Cooperative Relaying of Superposition Coding with Simple Feedback for Layered Source Transmission

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Abstract—We consider a relay network that delivers a Gaussian source by employing successive refinement source coding and superposition coding of layers at the source node and successive decoding at the relay and destination nodes. For the network, making use of the decoding results at the relay and destination nodes of the first transmission, an efficient relaying strategy of layers is proposed to minimize the expected distortion (ED) when only the average channel state information is available at the source node. Three types of the proposed scheme, defined as Prop-DF, using decode-and-forward signals, Prop-AF, using amplify-and-forward signals, and Prop-MF, using mixed-forward signals, are addressed and analyzed in terms of the outage probability and distortion exponent. Unlike other studies, we have also taken the relay location into account in deriving the distortion exponent showing the high SNR behavior of the ED. The results show that the proposed scheme increases the distortion exponent up to twice that of the conventional relaying schemes when the relay is close to the source node, and that Prop-MF provides the best performance for most relay locations.

Index Terms—Superposition coding, Amplify-and-forward, Decode-and-forward, Expected distortion, Distortion exponent

I. INTRODUCTION

WIRELESS relaying and cooperation of neighboring nodes can improve the communication reliability without increasing the transmit power or mounting multiple antennas in the communication nodes [1], [2]. These advantages have motivated the design of efficient relaying protocols and their performance analyses under various system conditions [3]–[6]. Relaying protocols are usually based on decode-and-forward (DF) which avoids noise amplification, or amplify-and-forward (AF) which makes the relay simple. In most

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studies, the performance has been investigated mainly from physical layer aspects such as the outage probability, symbol error rate, and average rate delivered reliably.

In the meantime, the advent of wireless multimedia services with a diversity of quality-of-services has initiated cross-layer designs between the application and physical layers for efficient utilization of wireless resources [7]–[9]. These designs attempt to improve the end-to-end performance by taking into account both source coding and channel transmission. Recently, without assuming the availability of channel state information (CSI) at the transmitter, cross-layer designs using layered transmission of successive refinement source coding have been extensively studied for various system models to minimize the expected distortion (ED) when the transmitted multimedia source is reconstructed at the receiver [8], [10]–[20].

In the designs, the transmitter encodes a source into multiple layers such that each layer refines the description in the previous layer successively, and transmits the layers sequentially or simultaneously by using superposition coding (SC) with an appropriate power allocation. The receiver decodes the layers successively, and combines the successfully decoded layers to reconstruct the source. Due to the difficulty in analyzing the finite signal-to-noise ratio (SNR) performance, the performance of such designs has often been analyzed in terms of the distortion exponent [8], [11], [13], [15], [17], [19] which quantifies the exponential decay rate of the ED in the high SNR regime. In some studies, efficient power and rate allocation algorithms have been investigated in the finite SNR regime to obtain the minimum ED [12], [14], [16]. The results have shown that SC transmission of layered sources tends to achieve better distortion performance than progressive transmission of layered sources (which transmits layered sources sequentially in time) [11], [13], [15].

The SC for layered sources has been applied in wireless relay networks also. In [10], layer-selective relaying based on the relay decoding result was proposed for a three-node relay network, but without any analysis of the distortion performance. In the same system model as in [10], the distortion exponents of DF and AF schemes were analyzed for the source-channel mismatch factor in [11], revealing that the distortion exponent of the relaying schemes gets larger than that of direct transmission when the source-channel mismatch factor is large. The analysis was extended to the case of multiple AF relay nodes [17] when relay selection or distributed beamforming is employed. The performance
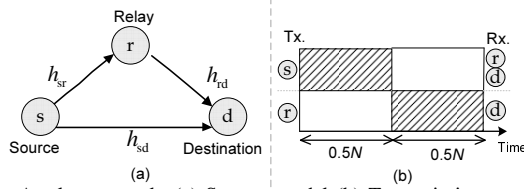
Fig. 1. A relay network: (a) System model (b) Transmission protocol.

with relay selection is also evaluated with multiple DF relay nodes in [18]. For the multiple AF relay network, power and rate allocation algorithms in the finite SNR regime were also proposed for the two-layer case in [19]. Although less relevant, the studies in [21]–[23] also apply SC transmission in relay networks and attempt to maximize the transmission rate with full CSI at the transmitter without considering successive refinement source coding.

In this paper, we consider a relaying scheme for the SC transmission of layered sources, by which the ED can be improved even with a single relay node. We extend the DF-based layer-selective relaying scheme using only the relay decoding result [10] into the case where the decoding result at the destination in the first slot is also incorporated in the design of relay signals. Specifically, making use of the decoding results at both the relay and destination nodes in the first slot, we propose three types of relay signal construction, one incorporating DF signals, another incorporating AF signals, and the third incorporating both DF and AF signals. Generalizing and extending the preliminary study in [20], we not only consider an arbitrary number of layers but also analyze the outage probability and successive decoding diversity order. In addition, the distortion exponent is derived in a closed form expression as a function of the source-channel mismatch factor. Unlike conventional studies, the effect of the relay location is also incorporated in the analysis of distortion performance.

II. System Model

Consider the relay network described in Fig. 1, where the source node s wishes to send an information source to the destination node d with the help of the relay node r. Each node is equipped with a single antenna, and the channels between any two nodes are Rayleigh fading, independent of each other, and quasi-static over N channel uses. Let $h_{uv} \sim \mathcal{CN}(0, \varsigma_{uv}^{-\nu})$ denote the complex channel gain between nodes u and v at distance ς_{uv} for $uv \in \{sd, sr, rd\}$, where $\sim$ stands for 'distributed as', $\mathcal{CN}(m, \sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with mean m and variance σ^2, and ν is the path loss exponent.

