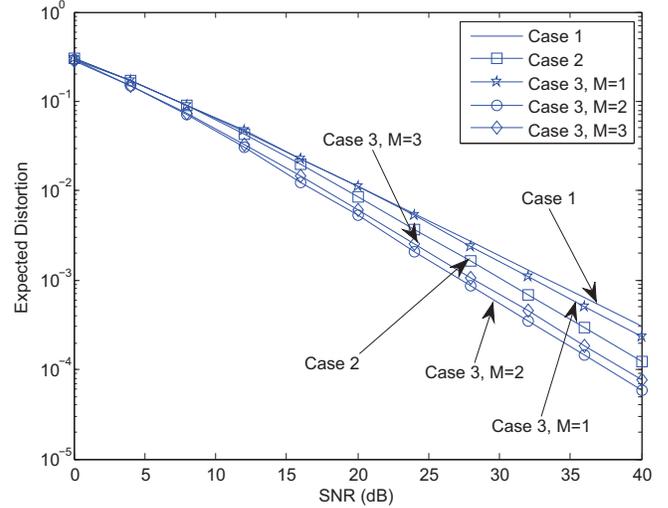


Fig. 7: Expected distortion with 4 relays and 1 receive antenna at the destination.

1 in terms of the expected throughput. The expected distortion is also presented in Fig. 7, where Case 3 with  $M = 2$  has the minimum distortion. Over most of the SNR range, Case 3 with  $M = 3$  has comparable performance to Case 3 with  $M = 2$ . However, if  $M = 1$ , the expected distortion becomes worse than that of Case 2, where the relay repeats the superposed signal to the destination. This shows that the number of relays assigned to the BL needs to be at least 2 to reduce the expected distortion.

#### IV. CONCLUSION

In this work, we consider the transmission of a Gaussian source over multiple relays. We consider three different protocols, which are based on the DF, with or without superposition coding. In the first protocol, a single layer source is transmitted and the relay repeats the source packet if it is decoded correctly. With superposition coding, the second protocol allows

relays to transmit either only the BL or both layers to the destination. We propose a simple protocol to assign dedicated relays to the BL and EL. The key point of the third protocol is that the superposition coding is used for the channel from the source to the multiple relays, which can be considered as a broadcast channel, and then specific relays are dedicated to the BL and the EL in the transmission from the relays to the destination, which is not a broadcast channel, to avoid the interference present in the superposition coding. Even though this simple protocol may not achieve the full diversity order for the superposed BL and EL at the destination, unequal diversity orders can be assigned to the layers. We also present numerical results based on an information-theoretic approach and show that there is a reduction in the expected distortion by using the protocol based on relay dedication to specific layers for both scenarios.

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