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WAVELET COMPRESSION OF 3D MEDICAL IMAGES USING CONDITIONAL ARITHMETIC CODING

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ABSTRACT

In this paper we investigate the coding of 3D medical images using three-dimensional wavelet decomposition and adaptive arithmetic coding. A highly efficient context arithmetic coding scheme is used in conjunction with classical lifting-based wavelet decompositions. The resulting coder is shown to attain superior performance in comparison to other compression schemes for 3D medical image coding.

1. INTRODUCTION

Lossless signal and image compression is necessary for many applications where perfect accuracy is needed. The efficient storage and communication of medical images is one of the main applications of lossless image compression.

Among the various losssless coding techniques, multiresolution coding techniques such as wavelet and pyramid decompositions [1] have gained much popularity during the past few years, mainly due to their progressive transmission capability.

In wavelet-based image representation the initial image is decomposed into a coarse image and a high pass image containing the detail information. By iterating the procedure described above to the lowpass channel, a logarithmically split filter-bank is formed consisting of wavelet coefficients at different scales and a lowpass image of very small dimensions.

Despite their excellent decorrelating performance, conventional wavelets and filter banks had not been widely used for lossless medical coding due to the need for the transmission of floating point coefficients. The lifting scheme [2] has recently attracted much interest as it provides the methodology to implement critically sampled filter banks which have integer output. In this paper we propose an efficient arithmetic coding scheme for the entropy coding of wavelet decompositions of 3D medical images.

2. THREE-DIMENSIONAL REVERSIBLE WAVELET TRANSFORMS

A wavelet transform in its simplest form is obtained by filtering an image with a filter bank such as that described in Fig. 1 composed of analysis filters h_k followed by subsampling by 2, upsampling and synthesis filters g_k. Exact recovery of the initial image is possible if proper relationships hold between the filters h and g [1].

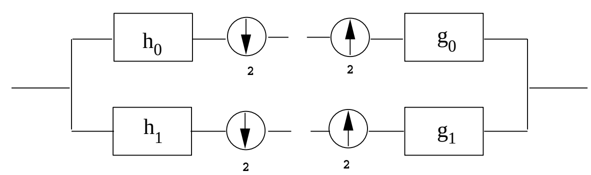

Figure 1: Conventional implementation of a two-channel filter bank.

Extension of the one-dimensional wavelet transform obtained using the filter bank in Fig. 1 can be trivialy made in the two- and three- dimensional space by using wavelet bases that are constructed as tensor-products of one-dimensional wavelet bases. A logarithmically-split three-dimensional filter bank decomposition is depicted in Fig. 2

A very efficient way to accomplish efficient integer wavelet decomposition is the implementation of a filter bank using the “lifting scheme”. In its most general form [2] this consists of splitting the image in separate components, estimating components from others and subsequently adding to components filtered versions of other components. The step associated with estimating the intensity of a coefficient is usually termed *prediction* whereas the step associated with smoothing the coefficients on which the initial prediction is based using the prediction errors is called *update*. A lifting decomposition is depicted in Fig. 3. The lifting scheme

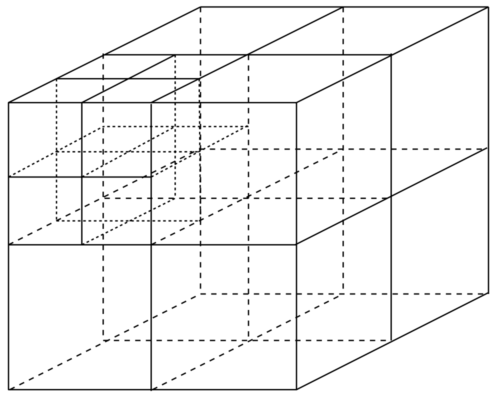

Figure 2: Three-dimensional wavelet decompostion of volume data. The wavelet bases used are constructed as tensor-products of one-dimensional wavelet bases.

guarantees perfect reconstruction of the original data. Such transforms seem to fit very well in the framework of 2D and 3D medical image coding where perfect reconstruction of the original data is necessary.

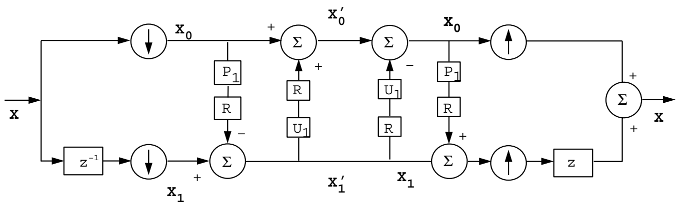

Figure 3: A simple lifting-based perfect reconstruction codec. P denotes the prediction filter, U the update filter and R the rounding operation.

