Adaptive Modulation for OFDM-based Multiple Description Progressive Image Transmission

S. S. Tan, M. J. Rim, P. C. Cosman and L. B. Milstein

***Abstract*—This paper addresses the use of adaptive modulation in progressive image transmission with multiple description coding in conjunction with an Orthogonal Frequency Division Multiplexing (OFDM) system. Specifically, two adaptive systems are considered: *variable rate with fixed power*, and *variable rate with variable power*. An algorithm is proposed to allocate power and constellation size at each subchannel by maximizing the throughput. Simulation results confirm that cross-layer optimization with adaptive modulation enhances system performance.**

***Index Terms*—Adaptive modulation, multiple description coding, Orthogonal Frequency Division Multiplexing (OFDM), power allocation**

I. INTRODUCTION

The growing demand for wireless multimedia services requires reliable and high-rate data communications over a wireless channel. Ideally, wireless multimedia systems have to be adaptive based on the channel conditions.

In this paper, we include the role of the application layer and investigate the use of adaptive modulation in an OFDM system used for transmitting progressively-coded images with multiple description coding. Specifically, each description is mapped into one of the subchannels of the OFDM waveforms [1]. In most of the literature, such as [2]–[4], temporal coding is used. In this paper, we use a cyclic redundancy check (CRC) to check the validity of each description, and erase all descriptions that do not pass the CRC. Then, Reed Solomon (RS) erasure decoding is used across the descriptions.

To achieve minimal image distortion, we need to optimize the constellation size and code rates jointly. However, due to the complexity of jointly optimizing adaptive modulation and channel coding, the problem is decomposed into two subproblems. First, we decide the constellation sizes to maximize the system throughput prior to RS decoding, then we decide the code rates to minimize distortion.

In much of the literature, the same constellation size is used for all the subchannels when applying adaptive modulation for image transmission [2], [3]. However, we propose adopting different constellations for different subchannels to avoid the problem of overwhelming some of the subchannels by imposing a higher order modulation size than the quality of their channels can sustain. Specifically, two schemes of M-QAM adaptive modulation are considered. The first is a variable rate, fixed power scheme; for each subchannel, a constellation size is assigned which maximizes the system throughput prior to RS decoding, with equal power allocation for all subchannels. The second is a variable rate, variable power scheme, which maximizes the system throughout prior to RS decoding by changing the constellation size and the allocated power at each subchannel.

The remainder of this paper is organized as follows: Section II outlines the system and channel models. Section III describes the adaptive modulation techniques, and Section IV presents the RS error protection framework. Section V demonstrates the simulation results and Section VI presents the conclusions.

II. SYSTEM AND CHANNEL MODELS

A. System Model

For the transmission of progressively-coded images over an OFDM system with L subchannels [1], an embedded bitstream is first converted into L descriptions using an FEC-based multiple description coder. Then, Reed-Solomon (RS) encoding is used to code across the descriptions and provide unequal error protection for the multiple descriptions, where the rates of the codes are a non-decreasing function of the level of importance of the data. Lastly, a cyclic redundancy check (CRC) is appended to each description for error detection. Note that the terms description and packet are used interchangeably in this paper.

Coding across the subchannels normally requires a consistent code alphabet. However, this paper proposes to have variable modulation alphabet sizes across the subchannels using adaptive modulation. Hence, a mapping from the modulated symbols to the RS code symbols is needed. For adaptive modulation, the constellation size M is restricted to 2^η, where η is an even number varying from 2 to N_b. For $N_b = 6$, the resulting constellation choices are 4-QAM, 16-QAM and 64-QAM. With adaptive modulation, the number of symbols modulating the subchannels is the same, but the number of bits may vary from subchannel to subchannel. As a $GF(2^{10})$ RS code is adopted, each RS code symbol contains 10 bits. We mapped the modulated symbols to the RS code symbols as shown in Fig.1. Five 4-QAM, 16-QAM, and 64-QAM modulated symbols are grouped as one, two, and three RS code symbols, respectively.