As shown in Fig. 1(b), N channel uses are divided into two time slots of length $\frac{N}{2}$, with the first and second slots used for the transmission of the source and relay, respectively. Over the N channel uses, a block of K source samples is transmitted, which leads to the source-channel mismatch factor (also called the bandwidth expansion ratio) [11], [12], [14]

$$b = \frac{N}{K} \quad \text{channel-uses per source-sample.} \tag{1}$$

The mismatch factor can be interpreted as a delay constraint in transmitting the source. The source samples are encoded by successive refinement source coding, which produces L

layers, each carrying the refinement information of a lower layer. Each layer is then channel-encoded with independent and identically distributed complex Gaussian codebooks. The channel code rate of layer l will be denoted by R_l bits/channel-use on average over N channel uses, which corresponds to the source code rate of bR_l bits/source-sample.

If the channel encoder outputs of L layers are transmitted by SC with power allocation, the transmit symbol of the source at the nth channel use in the first slot is given by

$$x(n) = \sum_{l=1}^{L} \sqrt{\mathcal{P}_s \alpha_l} x_l(n) \tag{2}$$

for $n \in \mathbb{N}_1^{\frac{N}{2}}$, where $x_l(n) \sim \mathcal{CN}(0, 1)$ is the channel encoder output of layer l, $\mathcal{P}_s$ is the transmit power of the source, and α_l is the power allocation factor to layer l, subject to $\sum_{l=1}^{L} \alpha_l = 1$ with $\mathbb{N}_i^j = \{i, i+1, \cdots, j\}$ denoting the set of integers from i to j. The corresponding received signal can be expressed as

$$y_{v,1}(n) = h_{sv} \sum_{l=1}^{L} \sqrt{\mathcal{P}_s \alpha_l} x_l(n) + w_{v,1}(n), \tag{3}$$

where $v \in \{r, d\}$ denotes the receiving node and $w_{v,i}(n) \sim \mathcal{CN}(0, \sigma^2)$ is the additive noise at node v in the ith slot for $i = 1, 2$. In the second slot, the relay transmits a signal $z(n)$, generated from $y_{r,1}(n)$ under the relay power constraint $\mathcal{P}_r$. Details of the proposed relay signal $z(n)$ will be described in Section III. The received signal at the destination in the second slot can then be written as

$$y_{d,2}(n) = h_{rd} z(n) + w_{d,2}(n). \tag{4}$$

In the network, receiving nodes perform successive decoding, decoding lower layers first and decoding the next higher layer only if all the lower layers are decoded successfully and removed from the received signal. Consequently, if layer l_o is in outage, so are layers $l \in \mathbb{N}_{l_o+1}^L$. Thus, with the successive decoding, we have $\{P_{\text{out},l} < P_{\text{out},l+1}\}_{l=0}^L$, where $P_{\text{out},l}$ denotes the outage probability of layer l averaged over all channel realizations with $P_{\text{out},0} = 0$ and $P_{\text{out},L+1} = 1$. Now, for a rate vector $\boldsymbol{R} = [R_1 \ R_2 \ \cdots \ R_L]$ and a power allocation vector $\boldsymbol{\alpha} = [\alpha_1 \ \alpha_2 \ \cdots \ \alpha_L]$, the ED is given by [11]

$$\mathbb{E}_D(\boldsymbol{R}, \boldsymbol{\alpha}) = \sum_{l=0}^{L} (P_{\text{out},l+1} - P_{\text{out},l}) D\left(b \sum_{m=1}^{l} R_m\right) \tag{5}$$

after reconstructing the source with successfully decoded layers at the destination, where $D(\cdot)$ is the distortion-rate function of source coding. In applying the SC, the source chooses vectors $\boldsymbol{R}$ and $\boldsymbol{\alpha}$ to minimize the ED as

$$\mathbb{E}_{D_o} = \min_{(\boldsymbol{R}, \boldsymbol{\alpha})} \mathbb{E}_D(\boldsymbol{R}, \boldsymbol{\alpha}). \tag{6}$$

In this paper, we assume a memoryless, zero-mean, unit-variance Gaussian source, of which the distortion-rate function is bounded by $D(R) = 2^{-R}$ under the squared-error distortion measure [24]. In addition, the numbers K and N are assumed to be large enough to achieve the distortion-rate bound and instantaneous channel capacity as in [11], [19].

III. THE PROPOSED RELAYING SCHEME

A. Overview

The conventional relaying schemes for SC signals adopt either DF or AF without any feedback information. Specifically, the relay signal is given by

$$z(n) = \sum_{l=1}^{q_{\mathrm{r}}} \sqrt{\frac{\mathcal{P}_{\mathrm{r}}\alpha_l}{\bar{\alpha}_{1,q_{\mathrm{r}}}}} x_l(n) \tag{7}$$

in SC-DF [10], and

$$
\begin{aligned}
z(n) &= \sqrt{\frac{\mathcal{P}_{\mathrm{r}}}{\mathcal{P}_{\mathrm{s}}|h_{\mathrm{sr}}|^2 + \sigma^2}} y_{\mathrm{r},1}(n) \\
&= \sqrt{\frac{\mathcal{P}_{\mathrm{r}}}{\mathcal{P}_{\mathrm{s}}|h_{\mathrm{sr}}|^2 + \sigma^2}} \left\{ h_{\mathrm{sr}} \sum_{l=1}^{L} \sqrt{\mathcal{P}_{\mathrm{s}}\alpha_l} x_l(n) + w_{\mathrm{r},1}(n) \right\}
\end{aligned} \tag{8}
$$

in SC-AF [11], where $q_{\mathrm{r}} \in \mathbb{N}_0^L$ is the number of layers decoded successfully at the relay in the first slot and $\bar{\alpha}_{i,j} = \sum_{m=i}^{j} \alpha_m$ is the power fraction allocated to layers $\{i, i+1, \cdots, j\}$ with $\bar{\alpha}_{i,j} = 0$ for $i > j$.

