First-order Markov Models for Packet Transmission on Rayleigh Fading Channels with DPSK/NCFSK Modulation

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Abstract— In this paper, we develop first-order Markov models that characterize the packet error processes on Rayleigh fading channels *considering binary DPSK/NCFSK modulation.* Such models available in the literature so far consider only the fading process ignoring the underlying modulation used. Our contribution in this paper is that we consider first-order Markov models for binary DPSK/NCFSK modulation. To derive the Markov model parameters, we first derive expressions for the second-order statistics of the channel error process (specifically, the auto-correlation function of the bit error process as well as the packet error process), and obtain the Markov model parameters, in closed-form, as a function of normalized Doppler bandwidth, average received SNR and packet length. We also verify the accuracy of the proposed Markov model by deriving closed-form expressions for the mutual information of the channel error process.

Index Terms— First-order Markov model, packet transmission, DPSK/NCFSK modulation, Rayleigh fading.

I. INTRODUCTION

MARKOV models for packet transmissions on mobile radio channels have been of interest for several reasons, including the resulting tractability of performance analysis of complex protocols on wireless channels, cross-layer design/analysis (e.g., channel prediction based protocols for wireless systems), and design of low complexity packet-based wireless channel simulators [1]-[6]. A popular idea in this regard is to develop first-order Markov representations of fading channels [1],[2]. For example, the idea in [2] is to define a packet success/error event depending on the instantaneous fade power being greater/less than a threshold, and to model the resulting packet success/failure process by a discrete-time first-order Markov chain. Efforts to develop K-state Markov models that partition the received signal-to-noise ratio (SNR) into a finite number of states have also been reported [3]-[6]. Owing to their simplicity, these models have been very widely adopted in the literature [7]-[9].

A main concern with the above models is that they consider only the fading process, ignoring the modulation/coding schemes employed. The shortcoming with this approach can be illustrated as follows. In the model in [2], if the fade values of two consecutive samples are the same (e.g., very slow fading) and if the first sample results in a success event, then with probability one the second sample also will result in a success event. In reality, however, there will be a non-zero probability with which the second sample will result in an error event, which depends on the additive noise and

the modulation/coding used. We point out that in order to overcome this inadequacy of the model, one should look at the statistics of the underlying *actual bit/symbol error process* for a given noise realization and modulation/coding used, rather than just looking at the statistics of a *modelled error process* obtained by comparing the instantaneous fade power with a threshold, as was done in [2].

Unlike the previous models, our focus in this paper is to develop Markov models that characterize the *actual bit/packet error* processes on Rayleigh fading channels *considering the modulation/coding scheme* used. Towards that end, the first problem that needs to be solved is to derive second-order statistics of the actual bit/symbol error process for a given modulation/coding at a given average received SNR. We point out that even for simple uncoded binary modulation schemes, the second-order statistics (autocorrelation function of the actual bit error process) are not available in the literature. Accordingly, our first contribution in this paper is that we derive closed-form expressions for the autocorrelation function (ACF) of the actual bit error process, as well as the actual packet error process, for uncoded DPSK and NCFSK modulation on time-correlated Rayleigh fading channels. Secondly, we approximate the actual packet error process by a first-order Markov model and obtain closed-form expressions for the Markov model parameters as a function of normalized Doppler bandwidth (f_dT), average received SNR ($\overline{\gamma}$) and packet length (L). We also verify the accuracy of the proposed Markov model by deriving closed-form expressions for the mutual information of the bit/packet error process.

To enable easy usage of the proposed model, we present expressions for the two defining parameters (p and q) of the Markov model as a function of f_dT, $\overline{\gamma}$, and L. The proposed model can serve as a more accurate model in various scenarios, including in packet error process simulation, in performance analysis of wireless protocols, in cross-layer design/analysis, etc. We point out that we have restricted ourselves to uncoded DPSK/NCFSK in this paper due to space limitation. In fact, we have developed these models for other modulations, including M-QAM/M-PSK with coding. Those results will be presented in a separate contribution.

The rest of this paper is organized as follows. In Sec. II, we present the system model. The ACF derivation is presented in Sec. III. The Markov model parameters are derived in Sec. IV, and the information theoretic model verification is presented in Sec. V. Numerical results and discussions are presented in

Sec. VI. Conclusions are given in Sec. VII.