B. Channel Model

In a frequency-selective OFDM channel, the entire frequency band of B_T Hz is assumed to be divided into L independent and identically distributed flat fading subchannels, with bandwidth approximately equal to the coherence bandwidth of the channel. Furthermore, slow Rayleigh fading

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S. S. Tan, P. C. Cosman, and L. B. Milstein are with the Electrical and Computer Engineering Department, University of California San Diego, USA (e-mail: shtan@ucsd.edu, pcosman@ece.ucsd.edu, milstein@ece.ucsd.edu). M. J. Rim is with Dongkuk University, Korea (e-mail: minjoong@dongguk.edu).

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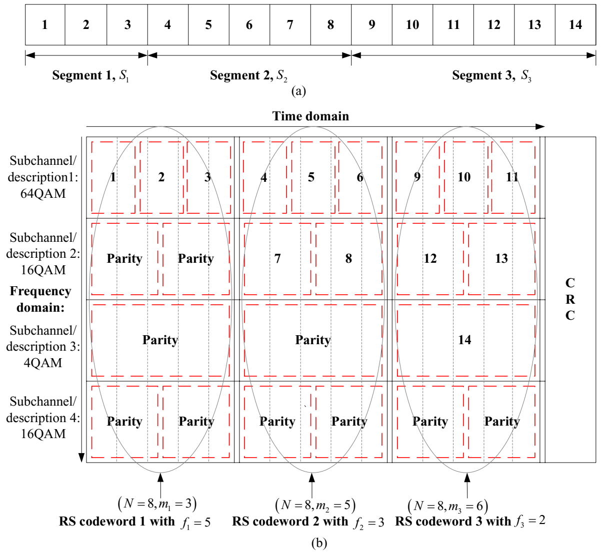

Fig. 1. Illustration of converting an embedded bitstream into multiple descriptions with $L = 4$ descriptions and $J = 3$ RS codewords. (a) An embedded biststream partitioned into 3 segments. (b) Mapping of the descriptions to RS codewords.

is assumed at each subchannel, such that the fading coefficient remains constant over a packet. We assume neither inter-symbol interference nor inter-carrier interference exists.

III. Adaptive Modulation

This section illustrates our two different adaptive modulation schemes for the transmission of progressive images via multiple descriptions. The constellation size is based upon the channel state. A pilot-symbol-assisted estimation technique can be used to determine the corresponding channel state. However, in this paper, perfect channel estimation and an error-free feedback channel are assumed such that both the transmitter and the receiver have the perfect knowledge of the channel state information.

A. Variable Rate, Fixed Power Scheme

This scheme responds to the channel fluctuations by varying the constellation size at each subchannel to maximize the system throughput prior to RS decoding, (average number of received bits from all the subchannels) with equal power allocation for all the subchannels. The system throughput prior to RS decoding, T_{uc}, is given as

$$T_{uc} = \sum_{l=1}^{L} \Gamma_{l,M}(\gamma) \tag{1}$$

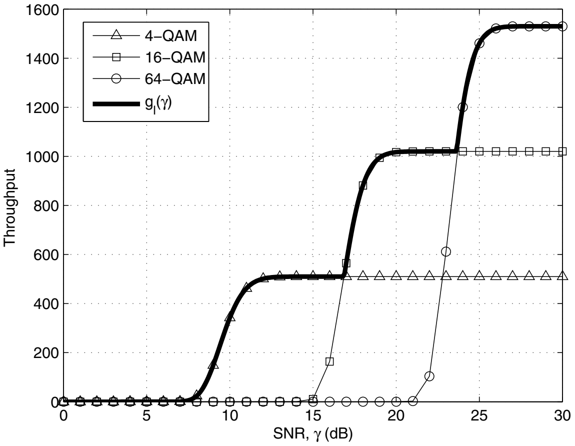

Fig. 2. Throughput of one subchannel prior to RS decoding, $\Gamma_{l,M}(\gamma)$, for M=4, 16, 64.

where $\Gamma_{l,M}(\gamma)$ is the throughput for the l^{th} subchannel prior to RS decoding with a constellation size of M, L is the number of subchannels and γ is the received instantaneous SNR.