The proposed relaying scheme, on the other hand, designs the relay signal by taking into account the decoding result $q_{\mathrm{d}} \in \mathbb{N}_0^L$ at the destination as well as the decoding result q_{r} at the relay in the first slot. Note that the decoding result q_{d} (q_{r}) indicates that the destination (relay) has successfully decoded all the layers up to and including layer q_{d} (q_{r}) but failed to decode higher layers. The information q_{d}, which can be made available at the relay through feedback from the destination using $\lceil \log_2(L+1) \rceil$ bits, will be used to avoid redundant transmission of the layers recovered successfully at the destination in the first slot. In the second slot, the destination attempts to decode layer l only for $l \in \mathbb{N}_{q_{\mathrm{d}}+1}^L$ (i.e., layers not decoded successfully in the first slot) and only when the relay signal contains the information on layer l (i.e., when combining the relay signal $z(n)$ with the directly received signal can improve the SNR).

The decoding result q_v at node $v \in \{\mathrm{r}, \mathrm{d}\}$ in the first slot is determined by the effective SNR $\eta_{v,l}$ at layer l when node v decodes layer l with the received signal (3). Assuming successive decoding with perfect cancellation of lower layers, we have [14]

$$\eta_{v,l} = \Phi_{l,L}(\gamma_{\mathrm{s}v}) \tag{9}$$

for $v \in \{\mathrm{r}, \mathrm{d}\}$, where $\gamma_{uv} = |h_{uv}|^2 \frac{\mathcal{P}_u}{\sigma^2}$ is the instantaneous SNR of the link between nodes u and v for $uv \in \{\mathrm{sd}, \mathrm{sr}, \mathrm{rd}\}$, and

$$\Phi_{i,j}(x) = \frac{\alpha_i x}{\bar{\alpha}_{i+1,j} x + 1} \tag{10}$$

is the effective SNR at layer i when layers $\{i, i+1, \cdots, j\}$ are received at SNR x.

Denoting by $\eta_{\mathrm{f},l}$ the final effective SNR at layer l when the destination performs the final decoding of layer l in the second slot, let us now design the relay signal $z(n)$ to improve $\eta_{\mathrm{f},l}$ for $l \in \mathbb{N}_{q_{\mathrm{d}}+1}^L$ by employing DF signals, AF signals, and both DF and AF signals.

B. Proposed DF (Prop-DF)

Prop-DF constructs the relay signal as

$$z(n) = \begin{cases} G_{q_{\mathrm{d}},q_{\mathrm{r}}}^D \displaystyle\sum_{l=q_{\mathrm{d}}+1}^{q_{\mathrm{r}}} \sqrt{\mathcal{P}_{\mathrm{r}}\alpha_l} x_l(n), & \text{if } q_{\mathrm{d}} < q_{\mathrm{r}}, \\ 0, & \text{if } q_{\mathrm{d}} \ge q_{\mathrm{r}}, \end{cases} \tag{11}$$

where $G_{i,j}^D = \sqrt{\frac{1}{\bar{\alpha}_{i+1,j}}}$ is the amplification factor to make the relay power $\mathcal{P}_{\mathrm{r}}$. Basically, Prop-DF scheme constructs the relay signals by selecting the layers required at the destination among the layers recovered at the relay.

In the second slot, to improve the final effective SNR for $l \in \mathbb{N}_{q_{\mathrm{d}}+1}^{q_{\mathrm{r}}}$, the destination performs maximal ratio combining (MRC) of (3) with $v = \mathrm{d}$ and (4). The corresponding final effective SNR is derived in Appendix I.A as

$$\eta_{\mathrm{f},l} = \Phi_{l,q_{\mathrm{r}}}\left(\lambda_{q_{\mathrm{d}},q_{\mathrm{r}}}^D\right), \tag{12}$$

where

$$\lambda_{i,j}^D = \frac{\Phi_{j,L}(\gamma_{\mathrm{sd}})}{\alpha_j} + \frac{\gamma_{\mathrm{rd}}}{\bar{\alpha}_{i+1,j}}. \tag{13}$$

C. Proposed AF (Prop-AF)

The relay signal for Prop-AF is expressed as

$$z(n) = G_{\min(q_{\mathrm{d}},q_{\mathrm{r}})}^A \left\{ y_{\mathrm{r},1}(n) - h_{\mathrm{sr}} \sum_{l=1}^{\min(q_{\mathrm{d}},q_{\mathrm{r}})} \sqrt{\mathcal{P}_{\mathrm{s}}\alpha_l} x_l(n) \right\}, \tag{14}$$

where $G_q^A = \sqrt{\frac{\mathcal{P}_{\mathrm{r}}}{\mathcal{P}_{\mathrm{s}}|h_{\mathrm{sr}}|^2 \bar{\alpha}_{q+1,L} + \sigma^2}}$ is the amplification factor to make the relay power $\mathcal{P}_{\mathrm{r}}$: Basically, Prop-AF scheme constructs the relay signals by removing, from the received signal, the layers which have been decoded successfully at the relay yet are not required at the destination. In this case, with MRC, the final effective SNR will be improved into

$$\eta_{\mathrm{f},l} = \Phi_{l,L}\left(\lambda_{\min(q_{\mathrm{d}},q_{\mathrm{r}})}^A\right) \tag{15}$$

for $l \in \mathbb{N}_{q_{\mathrm{d}}+1}^L$ as shown in Appendix I.B, where

$$\lambda_q^A = \gamma_{\mathrm{sd}} + \frac{\gamma_{\mathrm{sr}}\gamma_{\mathrm{rd}}}{\bar{\alpha}_{q+1,L}\gamma_{\mathrm{sr}} + \gamma_{\mathrm{rd}} + 1}. \tag{16}$$