II. SYSTEM MODEL

We consider packet data transmission over block fading Rayleigh channels. Each data packet is composed of L information bits. The bit duration is denoted by T_b, and hence the packet duration $T = LT_b$. We assume that the information bits are either binary DPSK modulated with ideal differential detection, or binary orthogonal FSK modulated with ideal noncoherent detection. As a result, the error probability of any bit, conditioned on the instantaneous SNR, is given by

$$P_b(\gamma) = a \exp(-b\gamma), \tag{1}$$

where γ is the instantaneous SNR random variable (r.v). In (1), $a = 1/2$ and $b = 1$ for DPSK modulation, whereas $a = 1/2$ and $b = 1/2$ for NCFSK modulation [10].

We assume a complex Gaussian fading channel where each fade is modelled as a zero-mean, unit-variance, circularly symmetric, complex Gaussian r.v. The fading process is assumed to be slowly varying, and is constant over the packet duration, T. If γ_i is the instantaneous SNR in the ith packet duration, then the joint probability density function (pdf) of γ_i and γ_j can be expressed as [11]

$$f_{\gamma_i,\gamma_j}(x,y) = \frac{e^{-\frac{x+y}{\overline{\gamma}(1-\rho_{i,j}^2)}}}{\overline{\gamma}^2(1-\rho_{i,j}^2)} I_0\left(\frac{2\rho_{i,j}\sqrt{xy}}{(1-\rho_{i,j}^2)\overline{\gamma}}\right), \ x \geq 0 \ y \geq 0, \tag{2}$$

where $\overline{\gamma} = E[\gamma_i]$ is the average received SNR per bit, $I_0(\cdot)$ is the zeroth order modified Bessel function of the first kind [12], and $\rho_{i,j} = J_0(2\pi f_d T|i-j|)$ is the correlation coefficient of the underlying complex-valued fading process [11]. Here, $J_0(\cdot)$ is the zeroth order Bessel function [12]. Since $\rho_{i,j}$ is only a function of $|i-j|$ we use $\rho_{|i-j|}$ instead of $\rho_{i,j}$.

Since the bits within a packets are not coded, a packet is declared erroneously decoded if at least one of the bits in that packet is incorrectly detected. Let us denote by $\text{PEP}(\gamma_j)$ the packet error probability (PEP) of the jth packet, conditioned on the instantaneous SNR, γ_j. Then,

$$\text{PEP}(\gamma_j) = 1 - \left(1 - ae^{-b\gamma_j}\right)^L = \sum_{k=1}^{L}(-1)^{k+1}\binom{L}{k}a^k e^{-bk\gamma_j}, \tag{3}$$

where in (3) we have expanded $(1 - ae^{-b\gamma_j})^L$ using the binomial theorem. The average PEP is denoted by $\overline{\text{PEP}}(\overline{\gamma})$, and is obtained by term-by-term averaging of (3) as

$$\overline{\text{PEP}}(\overline{\gamma}) = E\left[\text{PEP}(\gamma)\right] = \sum_{k=1}^{L}(-1)^{k+1}\binom{L}{k}\frac{a^k}{1+kb\overline{\gamma}}. \tag{4}$$

The average bit error probability (BEP) can be obtained by setting $L = 1$ in (4).

III. ACF DERIVATION

Let us denote by $\mathcal{R}(\overline{\gamma}, \rho_{|i-j|})$ the autocorrelation function of the packet error process. Mathematically, we have

$$\mathcal{R}(\overline{\gamma}, \rho_{|i-j|}) = E\left[\text{PEP}(\gamma_i)\text{PEP}(\gamma_j)\right]. \tag{5}$$

Upon substituting (3) in (5), we have

$$\mathcal{R}(\overline{\gamma}, \rho_{|i-j|}) = \sum_{k=1}^{L}\sum_{l=1}^{L}(-1)^{k+l}\binom{L}{k}\binom{L}{l}a^{k+l}E\left[e^{-bk\gamma_i-bl\gamma_j}\right]. \tag{6}$$