The SNR range is divided into $|\cdot|$ regions, where $|\eta|$ denotes the cardinality of the region set. When the received SNR, i.e., γ, is in the η^{th} region, a constellation of size $M = 2^n$ is transmitted. The region boundaries $\{\gamma_b\}$, $b = 0, 2, 4, \ldots, N_b$ are determined from the function $\Gamma_{l,M}(\gamma)$, which is given by

$$\Gamma_{l,M}(\gamma) = \Big(1 - PER_{l,M}(\gamma)\Big)\Big(zlog_2 M\Big) \tag{2}$$

where z is the total number of modulated symbols in one packet, $log_2 M$ is the number of bits in one symbol, and $PER_{l,M}(\gamma)$ is the conditional packet error rate of subchannel l for a constellation size of M, conditioned on the channel state, given by

$$PER_{l,M}(\gamma) = 1 - \Big(1 - SER_{l,M}(\gamma)\Big)^{z}. \tag{3}$$

In (3), $SER_{l,M}(\gamma)$ is the conditional symbol error probability of subchannel l, conditioned on the channel state, for a Gray-encoded $MQAM$ square constellation, and is given by (4) [5]. By substituting (3) and (4) into (2), $\Gamma_{l,M}(\gamma)$ can be rewritten as (5) at the bottom of the page.

Fig. 2 depicts the throughput of one subchannel prior to RS decoding, $\Gamma_{l,M}(\gamma)$, for $M = 2, 4, 6$. As is well known, the throughput for each constellation tends to flatten out for large SNR. Hence, switching to a higher constellation at the appropriate threshold is desirable to achieve higher throughput. The boundaries of the switching thresholds $\{\gamma_b\}$, $b = 0, 2, 4, .., N_b$ are the points where the throughput of the

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$$SER_{l,M}(\gamma) = 1 - \left(1 - \frac{4\left(\sqrt{M}-1\right)}{\sqrt{M}} Q\left(\sqrt{\frac{3\gamma}{M-1}}\right) + \left[\frac{2\left(\sqrt{M}-1\right)}{\sqrt{M}} Q\left(\sqrt{\frac{3\gamma}{M-1}}\right)\right]^2\right) \tag{4}$$

$$\Gamma_{l,M}(\gamma) = \left\{1 - \left[\frac{4\left(\sqrt{M}-1\right)}{\sqrt{M}} Q\left(\sqrt{\frac{3\gamma}{M-1}}\right)\right] + \left[\frac{2\left(\sqrt{M}-1\right)}{\sqrt{M}} Q\left(\sqrt{\frac{3\gamma}{M-1}}\right)\right]^2\right\}^{Z} \left(zlog_2 M\right) \tag{5}$$

two adjacent constellations cross. If the SNR of a particular subchannel is in the region bounded by γ_b and γ_{b+2}, then the subchannel will be assigned a constellation of size $M = 2^{b+2}$.

B. Variable Rate, Variable Power Scheme

For the variable rate, variable power scheme, both the constellation size and the allocated power for each subchannel are changed in order to maximize the system throughput prior to RS decoding, subject to a constraint of total transmission power, from all the subchannels. Mathematically, the problem can be formulated as

$$\text{Max: } \sum_{l=1}^{L} g_l\left(\gamma\right), \text{ subject to} \sum_{l=1}^{L} P_l = P_{total} \tag{6}$$

where P_l is the allocated power at the l^{th} subschannel, P_{total} is the total power constraint, and $g_l\left(\gamma\right)$ is the composite throughput function at the l^{th} subchannel, which is given in (7), and is illustrated in Fig. 2. As $N_b = 6$ is assumed in this paper, $g_l\left(\gamma\right)$ is thus a composite throughput function of 4-QAM, 16-QAM and 64-QAM:

$$\begin{aligned} g_l(\gamma) &= \Gamma_{l,M=4}\left(\gamma\right), \quad 0 \leq \gamma < \gamma_2 \\ g_l(\gamma) &= \Gamma_{l,M=16}\left(\gamma\right), \quad \gamma_2 \leq \gamma < \gamma_4 \\ g_l(\gamma) &= \Gamma_{l,M=64}\left(\gamma\right), \quad \gamma_4 \leq \gamma \end{aligned} \tag{7}$$

An algorithm has been developed that is summarized in the flowchart shown in Fig. 3. It is based upon a greedy approach to allocate power and choose constellation size.