D. Proposed Mixed-Forward (Prop-MF)

Noting that Prop-DF improves the SNR for $l \in \mathbb{N}_{q_{\mathrm{d}}+1}^{q_{\mathrm{r}}}$, we modify Prop-DF by sending AF signals when DF relaying is not possible. The relay signal is then defined as

$$z(n) = \begin{cases} G_{q_{\mathrm{d}},q_{\mathrm{r}}}^D \displaystyle\sum_{l=q_{\mathrm{d}}+1}^{q_{\mathrm{r}}} \sqrt{\mathcal{P}_{\mathrm{r}}\alpha_l} x_l(n), & \text{if } q_{\mathrm{d}} < q_{\mathrm{r}}, \\ G_{q_{\mathrm{r}}}^A \left\{ y_{\mathrm{r},1}(n) - h_{\mathrm{sr}} \displaystyle\sum_{l=1}^{q_{\mathrm{r}}} \sqrt{\mathcal{P}_{\mathrm{s}}\alpha_l} x_l(n) \right\}, & \text{if } q_{\mathrm{d}} \ge q_{\mathrm{r}}, \end{cases} \tag{17}$$

which leads to the final effective SNR (12) and (15) for $q_{\mathrm{d}} < l \le q_{\mathrm{r}}$ and $q_{\mathrm{r}} \le q_{\mathrm{d}} < l \le L$, respectively.

The final effective SNR for the three types of the proposed scheme is summarized in Table I. Note that the final effective SNR of a layer remains unchanged from $\eta_{\mathrm{d},l} = \Phi_{l,L}(\gamma_{\mathrm{sd}})$ when the MRC has not been performed for the layer.

IV. OUTAGE ANALYSIS IN THE FINITE SNR REGION

Given the rate and power allocation $(\boldsymbol{R}, \boldsymbol{\alpha})$, let us derive the outage probability $P_{\mathrm{out},l}$ of the proposed scheme.

TABLE I
FINAL EFFECTIVE SNR OF THE THREE TYPES OF THE PROPOSED SCHEME.

Case	Layer	Final effective SNR ($\eta_{\mathrm{f},l}$)		
		Prop-DF	Prop-AF	Prop-MF
$\mathcal{J}_1$	$1 \leq l \leq q_{\mathrm{d}}$	$\Phi_{l,L}(\gamma_{\mathrm{sd}})$	$\Phi_{l,L}(\gamma_{\mathrm{sd}})$	$\Phi_{l,L}(\gamma_{\mathrm{sd}})$
$\mathcal{J}_2$	$q_{\mathrm{d}} < l \leq q_{\mathrm{r}}$	$\Phi_{l,q_{\mathrm{r}}}(\lambda_{q_{\mathrm{d}},q_{\mathrm{r}}}^{D})$	$\Phi_{l,L}(\lambda_{q_{\mathrm{d}}}^{A})$	$\Phi_{l,q_{\mathrm{r}}}(\lambda_{q_{\mathrm{d}},q_{\mathrm{r}}}^{D})$
$\mathcal{J}_3$	$q_{\mathrm{d}} < q_{\mathrm{r}} < l \leq L$	$\Phi_{l,L}(\gamma_{\mathrm{sd}})$	$\Phi_{l,L}(\lambda_{q_{\mathrm{d}}}^{A})$	$\Phi_{l,L}(\gamma_{\mathrm{sd}})$
$\mathcal{J}_4$	$q_{\mathrm{r}} \leq q_{\mathrm{d}} < l \leq L$	$\Phi_{l,L}(\gamma_{\mathrm{sd}})$	$\Phi_{l,L}(\lambda_{q_{\mathrm{r}}}^{A})$	$\Phi_{l,L}(\lambda_{q_{\mathrm{r}}}^{A})$

A. General Framework for Analysis

Let $\mathcal{E}_{\mathrm{d},i}$ ($\mathcal{E}_{\mathrm{r},i}$) denote the event that the destination (relay) successfully decodes all the layers up to and including layer i but fails in decoding higher layers in the first slot. Assuming the power and rate allocation which allows successive decoding, we have

$$\mathcal{E}_{v,i} = \{\Gamma_{i,L} < \gamma_{sv} \leq \Gamma_{i+1,L}\} \qquad (18)$$

for $v \in \{\mathrm{d},\mathrm{r}\}$, where the threshold $\Gamma_{i,j}$, with $\Gamma_{0,j} = 0$ and $\Gamma_{L+1,j} = \infty$, is the SNR required for successful decoding of layer i with rate R_i when layers $\{i+1, i+2, \cdots, j\}$ are interfering with layer i for $i \in \mathbb{N}_1^L$ and $j \in \mathbb{N}_{i+1}^L$. Now, when the effective SNR is given by $\Phi_{i,j}(\gamma)$ (as in (9), (12), and (15)), by setting the mutual information equal to the rate as $\frac{1}{2}\log_2(1 + \Phi_{i,j}(\Gamma_{i,j})) = R_i$, we can obtain the threshold

$$\Gamma_{i,j} = \Phi_{i,j}^{-1}(\kappa_i) = \frac{\kappa_i}{\alpha_i - \bar{\alpha}_{i+1,j}\kappa_i} \qquad (19)$$

since $\Phi_{i,j}(x)$ is a monotonically increasing function of $x > 0$, where $\kappa_i = 2^{2R_i} - 1$. In addition, since the conditions $\{\Gamma_{l,L} < \Gamma_{l+1,L}\}_{l=0}^L$ are satisfied under the assumption of successive decoding, we also have $\{\Gamma_{l,j} < \Gamma_{l+1,j}\}_{l=0}^L$ for $j \in \mathbb{N}_1^{L-1}$.