To simplify the expectation in (6), we first obtain the conditional expectation of (6), conditioned on γ_i. Conditioned on γ_i, which is exponentially distributed with mean $\overline{\gamma}$, the pdf of γ_j can be written as

$$f_{\gamma_j|\gamma_i}(y|x) = \frac{f_{\gamma_i,\gamma_j}(x,y)}{f_{\gamma_i}(x)} = \frac{e^{-\frac{y+x\rho_{|i-j|}^2}{\overline{\gamma}(1-\rho^2|i-j|)}}}{\overline{\gamma}(1-\rho_{|i-j|}^2)} I_0\left(\frac{2\rho_{|i-j|}\sqrt{xy}}{(1-\rho_{|i-j|}^2)\overline{\gamma}}\right). \tag{7}$$

That is, $\gamma_j|\gamma_i = x$ is noncentral chi-square distributed with second moment $\overline{\gamma}(1 - \rho_{|i-j|}^2)$ and noncentrality parameter $x\rho_{|i-j|}^2$. The moment generating function (MGF) of $f_{\gamma_2|\gamma_1}(y|x)$ is [10]

$$\text{MGF}_{\gamma_2|\gamma_1=x}(ju) \triangleq E[e^{ju\gamma_2}|\gamma_1 = x] = \frac{\exp\left(\frac{jux\rho^2}{1-ju\overline{\gamma}(1-\rho_{|i-j|}^2)}\right)}{1-ju\overline{\gamma}(1-\rho_{|i-j|}^2)}. \tag{8}$$

Using (8), the conditional expectation $E\left[e^{-bk\gamma_i-bl\gamma_j}|\gamma_i\right]$ is

$$E\left[e^{-bk\gamma_i-bl\gamma_j}|\gamma_i\right] = \frac{e^{-bk\gamma_i}e^{-\frac{bl\gamma_i\rho^2}{1+bl\overline{\gamma}(1-\rho^2|i-j|)}}}{1+bl\overline{\gamma}(1-\rho_{|i-j|}^2)}, \tag{9}$$

and averaging (9) over γ_i gives us

$$E\left[e^{-bk\gamma_i-bl\gamma_j}\right] = \frac{1}{(1+bk\overline{\gamma})\left(1+bl\overline{\gamma}(1-\rho_{|i-j|}^2)\right)+bl\overline{\gamma}\rho_{|i-j|}^2}. \tag{10}$$

Finally, upon plugging in (10) in (6), a closed-form expression for $\mathcal{R}(\overline{\gamma}, \rho_{|i-j|}^2)$ is given by

$$\mathcal{R}(\overline{\gamma}, \rho_{|i-j|}) = \sum_{k=1}^{L}\sum_{l=1}^{L}\frac{(-1)^{k+l}\binom{L}{k}\binom{L}{l}a^{k+l}}{(1+bk\overline{\gamma})(1+bl\overline{\gamma}(1-\rho_{|i-j|}^2))+bl\overline{\gamma}\rho_{|i-j|}^2}. \tag{11}$$

With $L = 1$ in (11), the ACF of the bit error process is given by

$$\mathcal{R}_{\text{Bit}}(\overline{\gamma}, \rho_{|i-j|}) = \frac{a^2}{(1+b\overline{\gamma})(1+b\overline{\gamma}(1-\rho_{|i-j|}^2))+b\overline{\gamma}\rho_{|i-j|}^2}. \tag{12}$$

IV. COMPUTATION OF MODEL PARAMETERS

To derive the parameters of the first-order Markov model, we model the success/failure process of the transmitted packets as a binary random process. Let us denote by $\{\beta_j\}$ the success/failure process of the transmitted data packet. We denote by $\beta_j = 1$ the event that the jth packet is successful, and let $\beta_j = 0$ indicate it is a failure. Then we write $\text{Prob}(\beta_i = 1|\gamma_i) = 1 - \text{PEP}(\gamma_i)$, and $\text{Prob}(\beta_i = 0|\gamma_i) = \text{PEP}(\gamma_i)$. Or, in a compact form,

$$\text{Prob}(\beta_i = l|\gamma_i) = l + (1-2l)\text{PEP}(\gamma_i), \quad l \in \{0,1\}. \tag{13}$$

