Gray-Level Co-occurrence Matrices (GLCMs)

Consider the image (below left). If we use the position operator "1 pixel to the right and 1 pixel down" then we get the gray-level co-occurrence matrix (below right)

$$\begin{matrix} 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{matrix} \qquad\qquad C = \frac{1}{16}\begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

where an entry c_{ij} is a count of the number of times that $F(x, y) = i$ and $F(x+1, y+1) = j$. For example, the first entry comes from the fact that 4 times a 0 appears below and to the right of another 0. The factor 1/16 is because there are 16 pairs entering into this matrix, so this normalizes the matrix entries to be estimates of the co-occurrence probabilities.

For statistical confidence in the estimation of the joint probability distribution, the matrix must contain a reasonably large average occupancy level. Achieved either by (a) restricting the number of amplitude quantization levels (causes loss of accuracy for low-amplitude texture), or (b) using large measurement window. (causes errors if texture changes over the large window). Typical compromise: 16 gray levels and window size of 30 or 50 pixels on each side. Now we can analyze C:

- maximum probability entry
- element difference moment of order k: $\sum_i \sum_j (i-j)^k c_{ij}$

  This descriptor has relatively low values when the high values of C are near the main diagonal. For this position operator, high values near the main diagonal would indicate that bands of constant intensity running "1 pixel to the right and 1 down" are likely. When $k = 2$, it is called the contrast:
- Contrast $= \sum_i \sum_j (i-j)^2 c_{ij}$
- Entropy $= -\sum_i \sum_j c_{ij} \log c_{ij}$

  This is a measure of randomness, having its highest value when the elements of C are all equal. In the case of a checkerboard, the entropy would be low.

$$\begin{matrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{matrix} \qquad \rightarrow \qquad \begin{matrix} 8 & 0 \\ 0 & 8 \end{matrix}$$

- Uniformity (also called Energy) $= \sum_i \sum_j c_{ij}^2$ (smallest value when all entries are equal)
- Homogeneity $= \sum_i \sum_j \frac{c_{ij}}{1+|i-j|}$ (large if big values are on the main diagonal)

Problems associated with the co-occurrence matrix methods:

1. they require a lot of computation (many matrices to be computed)
2. features are not invariant to rotation or scale changes in the texture

Sample Question on GLCMs

Here are 4 different texture patches of size 96x96 pixels. All the pixels in the patch (quantized to 16 levels) were used to form GLCMs shown below. The position operator was “one down and one to the right.” Decide which texture patch gave rise to each GLCM.

Note that 3 of the plots show perspective views of the GLCM from the vantage point of the (0,0) position. However, one of the plots has the (0,0) matrix coordinate position placed in the upper left corner since that provides a better view. So check the axis labels.