For the construction of three-dimensional pyramids, the integer wavelets in [3] were used. The design of the integer wavelets in [3] is based on the polynomial preservation and annihilation properties of the resulting filter banks. These filter banks were shown to be capable of producing very parsimonious 2D [3] and 3D [4] image represenations.

Despite their excellent decorelating performance, wavelet transforms are not able to remove all existing redundancy in a still image. This can be easily confirmed by inspection of the wavelet representation of an image. It is seen that wavelet coefficients residing in edge areas have high variance at all scales of the decomposition. These coefficients are the most expensive to code. Using conditional arithmetic coding, lossless bitrates below first order entropy can be achieved.

3. CONDITIONAL ARITHMETIC CODING OF WAVELET COEFFICIENTS

For the coding of the coefficients produced by the transforms of the previous section, an entropy coder combining many features was implemented. The sequence of wavelet coefficients is first partitioned into a number of subsequences which consist of symbols having similar statistics. Each subsequence is coded using a separate arithmetic coder i.e. a variety of adaptive probability models are employed. The lowpass image is also coded using a separate adaptive model. Each coefficient is represented by the class in which it belongs, its sign and the residual bits specifying its exact magnitude. The classes used are shown in Table 1.

This representation is similar to that used in JPEG [5] as well as in [6] for lossy compression. The whole range of the coefficients is divided into several classes. Due to the fact that the values of wavelet coefficients are concentrated around zero, more classes are allotted to near-zero magnitudes (Table 1). Some classes (0,1,2,3) include only one coefficient magnitude. This means that a coefficient belonging to such a class does not require residual bits for its representation since the class itself identifies the exact magnitude of the coefficient.

Each coefficient in the highpass bands is conditioned using the values of adjacent coefficients. The causal contexts used presume that the exact value of these wavelet coefficients is known i.e. all class, residual and sign bits of the previously coded coefficients are known. This means that the decoder while decoding the current coefficient has fully recovered the past coefficients. In this way, past coefficients that were used as a conditioning context during encoding, are also *explicitly* known by the decoder during the decoding process. This fact makes the formation of modeling contexts more flexible in comparison to lossy coders since in the later case only quantized and not the original coefficient values are known.

An arithmetic coder [7] achieves significant compression by transmitting the more probable symbols in fewer bits than the less probable ones. For example, the model may assign a predetermined probability to each possible symbol. These probabilities may be determined by counting frequencies in representative samples of the input source to be transmitted. Such a *fixed* model is communicated in advance to both the encoder and decoder, after which it is used for many images. Alternatively, the probabilities the model assign may change as each symbol is transmitted, based on the symbol frequencies seen *so far* in this message. In such an *adaptive model* there is no need for a representative sample of input data, because each message is treated as an independent unit, starting from scratch. The encoder's model changes with each symbol transmitted, and the decoder's changes with each symbol received, in precisely the same manner. Adaptive models are used in all our experiments.

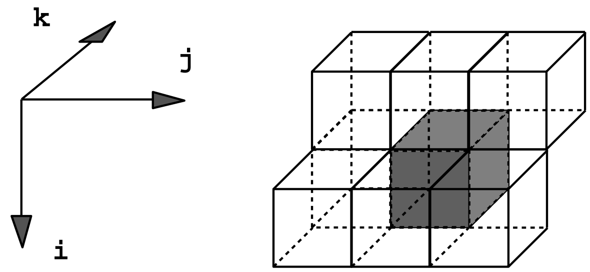

Figure 4: Causally available coefficients that are used for the class coding of the current coefficient (grey coloured).

Splitting of the information of a coefficient in three components, namely class, residual bits and sign, requires the use of different adaptive arithmetic coders (adaptive models) for the arithmetic coding of each component, since it is to be expected that the symbols corresponding to the three components have different statistics. The adaptivity of the coder means that initially all symbols are considered of having equal probability. During encoding, however, the probability tables are updated and soon converge to the actual statistics of the source maximizing the coding efficiency of the arithmetic coder. In order to further enhance performance, we used many adaptive models for coding each of the three components in which a coefficient is split.

In each quadrant, the wavelet coefficients are visited in a lexicographical order. For each coefficient, a "context" C_j is derived based on the values of its adjacent coefficients which have already been visited and its parent coefficient. For example, for coding the class of a coefficient, the index of the adaptive model that will be used is derived based on its causal neighbourhood and then the corresponding adaptive model is used for the coding of the class. In this way the activity in the vinicity of the current coefficient provides some information about the magnitude of the current coefficient. By taking this into consideration, coefficients of different activity are treated differently by the encoder, by being fed to different adaptive coders. This higher order approach reduces the eventual errorless encoding cost below the first order entropy of the initial coefficient sequence. Similar conclusions hold for residual bit and sign coding.