Denote by $\mathcal{S}_{\mathrm{f},l}(i,j)$ the event that the decoding results $(q_{\mathrm{d}}, q_{\mathrm{r}})$ of the first slot are (i,j) and layer l is reconstructed successfully in the second slot. If we write the final effective SNR of layer l when $(q_{\mathrm{d}}, q_{\mathrm{r}}) = (i,j)$ as $\eta_{\mathrm{f},l}(i,j)$ by showing the dependence on (i,j) explicitly, the event $\mathcal{S}_{\mathrm{f},l}(i,j)$ can be written as

$$\mathcal{S}_{\mathrm{f},l}(i,j) = \mathcal{E}_{\mathrm{d},i} \bigcap \mathcal{E}_{\mathrm{r},j} \bigcap \{\eta_{\mathrm{f},l}(i,j) > \kappa_l\}. \qquad (20)$$

The outage probability of layer l is then given by

$$P_{\mathrm{out},l} = 1 - \sum_{i=0}^{L}\sum_{j=0}^{L}\mathbb{P}[\mathcal{S}_{\mathrm{f},l}(i,j)], = 1 - \sum_{k=1}^{4}P_{\mathrm{suc},l}(\mathcal{J}_k), \quad (21)$$

where $\mathbb{P}[\cdot]$ denotes the probability of an event, $\{\mathcal{J}_k\}_{k=1}^4$ denote the four cases shown in Table I,

$$P_{\mathrm{suc},l}(\mathcal{J}_k) = \sum_{(i,j)\in\mathcal{Q}_l(\mathcal{J}_k)}\mathbb{P}[\mathcal{S}_{\mathrm{f},l}(i,j)] \qquad (22)$$

is the probability of decoding success at layer l when $\mathcal{J}_k$ occurs, and

$$\begin{cases} \mathcal{Q}_l(\mathcal{J}_1) = \{(i,j) : l \leq i \leq L, 0 \leq j \leq L\}, \\ \mathcal{Q}_l(\mathcal{J}_2) = \{(i,j) : 0 \leq i < l \leq j \leq L\}, \\ \mathcal{Q}_l(\mathcal{J}_3) = \{(i,j) : 0 \leq i < j < l\}, \\ \mathcal{Q}_l(\mathcal{J}_4) = \{(i,j) : 0 \leq j \leq i < l\} \end{cases} \qquad (23)$$

denote the sets of $(q_{\mathrm{d}}, q_{\mathrm{r}})$ belonging to $\{\mathcal{J}_k\}_{k=1}^4$ for layer l. Here, the probability $\mathbb{P}[\mathcal{S}_{\mathrm{f},l}(i,j)]$ is derived over the distribution of $\boldsymbol{\gamma} = [\gamma_{\mathrm{sd}}\ \gamma_{\mathrm{sr}}\ \gamma_{\mathrm{rd}}]$: Note that $\gamma_{uv} = |h_{uv}|^2 \frac{\mathcal{P}_u}{\sigma^2}$

for $uv \in \{\mathrm{sd}, \mathrm{sr}, \mathrm{rd}\}$ are independent and exponentially distributed with mean $\Omega_{uv} = \varsigma_{uv}^{-\nu}\frac{\mathcal{P}_u}{\sigma^2}$ since $h_{uv} \sim \mathcal{CN}(0, \varsigma_{uv}^{-\nu})$.

Let us next derive $\mathcal{S}_{\mathrm{f},l}(i,j)$ and then $P_{\mathrm{suc},l}(\mathcal{J}_k)$ for the three types of the proposed scheme, which would allow us to evaluate (21).

B. Prop-DF

When $(i,j) \in \mathcal{Q}_l(\mathcal{J}_1)$, we have $\eta_{\mathrm{f},l}(i,j) = \Phi_{l,L}(\gamma_{\mathrm{sd}})$ from Table I so that

$$\mathcal{S}_{\mathrm{f},l}(i,j) = \mathcal{E}_{\mathrm{d},i} \bigcap \mathcal{E}_{\mathrm{r},j} \qquad (24)$$

since $\{\eta_{\mathrm{f},l}(i,j) > \kappa_l\} = \{\gamma_{\mathrm{sd}} > \Gamma_{l,L}\}$ in (20). We then have

$$P_{\mathrm{suc},l}(\mathcal{J}_1) = \sum_{i=l}^{L}\mathbb{P}[\mathcal{E}_{\mathrm{d},i}]\sum_{j=0}^{L}\mathbb{P}[\mathcal{E}_{\mathrm{r},j}] = e^{-\frac{\Gamma_{l,L}}{\Omega_{\mathrm{sd}}}} \qquad (25)$$

from (22), $\mathcal{Q}_l(\mathcal{J}_1)$ in (23), (24), and $\mathbb{P}[\mathcal{E}_{v,i}] = e^{-\frac{\Gamma_{i,L}}{\Omega_{sv}}} - e^{-\frac{\Gamma_{i+1,L}}{\Omega_{sv}}}$ for $v \in \{\mathrm{r},\mathrm{d}\}$.

When $(i,j) \in \mathcal{Q}_l(\mathcal{J}_2)$, since $\eta_{\mathrm{f},l}(i,j) = \Phi_{l,j}(\lambda_{i,j}^D)$ from Table I, we have

$$\mathcal{S}_{\mathrm{f},l}(i,j) = \mathcal{E}_{\mathrm{d},i} \bigcap \mathcal{E}_{\mathrm{r},j} \bigcap \{\lambda_{i,j}^D > \Gamma_{l,j}\}. \qquad (26)$$

Thus, recollecting $\mathcal{Q}_l(\mathcal{J}_2)$ in (23) and (26), we have

$$P_{\mathrm{suc},l}(\mathcal{J}_2) = \sum_{j=l}^{L}\mathbb{P}[\mathcal{E}_{\mathrm{r},j}]\sum_{i=0}^{l-1}P_{l,i}^D(j) \qquad (27)$$

from (22), where

$$\begin{aligned} P_{l,i}^D(j) &= \mathbb{P}\left[\mathcal{E}_{\mathrm{d},i}\bigcap\{\lambda_{i,j}^D \geq \Gamma_{l,j}\}\right] \\ &= \mathbb{P}\left[\mathcal{E}_{\mathrm{d},i}\bigcap\left\{\frac{\Phi_{j,L}(\gamma_{\mathrm{sd}})}{\alpha_j} + \frac{\gamma_{\mathrm{rd}}}{\bar{\alpha}_{i+1,j}} > \Gamma_{l,j}\right\}\right]. \end{aligned} \qquad (28)$$