The joint probability mass function (pmf) of β_i and β_j can be derived as

$$
\begin{aligned}
\mathrm{Prob}(\beta_i = l, \beta_j = k) &= E\left[\mathrm{Prob}(\beta_i = l, \beta_j = k | \gamma_i, \gamma_j)\right] \\
&= E\left[\mathrm{Prob}(\beta_i = l | \gamma_i)\mathrm{Prob}(\beta_j = k | \gamma_j)\right] \\
&= lk + (l + k - 4lk)\overline{\mathrm{PEP}}(\overline{\gamma}) + \\
&\quad (1 - 2l)(1 - 2k)\mathcal{R}(\overline{\gamma}, \rho_{|i-j|}),
\end{aligned} \tag{14}
$$

where, in (14), $l, k \in \{0, 1\}$, $\overline{\mathrm{PEP}}(\overline{\gamma})$ is given by (4) and $\mathcal{R}(\overline{\gamma}, \rho_{|i-j|})$ is given by (11). Note that, when $\rho_{|i-j|} \to 1$ (i.e., for the case of fully correlated Rayleigh fading) the joint pdf of γ_i and γ_j of (2) degenerates to

$$
f_{\gamma_i, \gamma_j}(x, y) = f_{\gamma_i}(x)\delta(x - y) \ (\rho_{|i-j|} \to 1), \tag{15}
$$

where $\delta(x)$ is the Dirac delta function. Accordingly, the ACF $\mathcal{R}(\overline{\gamma}, \rho_{|i-j|} \to 1)$ of (5) reduces to

$$
\mathcal{R}(\overline{\gamma}, \rho_{|i-j|} \to 1) = E[\mathrm{PEP}(\gamma_i)^2], \tag{16}
$$

which is equal to the second moment of the PEP. Upon using this in (14), we have

$$
\begin{aligned}
\mathrm{Prob}(\beta_i = l, \beta_j = k)(\rho_{|i-j|} \to 1) &= lk + \\
(l + k - 4lk)\overline{\mathrm{PEP}}(\overline{\gamma}) + (1 - 2l)(1 - 2k)&E[\mathrm{PEP}(\gamma)^2],
\end{aligned} \tag{17}
$$

which is not equal to $\mathrm{Prob}(\beta_i = l)\delta_{l-k}$, where δ_n is the Kronecker delta function: $\delta_n = 1$ for $n = 0$ and is equal to zero for $n \neq 0$. That is, our proposed model shows that on a perfectly time-correlated fading channel, the packet success (failure) event in the ith time interval does not lead to a packet success (failure) event in the jth time interval with probability one. Our definition of the success/failure process takes into account not only the fading channel variations, but also the underlying modulation/coding characteristics and the additive noise effects of the channel. On the contrary, the first order model of [2] and the finite-state Markov model of [3] conclude that $\mathrm{Prob}(\beta_j = 1 | \beta_i = 1) = 1$ and $\mathrm{Prob}(\beta_j = 0 | \beta_i = 0) = 1$.

The marginal pmf of $\beta = 1$ can easily be shown to equal

$$
\mathrm{Prob}(\beta_i = 1) = 1 - \overline{\mathrm{PEP}}(\overline{\gamma}) \tag{18}
$$
$$
\text{and} \quad \mathrm{Prob}(\beta_i = 0) = \overline{\mathrm{PEP}}(\overline{\gamma}). \tag{19}
$$

Using (14), (18) and (19), it is now straightforward to derive the parameters of the first-order Markov model. Conditioned on the event that the $(i - 1)$th packet is successful, the probability of the ith packet being successful is denoted by p, and is given by

$$
p = \mathrm{Prob}(\beta_i = 1 | \beta_{i-1} = 1) = \frac{1 - 2\overline{\mathrm{PEP}}(\overline{\gamma}) + \mathcal{R}(\overline{\gamma}, \rho_1)}{1 - \overline{\mathrm{PEP}}(\overline{\gamma})}, \tag{20}
$$

where $\rho_1 = \rho_{i-1,i} = J_0(2\pi f_d T)$. Conditioned on the event that the $(i - 1)$th packet is a failure, the probability of the ith packet being unsuccessful is denoted by q, and is given by

$$
q = \mathrm{Prob}(\beta_i = 0 | \beta_{i-1} = 0) = \frac{\mathcal{R}(\overline{\gamma}, \rho_1)}{\overline{\mathrm{PEP}}(\overline{\gamma})}. \tag{21}
$$

Let us denote by $\underline{\pi} = [\pi_s, \pi_f]$ the steady-state probability vector of the Markov chain, where π_s and π_f are the steady-state probabilities of the packet being in the success and the failure states, respectively. Assuming that the Markov chain is stationary, the $\underline{\pi}$ vector can be obtained by solving $\underline{\pi} = \underline{\pi}\mathbf{P}$, where $\mathbf{P}$ is the channel transition probability matrix, which is given in terms of the Markov chain parameters p and q as

$$
\mathbf{P} = \begin{bmatrix} p & 1 - p \\ 1 - q & q \end{bmatrix}. \tag{22}
$$