A. Coding the class

Using the conditioning contexts described above, the adaptive probability model which will be used for the coding of the the current coefficient is determined. This is done as follows:

$$C_p = CLASS(A) \tag{1}$$

where the parameter A is equal to the average of the absolute causal, adjacent coefficients of the current coefficient (Fig. 4). Equation (1) gives the index of the adaptive probability model which will be used for the coding of the class of the current coefficient.

For example consider a coefficient valued -80. This coefficient is represented by the triad $(10, -, 16)$ The first number in the triad is the class number. Thus, a coefficient belonging to the $10th$ class is valued between 64-127. The symbol "-" is its sign and the number 16 is the residual difference between the number being depicted and the lowest number in its class: 80=64+16. If, then, equation (1) gives CLASS(70)=10, the 10th arithmetic model will be used to code the class of the current coefficient. In a similar way, all coefficient classes (0-11) are coded using the adaptive model whose index is derived using (1).

Class	magnitude
0	0
1	1
2	2
3	3
4	4-5
5	6-7
6	8-11
7	12-15
8	16-31
9	32-63
10	64-127
11	128-255

Table 1: Classes used for the coding of the magnitude information of coefficients

B. Coding the residual bits

The coding of the residual bits uses a different adaptive arithmetic model for each class. Since the decoder is aware of the class in which the current coefficient belongs, it decodes the residual bits using the adaptive model which has been formulated for the decoding of the residual bits of this class.

C. Coding the sign

Efficient coding of the signs is probably the most challenging of the three tasks. We code the signs coefficients using a very simple context selector and two adaptive probability models. We used the available causally adjacent pixels lying on the horizontal and vertical directions. If the sum of these two coefficients is positive, we use the first adaptive arithmetic coder. Otherwise, we use the second arithmetic coder for the coding of the current sign. This intuitive approach produces a small gain in coding. This gain is due to the fact that the partitioning of the sign stream, using the aforementioned rule, produces two streams which have different statistics. Note, that since adaptive arithmetic coders are used, these statistics need not be computed, stored or transmitted. The adaptive arithmetic coder simply adapts to the statistics of each of the two input streams.

As a whole, our coder uses 12 adaptive probability models (0-11) for the coding of the class of the current pixel, 7 models for the coding of the residual bits and 2 models for the coding of the sign. It is important to note that all these models are adaptively updated while coding and decoding. This means that the arithmetic coders learn the statistics of the input sequences.

4. EXPERIMENTAL RESULTS

The efficiency of the proposed method for lossless coding was evaluated using a variety of 3D medical images. The comparison is based on the exact lossless compression rates. The algorithms compared were the state-of-the-art CALIC method [8] (applied independently to each slice of the volume), the lifting-based 3D CB-EZW method [4] (the result produced by the best-performing filter is reported), and a variety of integer wavelets [3] coupled with the proposed entropy coder.

The lossless compression performance of these methods in terms of exact bitrates is reported in Table 2. As seen from this table, the proposed entropy coding scheme, when applied to three-dimensional integer wavelet decompositions, outperforms in most cases the examined lossless compression algorithms.

The results shown in Table 2 confirm that, in general, non-EZW arithmetic coding techniques are more efficient for the lossless coding of wavelet coefficients in comparison to their EZW-based counterparts. This superior lossless performance does not come at the expence of sacrificing the intermediate view capability of multiresolution coding techniques. Despite the fact that the non-EZW entropy coding scheme proposed in this paper is intended to serve lossless medical image coding applications, intermediate view capability is always possible if, during decoding, inverse wavelet transform is applied to the coefficients so far decoded.

5. CONCLUSIONS

In this paper we investigated the coding of 3D medical images using three-dimensional wavelet decomposition and adaptive arithmetic coding. An efficient context arithmetic coding strategy was described and applied in conjunction with classical lifting-based wavelet decompositions. The resulting coding scheme was shown to perform better in comparison to other compression schemes for 3D medical image coding.

Volume	[4]	[8]	(4,2)	(2+2,2)	(6,2)
MRlivert1	2.398	3.049	2.199	**2.172**	2.194
MRlivert2e1	1.822	2.243	**1.774**	1.787	1.785
MRsaghead	2.228	2.494	2.181	**2.176**	2.188
MRpedchest	2.022	2.810	1.914	**1.763**	1.934
CTcarotid	**1.528**	1.654	1.543	1.575	1.540
CTAperts	**0.988**	1.047	1.043	1.035	1.061
CTskull	2.200	2.725	2.066	**2.051**	2.073

Table 2: Comparison of the proposed coding scheme (applied to the (4,2), (2+2,2) and (6,2) integer wavelet decomposition of the test volumes) with the 3D CB-EZW scheme (result for the best performing integer wavelet is reported) in [4] as well as the separate coding of each volume slice using the CALIC [8]. Exact bit rates are reported.

6. ACKNOWLEDGEMENT

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