Similarly, for $(i,j) \in Q(\mathcal{J}_3) \cup Q(\mathcal{J}_4)$, we have $\eta_{\mathrm{f},l}(i,j) = \Phi_{l,L}(\gamma_{\mathrm{sd}})$ and $\mathcal{S}_{\mathrm{f},l}(i,j) = \mathcal{E}_{\mathrm{d},i}\bigcap\mathcal{E}_{\mathrm{r},j}\bigcap\{\gamma_{\mathrm{sd}} > \Gamma_{l,L}\} = \emptyset$: Consequently, $P_{\mathrm{suc},l}(\mathcal{J}_3) = P_{\mathrm{suc},l}(\mathcal{J}_4) = 0$, which produces the outage probability

$$P_{\mathrm{out},l} = 1 - e^{-\frac{\Gamma_{l,L}}{\Omega_{\mathrm{sd}}}} - \sum_{j=l}^{L}\mathbb{P}[\mathcal{E}_{\mathrm{r},j}]\sum_{i=0}^{l-1}P_{l,i}^D(j) \qquad (29)$$

of Prop-DF when combined with (25) and (27).

C. Prop-AF

In the case $\mathcal{J}_1$, with $\eta_{\mathrm{f},l}(i,j)$ being identical to that of Prop-DF, we have $P_{\mathrm{suc},l}(\mathcal{J}_1) = e^{-\frac{\Gamma_l}{\Omega_{\mathrm{sd}}}}$ from (25). Next, for $(i,j) \in \mathcal{Q}_l(\mathcal{J}_2)\cup\mathcal{Q}_l(\mathcal{J}_3)$, the relation $\eta_{\mathrm{f},l}(i,j) = \Phi_{l,L}(\lambda_i^A)$ results in $\mathcal{S}_{\mathrm{f},l}(i,j) = \mathcal{E}_{\mathrm{d},i}\bigcap\mathcal{E}_{\mathrm{r},j}\bigcap\{\lambda_i^A > \Gamma_{l,L}\}$, from which we have

$$\begin{aligned} \sum_{k=2}^{3}P_{\mathrm{suc},l}(\mathcal{J}_k) &= \sum_{i=0}^{l-1}\sum_{j=i+1}^{L}\mathbb{P}\left[\mathcal{E}_{\mathrm{d},i}\bigcap\mathcal{E}_{\mathrm{r},j}\bigcap\{\lambda_i^A > \Gamma_{l,L}\}\right] \\ &= \sum_{i=0}^{l-1}P_{l,i*}^A \end{aligned} \qquad (30)$$

since $\mathcal{Q}_l(\mathcal{J}_2) \cup \mathcal{Q}_l(\mathcal{J}_3) = \{(i,j) : 0 \leq i < l, i < j \leq L\}$, where

$$\begin{aligned} P_{l,i*}^A &= \sum_{j=i+1}^{L}\mathbb{P}\left[\mathcal{E}_{\mathrm{d},i}\bigcap\mathcal{E}_{\mathrm{r},j}\bigcap\{\lambda_i^A > \Gamma_{l,L}\}\right] \\ &= \mathbb{P}\left[\Gamma_{i,L} < \gamma_{\mathrm{sd}} \leq \Gamma_{i+1,L},\ \Gamma_{i+1,L} < \gamma_{\mathrm{sr}}, \right. \\ &\qquad \left. \gamma_{\mathrm{sd}} + \frac{\gamma_{\mathrm{sr}}\gamma_{\mathrm{rd}}}{\gamma_{\mathrm{sr}}\bar{\alpha}_{i+1,L} + \gamma_{\mathrm{rd}} + 1} > \Gamma_{l,L}\right]. \end{aligned} \qquad (31)$$

In the case $\mathcal{J}_4$, with $\eta_{\mathrm{f},l}(i,j) = \Phi_{l,L}\left(\lambda_i^A\right)$, we have $\mathcal{S}_{\mathrm{f},l}(i,j) = \mathcal{E}_{\mathrm{d},i}\bigcap\mathcal{E}_{\mathrm{r},j}\bigcap\{\lambda_j^A > \Gamma_{l,L}\}$, and consequently,

$$
\begin{aligned}
P_{\mathrm{suc},l}(\mathcal{J}_4) &= \sum_{j=0}^{l-1}\sum_{i=j}^{l-1}\mathbb{P}\left[\mathcal{E}_{\mathrm{d},i}\bigcap\mathcal{E}_{\mathrm{r},j}\bigcap\{\lambda_j^A > \Gamma_{l,L}\}\right] \\
&= \sum_{j=0}^{l-1} P_{l,*j}^A,
\end{aligned}
\tag{32}
$$

where

$$
\begin{aligned}
P_{l,*j}^A &= \sum_{i=j}^{l-1}\mathbb{P}\left[\mathcal{E}_{\mathrm{d},i}\bigcap\mathcal{E}_{\mathrm{r},j}\bigcap\{\lambda_j^A > \Gamma_{l,L}\}\right] \\
&= \mathbb{P}\Big[\Gamma_{j,L} < \gamma_{\mathrm{sd}} \leq \Gamma_{l,L},\ \Gamma_{j,L} < \gamma_{\mathrm{sr}} \leq \Gamma_{j+1,L}, \\
&\qquad \gamma_{\mathrm{sd}} + \tfrac{\gamma_{\mathrm{sr}}\gamma_{\mathrm{rd}}}{\gamma_{\mathrm{sr}}\bar{\alpha}_{j+1,L}+\gamma_{\mathrm{rd}}+1} > \Gamma_{l,L}\Big].
\end{aligned}
\tag{33}
$$

Thus, from (25), (30), and (32), we finally have the outage probability of Prop-AF as

$$
P_{\mathrm{out},l} = 1 - e^{-\frac{\Gamma_{l,L}}{\Omega_{\mathrm{sd}}}} - \sum_{i=0}^{l-1}\left(P_{l,i*}^A + P_{l,*i}^A\right).
\tag{34}
$$