The steady-state probabilities, π_s and π_f, are given by

$$
\pi_s = \frac{1 - q}{2 - p - q} \tag{23}
$$
$$
\text{and} \quad \pi_f = \frac{1 - p}{2 - p - q}. \tag{24}
$$

The average burst error length, denoted by $\overline{L}_B$, of the above two-state Markov chain is given by $(1 - q)^{-1}$, which, for the proposed bit/packet error probability-based model, is given by

$$
\overline{L}_B = \frac{\overline{\mathrm{PEP}}(\overline{\gamma})}{\overline{\mathrm{PEP}}(\overline{\gamma}) - \mathcal{R}(\overline{\gamma}, \rho_1)} \tag{25}
$$

which, when $\rho_1 \to 1$, from (16), reduces to

$$
\overline{L}_B(\rho_1 \to 1) = \frac{\overline{\mathrm{PEP}}(\overline{\gamma})}{\overline{\mathrm{PEP}}(\overline{\gamma}) - E\left[\mathrm{PEP}(\gamma)^2\right]}. \tag{26}
$$

When $L = 1$, using (4) and (12), (25) simplifies to

$$
\overline{L}_B = \frac{(1 + b\overline{\gamma})(1 + b\overline{\gamma}(1 - \rho_1^2)) + b\overline{\gamma}\rho_1^2}{(1 + b\overline{\gamma})(1 - a + b\overline{\gamma}(1 - \rho_1^2)) + b\overline{\gamma}\rho_1^2}. \tag{27}
$$

Equation (27) shows that, when $\rho_1 = 1$, as is the case for a fully correlated fading channel, $\overline{L}_B \to (1 + 2b\overline{\gamma})/(1 - a + (2 - a)b\overline{\gamma})$, which when $\overline{\gamma} \to \infty$ gives us $\overline{L}_B = 4/3$ for both DPSK and NCFSK modulations. On the other hand, when $\rho_1 < 1$, (27) shows that $\overline{L}_B \to 1$ as $\overline{\gamma} \to \infty$, irrespective of the actual value of ρ_1. This is due to the fact that, from (12), the ACF decays inversely with $\overline{\gamma}$ when $\rho_1 = 1$, and it decays as $\overline{\gamma}^{-2}$ for any value of $\rho_1 < 1$. As $\rho_1 \to 0$, $\overline{L}_B \to (1 - \overline{\mathrm{PEP}}(\overline{\gamma}))^{-1}$. Unless $\overline{\gamma} \to 0$ (in which case $\mathrm{PEP}(\gamma)$ is unity with probability one) (25) shows that the average burst error length is *finite*. In contrast, the threshold-based model of [2] shows that $\overline{L}_B \to \infty$ as $\rho_1 \to 1$ (since $q \to 1$ as $\rho_1 \to 1$ in a threshold model).

V. MUTUAL INFORMATION ANALYSIS

In [2], a mutual information-based argument is presented for the accuracy of a first order Markov model derived from the threshold-based success/failure process. In this section, using MI-based analysis, we argue that the proposed first-order Markov model is reasonably accurate for moderate to high values of normalized Doppler bandwidth. The MI between the current sample, β_i, of the success/failure process and the past two samples is[1] [13]

$$
\begin{aligned}
I\left(\beta_i; (\beta_{i-1}, \beta_{i-2})\right) &= I(\beta_i; \beta_{i-1}) + I(\beta_i; \beta_{i-2} | \beta_{i-1}) \\
&= I(\beta_i; \beta_{i-1})(1 + \zeta),
\end{aligned} \tag{28}
$$

where $\zeta = I(\beta_i; \beta_{i-2} | \beta_{i-1})/I(\beta_i; \beta_{i-1})$. To show that the first-order Markov model is sufficient to characterize the actual bit/packet error process, we would like to show that the MI between β_i and $(\beta_{i-1}, \beta_{i-2})$ is approximately the same as the

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[1]For analytical tractability, in this paper we consider only the past two samples.