We employ a utility-cost function as in [6]. The utility measures the amount of benefit that the receiver is likely to achieve from receiving the packet, and the cost measures how much one has to pay to achieve a certain utility. The throughput prior to RS decoding is taken to be the utility value, and the required power is the cost. Overall, the function $g_l\left(\gamma\right)$ depicted in Fig. 2 (given in (7)) contains both the utility and the cost. Based on the function $g_l\left(\gamma\right)$, the algorithm assigns the power successively to maximize the argument of the utility function per unit power on each step. The constellation size of each subchannel is determined by its final power assignment.

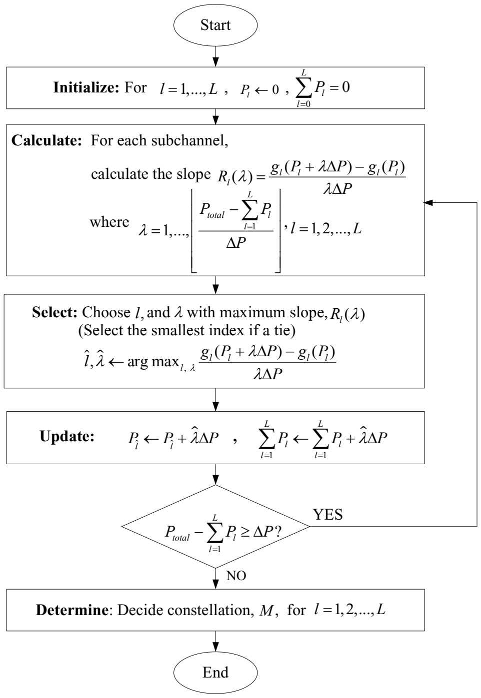

Fig. 3. Flow chart of variable rate, variable power algorithm

Before we describe the algorithm in detail, we define λ as

$$\lambda = 1, ..., \left\lfloor \left(P_{total} - \sum_{l=1}^{L} P_l \right) \Big/ \Delta P \right\rfloor \tag{8}$$

where $\sum_{l=1}^{L} P_l$ is the current total power allocated for all the subchannels, hence, $P_{total} - \sum_{l=1}^{L} P_l$ is the current remaining power budget. The floor $\lfloor x \rfloor$ denotes the greatest integer $\leq x$, and ΔP is a fixed, small increment of power. Several values for ΔP have been tested and 0.1 appeared to be an appropriate value.

The algorithm is described as follows:

1. Initialization: Set the power of all subchannels to zero.

2. Calculate the slope: For each subchannel, calculate the slope of the throughput when an increment $\lambda\Delta P$ of power is applied. The slope $R_l(\lambda)$ is defined as

$$R_l(\lambda) = \frac{g_l(P_l + \lambda\Delta P) - g_l(P_l)}{\lambda\Delta P} \tag{9}$$

where $0 \leq \lambda\Delta P \leq P_{total} - \sum_{l=1}^{L} P_l$, $l \in \{1, 2, ..., L\}$, $\lambda \in \left\{1, ..., \left\lfloor \left(P_{total} - \sum_{l=1}^{L} P_l \right) \Big/ \Delta P \right\rfloor \right\}$, and P_l is the current power at the l^{th} subcarrier. The incremental throughput $g_l(P_l + \lambda\Delta P) - g_l(P_l)$ represents the utility achieved at a cost of $\lambda\Delta P$ units of power applied to the l^{th} subchannel.