D. Prop-MF

For Prop-MF, the event $\mathcal{S}_{\mathrm{f},l}(i,j)$ is identical to that for Prop-DF in the three cases $\{\mathcal{J}_j\}_{j=1}^3$ and to that for Prop-AF in the case $\mathcal{J}_4$. Therefore, we have the outage probability

$$
P_{\mathrm{out},l} = 1 - e^{-\frac{\Gamma_{l,L}}{\Omega_{\mathrm{sd}}}} - \sum_{j=l}^{L}\mathbb{P}[\mathcal{E}_{\mathrm{r},j}]\sum_{i=0}^{l-1}P_{l,i}^D(j) - \sum_{j=0}^{l-1}P_{l,*j}^A
\tag{35}
$$

using (25), (27), $P_{\mathrm{suc},l}(\mathcal{J}_3) = 0$ as in Section IV-B, and (32).

E. Discussion

With the outage probabilities derived analytically so far, we next find the minimum ED shown in (6), which can be restated as an optimization problem:

$$
\mathbb{E}_{D_o} = \min_{\{(\boldsymbol{R},\boldsymbol{\alpha}):\,\boldsymbol{R}\succeq 0,\,\boldsymbol{\alpha}\succeq 0\}}\mathbb{E}_D(\boldsymbol{R},\boldsymbol{\alpha})
\tag{36}
$$

$$
\text{subject to C1: } \sum_{l=1}^{L}\alpha_l = 1
\tag{37}
$$

$$
\text{C2: } \Gamma_{L,L} > \Gamma_{L-1,L} > \cdots > \Gamma_{1,L} > 0,
\tag{38}
$$

where $\succeq$ denotes the element-wise inequality. The problem can be solved by first minimizing the objective function $\mathbb{E}_D(\boldsymbol{R},\boldsymbol{\alpha})$ over all feasible $\boldsymbol{\alpha}$ at each feasible value of $\boldsymbol{R}$ as

$$
\mathbb{E}_{D,\mathrm{min}}(\boldsymbol{R}) = \min_{\{\boldsymbol{\alpha}:\,\boldsymbol{\alpha}\succeq 0,\,\mathrm{C1},\,\mathrm{C2}\}}\mathbb{E}_D(\boldsymbol{R},\boldsymbol{\alpha}),
\tag{39}
$$

and then, finding the minimum of (39) over all feasible $\boldsymbol{R}$ as

$$
\mathbb{E}_{D_o} = \min_{\{\boldsymbol{R}:\,\boldsymbol{R}\succeq 0\}}\mathbb{E}_{D,\mathrm{min}}(\boldsymbol{R}).
\tag{40}
$$

Since the optimization problem is non-convex and non-linear, no existing algorithm may be employed to obtain the solution effectively. We thus resort to exhaustive search for the solution, in which the following linearization of the constraints is helpful for obtaining $\mathbb{E}_{D,\mathrm{min}}(\boldsymbol{R})$ in (39). By changing variables as

$$
a_l = \frac{1}{\Gamma_{l,L}} = \frac{1}{\kappa_l}\alpha_l - \bar{\alpha}_{l+1,L}
\tag{41}
$$

for $l \in \mathbb{N}_1^L$ [12], we can express α_l as a linear combination $\alpha_l = \sum_{m=1}^{L} t_{l,m}a_m$ of $\boldsymbol{a} = [a_1\ a_2\ \cdots\ a_L]$, where $t_{l,l} = \kappa_l$, $t_{l,m} = 0$ if $l \in \mathbb{N}_{m+1}^L$, and $t_{l,m} = \kappa_l\sum_{k=l+1}^{L}t_{k,m}$ if $l \in \mathbb{N}_1^{m-1}$. Replacing $\boldsymbol{\alpha}$ with $\boldsymbol{Ta}$, the problem (39) can be transformed into $\mathbb{E}_{D,\mathrm{min}}(\boldsymbol{R}) = \min_{\{\boldsymbol{a}:\,\mathrm{C1}',\,\mathrm{C2}'\}}\mathbb{E}_D(\boldsymbol{R},\boldsymbol{Ta})$, where C1': $\sum_{m=1}^{L}T_m a_m = 1$ with $T_m = \sum_{l=1}^{L}t_{l,m}$, C2': $a_1 > a_2 > \cdots > a_L > 0$, and $\boldsymbol{T}$ is an $L \times L$ matrix with $t_{l,m}$ as the (l,m)th element. With the linear constraints C1' and C2', we can set the search range of $\boldsymbol{a}$ conveniently, since $\left\{\sum_{m=1}^{M}T_m\right\}^{-1} < a_1 < T_1^{-1}$, $a_l < a_{l-1}$ for $l = 2, 3, \cdots, L-1$, and $a_L = T_L^{-1} - T_L^{-1}\sum_{m=1}^{L-1}T_m a_m$. Next, in solving (40), we employ an exhaustive search over $0 \leq R_l \leq R_{\mathrm{max}}$, where R_{max} is empirically chosen to be sufficiently large to include the optimal point.

Normally, numerical integrations would be required when searching for the minimum ED, incurring an excessive amount of computation time. To reduce the search time in the finite SNR regime, we have derived upper bounds $\breve{P}_{l,i}^D(j)$ (given in (65)), $\breve{P}_{l,i*}^A$ (given in (67)), and $\breve{P}_{l,*j}^A$ (given in (68)) on $P_{l,i}^D(j)$, $P_{l,i*}^A$, and $P_{l,*j}^A$ shown in (28), (31), and (33), respectively, in Appendix II, which lead to a lower bound $\tilde{P}_{\mathrm{out},l}$ on $P_{\mathrm{out},l}$, shown in (29), (34), and (35): Let us just mention that the computation time for evaluating the minimum ED of Prop-MF using $\tilde{P}_{\mathrm{out},l}$ is, for example, only 0.26% that of using numerical integration when $L = 2$ on Windows7 with a 3 GHz processor.