MI between β_i and β_{i-1}. That is, conditioned on β_{i-1}, we would like to show that the conditional MI between β_i and β_{i-2} is negligible in comparison with $I(\beta_i; \beta_{i-1})$. In other words, we want ζ to be as small as possible. We now calculate the individual MI expressions of (28).

The average MI, $I(\beta_i; \beta_{i-1})$, between β_i and β_{i-1} is given by [13]

$$
\begin{aligned}
I(\beta_i; \beta_{i-1}) &= E_{(\beta_i, \beta_{i-1})}\left[\log_2\left(\frac{\text{Prob}(\beta_i, \beta_{i-1})}{\text{Prob}(\beta_i)\text{Prob}(\beta_{i-1})}\right)\right] \\
&= \sum_{l=0}^{1}\sum_{m=0}^{1}\text{Prob}(\beta_i = l, \beta_{i-1} = m) \times \\
&\quad \log_2\left(\frac{\text{Prob}(\beta_i = l, \beta_{i-1} = m)}{\text{Prob}(\beta_i = l)P(\beta_{i-1} = m)}\right).
\end{aligned}
\tag{29}
$$

Note that computation of $I(\beta_i; \beta_{i-1})$ requires the joint pmf of β_i and β_{i-1}, which is derived in closed-form in (14). The conditional average MI between β_i and β_{i-2}, conditioned on β_{i-1}, is given by [13]

$$
\begin{aligned}
I(\beta_i; \beta_{i-2}|\beta_{i-1}) &= E_{(\beta_i, \beta_{i-1}, \beta_{i-2})}\Bigg[\\
&\log_2\left(\frac{\text{Prob}(\beta_i, \beta_{i-2}|\beta_{i-1})}{\text{Prob}(\beta_i|\beta_{i-1})\text{Prob}(\beta_{i-2}|\beta_{i-1})}\right)\Bigg] \\
&= \sum_{l=0}^{1}\sum_{m=0}^{1}\sum_{n=0}^{1}\text{Prob}(\beta_i = l, \beta_{i-1} = m, \beta_{i-2} = n) \times \\
&\log_2\left(\frac{\text{Prob}(\beta_i = l, \beta_{i-1} = m, \beta_{i-2} = n)\text{Prob}(\beta_{i-1} = m)}{\text{Prob}(\beta_i = l, \beta_{i-1} = m)\text{Prob}(\beta_{i-2} = n, \beta_{i-1} = m)}\right)
\end{aligned}
\tag{30}
$$

Computation of (30) requires the joint pmf of β_i, β_{i-1} and β_{i-2}, which can be shown to be

$$
\begin{aligned}
\text{Prob}(\beta_i = l_0, \beta_{i-1} = l_1, \beta_{i-2} = l_2) &= E_{(\gamma_i, \gamma_{i-1}, \gamma_{i-2})}\Bigg[\\
(l_0 + \text{PEP}(\gamma_i)(1 - 2l_0)) &\times \\
(l_1 + \text{PEP}(\gamma_{i-1})(1 - 2l_1))\,(l_2 + \text{PEP}(\gamma_{i-2})(1 - 2l_2))&\Bigg] \\
= \overline{\text{PEP}}(\overline{\gamma})\,(l_0 l_1 + l_1 l_2 + l_2 l_0 - 6l_0 l_1 l_2) &+ \\
l_0(1 - 2l_1)(1 - 2l_2)\mathcal{R}\,(\overline{\gamma}, \rho_1) &+ \\
l_1(1 - 2l_0)(1 - 2l_2)\mathcal{R}\,(\overline{\gamma}, \rho_2) &+ \\
l_2(1 - 2l_0)(1 - 2l_1)\mathcal{R}\,(\overline{\gamma}, \rho_1) &+ \\
(1 - 2l_0)(1 - 2l_1)(1 - 2l_2)\mathcal{K}\,(\overline{\gamma}, \rho_1, \rho_2)\,,
\end{aligned}
\tag{31}
$$

where

$$
\mathcal{K}\,(\overline{\gamma}, \rho_1, \rho_2,) \triangleq E\left[\text{PEP}(\gamma_i)\text{PEP}(\gamma_{i-1})\text{PEP}(\gamma_{i-2})\right],
\tag{32}
$$

is the *third order statistic* of the packet error process. In (31) we have $l_0, l_1, l_2 \in \{0, 1\}$. Using (3), $\mathcal{K}\,(\overline{\gamma}, \rho_1, \rho_2,)$ can be simplified as

$$
\begin{aligned}
\mathcal{K}\,(\overline{\gamma}, \rho_1, \rho_2,) &= \sum_{m=1}^{L}\sum_{n=1}^{L}\sum_{r=1}^{L}(-1)^{m+n+r+1}a^{m+n+r} \times \\
&\binom{L}{m}\binom{L}{m}\binom{L}{r}E\left[e^{-bm\gamma_i - bn\gamma_{i-1} - br\gamma_{i-2}}\right].
\end{aligned}
\tag{33}
$$