3. Select and update: Selection is made by choosing subchannel $\widehat{l}$, with corresponding value $\widehat{\lambda}$, that corresponds to the steepest slope $R_l\left(\lambda\right)$ (i.e., the largest throughput increment per unit power). If there is a tie, select the subchannel with the smaller index. The corresponding increment of power, $\widehat{\lambda}\Delta P$, is then assigned to the $\widehat{l}^{th}$ selected subchannel, and the total power budget is reduced by the allocated amount. This iterative power allocation process terminates either when the total transmission power constraint is reached, or when the remaining power is less than ΔP.

4. Determine the constellation size: The final step of the algorithm is to decide the constellation size for each subchannel based on its final allocated power. The assigned power determines the value of γ, and thus the constellation size, for $N_b = 6$, is indicated below:

$$\begin{array}{rl} \gamma = 0, & \text{No Transmission} \\ 0 < \gamma < \gamma_2, & M = 4 \\ \gamma_2 \leq \gamma < \gamma_4, & M = 16 \\ \gamma_4 \leq \gamma, & M = 64 \end{array} \tag{10}$$

IV. RS ERROR PROTECTION FRAMEWORK

After determining the constellation sizes, the channel code rates have to be determined. This section presents the error protection framework by considering the RS rate assignment in order to minimize the distortion of the image.

For the transmission of a progressively-coded image, an embedded bitstream is first converted into L descriptions using multiple description coding. Descriptions for an embedded bitstream are then protected by J RS codewords, where each codeword contains a segment $(S_j, j \in [1, J])$ of the information data (see Fig. 1), and each segment consists of $m_j \in \{1, 2, ..., N\}$ source symbols. Without loss of generality, we let one source symbol correspond to one RS code symbol. Let $f_j = N - m_j$ denote the number of RS parity symbols that protects the segment S_j. Then, by adding a set of constraints $f_1 \geq f_2 \geq ... \geq f_J$ and $f_j \in \{1, 2, ..., N-1\}$, the receiver can recover at least the first j segments, given that no more than f_j RS code symbols are erased. Fig. 1 illustrates a mechanism for mapping an embedded bitstream from a source encoder into multiple descriptions. For a progressively-coded source, prefixes of the bitstream can be used to reconstruct the source with a certain fidelity. However, occurrence of an error along the bitstream will result in the loss of the subsequent bits. For the example in Fig. 1, if the number of erased code symbols is no more than 3, 5 and 6, source segments up to 1, 2 and 3 can be reconstructed, respectively.

For a given $\underline{F} = (f_1, ..., f_J)$, the conditional distortion, $D\left(\underline{F}, \underline{\gamma}\right)$, conditioned on the received SNR, $\underline{\gamma} = [\gamma_1,\ \gamma_2, ..., \gamma_L]$, can be written as [7]

$$D\left(\underline{F}, \underline{\gamma}\right) = \sum_{j=0}^{J} P_j\left(\underline{F}, \underline{\gamma}\right) \phi\left(T_j\left(\underline{F}\right)\right) \tag{11}$$

where

$$P_j(\underline{F}, \underline{\gamma}) = \begin{cases} \text{Prob}\left(X > f_1 | \underline{\gamma}\right) & j = 0 \\ \text{Prob}\left(f_{j+1} < X \leq f_j | \underline{\gamma}\right) & j = 1, ..., J-1 \\ \text{Prob}\left(X \leq f_J | \underline{\gamma}\right) & j = J \end{cases} \tag{12}$$

In above equations, ϕ represents the operational rate-distortion function of the source encoder, X is the number of erased RS code symbols and $T_j\left(\underline{F}\right) = \sum_{k=1}^{j} m_k$ is the number of source symbols in the first j segments. Note that $T_0(\underline{F}) = 0$, and thus $\phi\left(T_0(\underline{F})\right) = \phi\left(0\right)$ corresponds to the distortion when no transmitted source symbol is correctly decoded, and so the decoder must reconstruct the source without using any of the transmitted information.