To simplify the expectation in (33) we use a result from [14]. In [14], Mallik presented a simple closed-form expression for the MGF of correlated exponential r.vs when the underlying Gaussian r.vs are circularly symmetric. Since the underlying Gaussian r.vs in our model are also assumed to be circularly symmetric, using (77) in [14] the expectation in (33) can be simplified as

$$
E\left[e^{-bm\gamma_i - bn\gamma_{i-1} - br\gamma_{i-2}}\right] = \frac{1}{\text{det}\,(I_3 + j2b\text{diag}(m, n, r)\overline{\gamma}\text{R})}
\tag{34}
$$

where $\text{det}(\cdot)$ is the determinant operator, I_3 is the 3-by-3 identity matrix, $\text{diag}(a_1, a_2, \ldots, a_n)$ is the n-by-n diagonal matrix with the diagonal elements $a_1, \ldots, a_n$, and R is the covariance matrix of the underlying complex Gaussian matrix which is given by

$$
\text{R} = \frac{1}{2}\begin{bmatrix} 1 & \rho_1 & \rho_2 \\ \rho_1 & 1 & \rho_1 \\ \rho_2 & \rho_1 & 1 \end{bmatrix},
\tag{35}
$$

where $\rho_2 = \rho_{i-2,i} = J_0(4\pi f_d T)$. Upon substituting (34) in (33) we arrive at

$$
\begin{aligned}
\mathcal{K}\,(\overline{\gamma}, \rho_1, \rho_2) &= \sum_{m=1}^{L}\sum_{n=1}^{L}\sum_{r=1}^{L}(-1)^{m+n+r+1} \times \\
&\frac{a^{m+n+r}\binom{L}{m}\binom{L}{n}\binom{L}{r}}{\text{det}\,(I_3 + j2b\text{diag}(m, n, r)\overline{\gamma}\text{R})}.
\end{aligned}
\tag{36}
$$

Substituting (36) in (31) gives us a closed-form expression for $\text{Prob}(\beta_i = l_0, \beta_{i-1} = l_1, \beta_{i-2} = l_2)$.

VI. RESULTS AND DISCUSSION

In this section, we present some numerical results illustrating the design of the proposed Markov model. First, in Fig. 1 the correlation coefficient (CorrCoeff) function of the BEPs of both DPSK and NCFSK modulations are compared as a function of normalized Doppler bandwidth $f_d T$. These curves are parameterized by the average received SNR $\overline{\gamma} \in \{10, 15, 20\}$ dB. From Fig. 1, we conclude that, at a given $f_d T$ and average SNR, $\overline{\gamma}$, the CorrCoeff between the BEP of DPSK is much smaller in comparison with the BEP CorrCoeff of NCFSK. This is due to the fact that DPSK is superior to NCFSK by 3 dB. The CorrCoeff decreases with both the average received SNR and the normalized Doppler bandwidth. This can be explained by the fact that at high SNR, due to the improved reliability, the bit-to-bit error dependence is reduced, whereas increased normalized channel bandwidth directly leads to a reduction in channel correlation. It is to be noted that the nonmonotonic behavior of the BEP correlation coefficient is due to the nonmonotonic nature of the underlying fading correlation coefficient, $J_0(2\pi f_d T)$, with $f_d T$.

The MI and conditional MI of the packet success/failure process is compared in Fig. 2 as a function of the average received SNR. The MI and conditional MI expressions are evaluated from the analysis presented in Section VI. For $f_d T = 0.05$, Fig. 2 shows that the conditional MI, $I(\beta_i; \beta_{i-2}|\beta_{i-1})$, is smaller, in most cases by an order of magnitude or half an order of magnitude, than $I(\beta_i; \beta_{i-1})$, and can be ignored for simplicity. The contribution of $I(\beta_i; \beta_{i-2}|\beta_{i-1})$
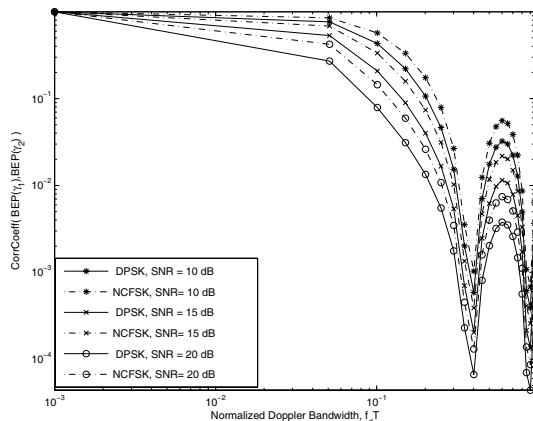
Fig. 1.   Normalized correlation coefficient (i.e., the covariance normalized by the variance) of the BEP process as a function of the normalized Doppler bandwidth f_dT.
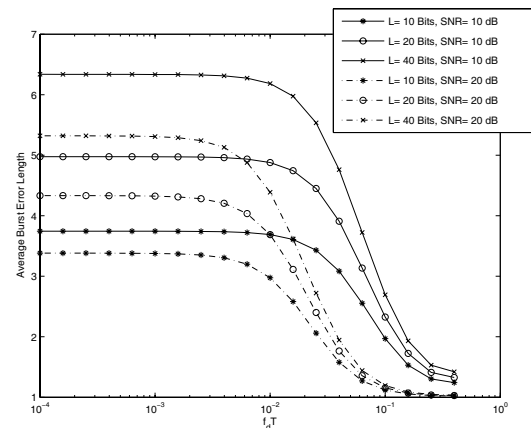

Fig. 3.   Average burst error length as a function of normalized Doppler bandwidth f_dT for DPSK packet transmission.
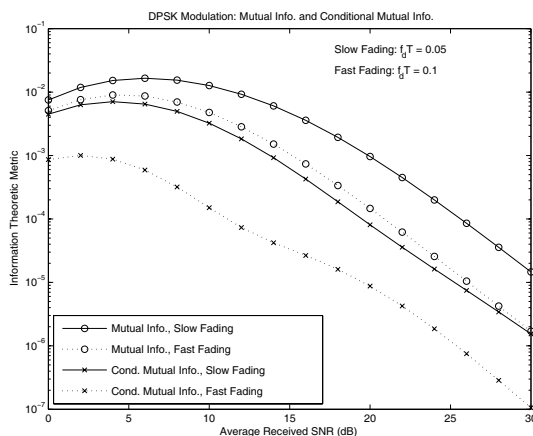

Fig. 2.   Average MI, $I(\beta_i; \beta_{i-1})$, and conditional average MI, $I(\beta_i; \beta_{i-2}|\beta_{i-1})$, of the channel error process β_i with binary DPSK modulation on time-correlated Rayleigh fading channels.

to $I(\beta_i; (\beta_{i-1}, \beta_{i-2})$ becomes much less, in comparison with $I(\beta_i; \beta_{i-1})$, when $f_dT = 0.1$, thus justifying the validity of the proposed model for moderate to high values of f_dT.

In Fig. 3, the average burst error length $\overline{L}_B$ is plotted as a function of f_dT, and parameterized by the average SNR and the packet length. For a given average SNR, Fig. 3 shows that the burst length increases with the packet length. As the average SNR increases, the probability of a packet failure, given a packet failure, q, decreases, which leads to a reduction in the average burst error length. Fig. 3 also shows that for a given packet length and average received SNR, the average burst error length is finite as $f_dT \to 0$.

VII. CONCLUSION

We presented a new approach to first-order Markov modelling of slowly varying Rayleigh channel by explicitly taking into account the bit/frame error probability variation as a function of the channel fading, the underlying modulation employed, and the effect of additive noise. To arrive at the model parameters, we presented closed-form analysis of the second-order statistics of the packet error probability with DPSK/NCFSK modulation. A simple mutual information-based analysis also was presented to validate the accuracy of the proposed model. On a slowly varying Rayleigh channel, the proposed model was shown to be superior to an earlier threshold-based model in capturing the realistic behavior of the packet success/failure process.

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