The FEC optimization goal is to determine the set of RS parity assignments that minimizes the conditional average distortion expressed in (11). Given a set of received SNRs, $\underline{\gamma}$, and given the operational distortion-throughput function $\phi\left(T_j\left(\underline{F}\right)\right)$, the problem is as follows:

$$D^*\left(\underline{\gamma}\right) = \min_{\underline{F} \in \mathbb{F}} \left\{D\left(\underline{F}, \underline{\gamma}\right)\right\} = \min_{\underline{F} \in \mathbb{F}} \left\{\sum_{j=0}^{J} P_j\left(\underline{F}, \underline{\gamma}\right) \phi\left(T_j\left(\underline{F}\right)\right)\right\} \tag{13}$$

where $\mathbb{F}$ denotes the set of J-tuples $(f_1, f_2, ..., f_J)$ such that $f_1 \geq f_2 \geq ... \geq f_J$.

The hill climbing approach proposed in [8] is adopted to find the optimal FEC assignment for the RS codewords. At each iteration, the algorithm examines $2QJ$ possible assignments; Q is the maximum number of parity symbols that can be added or subtracted to a codeword in one iteration, and J is the number of codewords. The conditional distortion is evaluated using (11) after adding or subtracting one parity symbol to each codeword. Lastly, the FEC allocation, $\underline{F}$, with the lowest distortion is chosen.

V. SIMULATION RESULTS AND DISCUSSIONS

In this section, simulations are performed on a 128×128 gray-scale Lena image, which is encoded using the Set Partitioning in Hierarchical Trees (SPIHT) [9] algorithm to produce an embedded bitstream. To analyze the image quality, the peak-signal-to-noise ratio (PSNR) performance metric is used, which defined as

$$PSNR = 10 \log \frac{255^2}{E_{\underline{\gamma}}\left[D^*\left(\underline{\gamma}\right)\right]} \text{dB} \tag{14}$$

where $E_{\underline{\gamma}}\left[D^*\left(\underline{\gamma}\right)\right]$ is the expectation of the distortion in (13).

We compare the two proposed schemes against a fixed rate, fixed power baseline scheme. For the latter scheme, adaptive modulation is not used, but rather a 4-QAM constellation is used for all the subchannels. The parameters for the simulations are as follows: $L = 16$, $z = 255$ symbols, $J = 51$ and $P_{total} = 16$.

Fig. 4 compares the PSNR of the received image for the three systems, and the variable rate, variable power performs the best. The PSNR for the non-adaptive scheme saturates at high SNR, as the low constellation size limits the maximum source data that can be transmitted. For the adaptive modulation systems, the amount of transmitted source data varies according to the channel conditions, which, in turn, results in a higher PSNR. In particular, at low SNR, the variable rate, variable power scheme gives a significant gain. When the channel conditions are bad, system performance can be improved by transmitting fewer descriptions (descriptions with a zero power assignment are, by definition, not transmitted), each with a higher probability of being received. At high SNR, when the channels are all relatively good, changing the constellation (variable rate, fixed power) alone is sufficient to enhance the system performance.

To obtain a better insight for the performance of the three schemes, Fig. 5 illustrates the packet error rate of the three systems. It can be observed that the packet error rate of the variable rate, fixed power scheme is similar to the fixed rate, fixed power scheme at low SNR. This is because the variable rate, fixed power scheme has a high probability of choosing the lowest order constellation (i.e., 4QAM) at low SNR, namely the same constellation that the fixed rate, fixed power scheme uses. When the SNR increases, the variable rate, fixed power scheme will switch to a constellation size higher than 4-QAM, which results in a higher packet error rate. The packet error rate for the variable rate, variable power system is the lowest among the three systems. This is because if one particular subchannel is bad, this scheme will not assign power to it. Instead, the power will be allocated to the relatively good channels, and thus increase the probability of correct detection.

Fig. 6 compares the performance between a system with a uniform constellation for all the subchannels and a system with variable constellation for each subchannel (variable rate, fixed power). For the system with a uniform constellation, the system throughput prior to RS decoding for three different constellation sizes, 4-QAM, 16-QAM and 64-QAM, are evaluated, and the constellation size with the highest throughput is chosen for all subchannels. That is, for both of the schemes, the constellation size is changed based on the channel conditions. However, for the uniform constellation scheme, all subchannels have to adopt the same constellation, whereas for the variable constellation scheme, each subchannel can adapt its constellation size to its channel conditions. It can be observed that when SNR is low, the two systems perform similarly, for the same reason as discussed relative to Fig. 5. In addition, similar to Fig. 5, when SNR is high, each subchannel in the variable constellation scheme will choose a constellation size that fits its current channel condition, thus yielding better performance than the uniform scheme.

VI. CONCLUSIONS

In this paper, we proposed an adaptive modulation technique for transmitting progressive images with multiple description coding. The constellation size and power allocation for each description were chosen to maximize throughput. Compared with a system of fixed power and fixed constellation assignment, our proposed adaptive scheme resulted in superior performance. In particular, in a low SNR regime, changing both the power and constellation size will give us significant gain. In a high SNR regime, the extra gain achieved from allocating different power to each subchannel in addition to changing the constellation size is relatively small compared to the gain achieved from changing the constellation size alone. This suggests that rate adaptation is the key to achieve high performance at high SNR.

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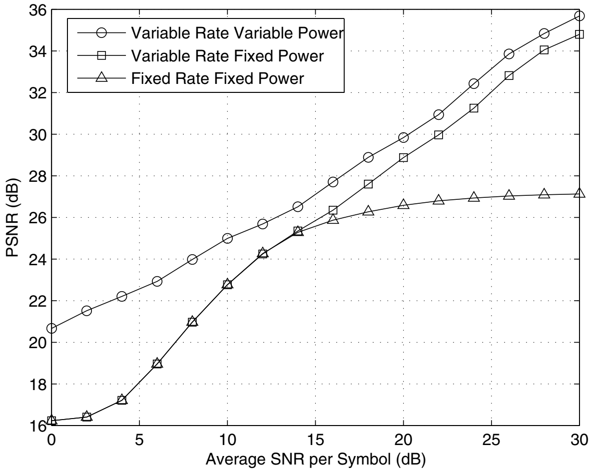

Fig. 4. Comparison of PSNR for three schemes: Variable rate variable power, variable rate fixed power and fixed rate fixed power

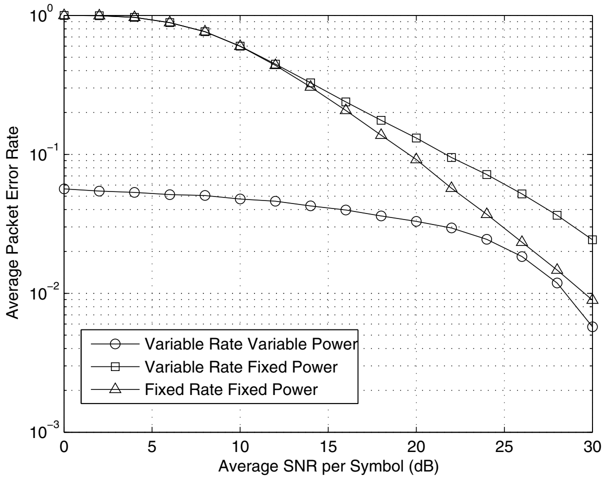

Fig. 5. Average packet error rate

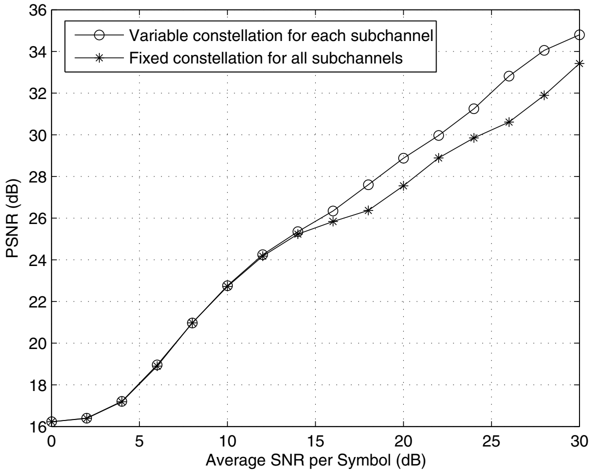

Fig. 6. Comparison between variable constellation size at different subcarriers and same constellation size for all subcarriers