V. ASYMPTOTIC DISTORTION PERFORMANCE

The high SNR behavior of the ED of the proposed scheme will now be investigated in terms of the distortion exponent defined as

$$
\Delta = -\lim_{\Omega\to\infty}\frac{\log\mathbb{E}_D(\boldsymbol{R},\boldsymbol{\alpha})}{\log\Omega}
\tag{42}
$$

for a single link system [8], where Ω is the average SNR of the link. In [11], the distortion exponent (42) is investigated for the relay system assuming that $\Omega_{uv} = \Omega$ for $uv \in \{\mathrm{sd},\mathrm{sr},\mathrm{rd}\}$. Taking the relay location also into account, we will investigate the distortion exponent behavior in terms of $\Omega = \Omega_{\mathrm{sd}}$ when $\Omega_{\mathrm{sr}} = \rho_1\Omega$ and $\Omega_{\mathrm{rd}} = \rho_2\Omega$, where $\rho_1 = \left(\frac{\varsigma_{\mathrm{sd}}}{\varsigma_{\mathrm{sr}}}\right)^{\nu}$ and $\rho_2 = \frac{\mathcal{P}_{\mathrm{r}}}{\mathcal{P}_{\mathrm{s}}}\left(\frac{\varsigma_{\mathrm{sd}}}{\varsigma_{\mathrm{rd}}}\right)^{\nu}$. We assume $\mathcal{P}_{\mathrm{r}} = \mathcal{P}_{\mathrm{s}}$, $0 < \varsigma_{\mathrm{sr}} < \varsigma_{\mathrm{sd}}$, and $0 < \varsigma_{\mathrm{rd}} < \varsigma_{\mathrm{sd}}$, so that $\rho_1 > 1$ and $\rho_2 > 1$. In obtaining distortion exponents of the proposed scheme at two extreme relay locations of $\varsigma_{\mathrm{sr}} \to 0$ and $\varsigma_{\mathrm{sr}} \to \varsigma_{\mathrm{sd}}$, we employ the diversity-multiplexing tradeoff approach [11], [17], in which the successive decoding diversity gain (SDDG) at multiplexing gain $\boldsymbol{r} = [r_1\ r_2\ \cdots\ r_L] = \frac{\boldsymbol{R}}{\log_2\Omega}$ is defined as

$$
d_l(\boldsymbol{r}) = -\lim_{\Omega\to\infty}\frac{\log P_{\mathrm{out},l}}{\log\Omega}
\tag{43}
$$

with $d_0(\boldsymbol{r}) = \infty$ and $d_{L+1}(\boldsymbol{r}) = 0$ since $P_{\mathrm{out},0} = 0$ and $P_{\mathrm{out},L+1} = 1$.

We employ the exponential power allocation, assigning $\bar{\alpha}_{1,L} = 1$ and $\bar{\alpha}_{l,L} = \Omega^{-2\sum_{m=1}^{l-1} r_m - \epsilon_{l-1}}$ for $l \in \mathbb{N}_2^L$ with some infinitesimal numbers $\{\epsilon_j\}_{j=1}^{L-1}$ such that $0 < \epsilon_1 < \epsilon_2 < \cdots < \epsilon_{L-1}$ [11], [17]. The exponential power allocation is selected based on the following rationale of an attempt to make the effective SNR (9) as high as possible. First, the vector $\boldsymbol{\alpha}$ should be chosen to satisfy the condition $P_{\text{out},l} < 1$, or equivalently, $\Gamma_{l,L} > 0$: This implies we should have $\bar{\alpha}_{l+1,L}\kappa_l < \alpha_l$ in (19), which can be expressed as (i) $\bar{\alpha}_{l+1,L} < \bar{\alpha}_{l,L}\Omega^{-2r_l}$. Now, from (i), we can show that $\alpha_l = \bar{\alpha}_{l,L} - \bar{\alpha}_{l+1,L} \approx \bar{\alpha}_{l,L}$. Therefore, to make the effective SNR (9) as high as possible, $\bar{\alpha}_{l,L}$ should be chosen as large as possible under the constraint (i), which can be achieved by the allocation $\bar{\alpha}_{l,L} = \bar{\alpha}_{l-1,L}\Omega^{-2r_{l-1}-\varepsilon_{l-1}}$ for some infinitesimal $\varepsilon_{l-1} > 0$ for $l \in \mathbb{N}_2^L$. In the sequel, since $\bar{\alpha}_{1,L} = 1$, we have $\bar{\alpha}_{l,L} = \Omega^{-2\sum_{m=1}^{l-1} r_m - \epsilon_{l-1}}$, where $\epsilon_l = \sum_{m=1}^{l} \varepsilon_m$ for $l \in \mathbb{N}_1^{L-1}$.

With the exponential power allocation, the SNR thresholds in the high SNR regime can be approximated, from $\kappa_l \approx \Omega^{2r_l}$, $\alpha_l \approx \bar{\alpha}_{l,L} = \Omega^{-2\sum_{m=1}^{l-1} r_m - \epsilon_{l-1}}$, and $\alpha_l - \bar{\alpha}_{l+1,j}\kappa_l \approx \bar{\alpha}_{l,L}\left(1 - \Omega^{-\varepsilon_l}\right) \approx \Omega^{-2\sum_{m=1}^{l-1} r_m - \epsilon_{l-1}}$ in (19), as

$$\Gamma_{l,j} \approx \frac{\kappa_l}{\bar{\alpha}_{l,L}} \approx \Omega^{2\sum_{m=1}^{l} r_m + \epsilon_{l-1}}. \tag{44}$$

To ensure $P_{\text{out},l} \to 0$ as $\Omega \to \infty$ in (29), (34), and (35), the multiplexing gain vector $\boldsymbol{r}$ should be chosen to satisfy $\frac{\Gamma_{l,L}}{\Omega} \to 0$, which results in $2\sum_{m=1}^{l} r_m + \epsilon_{l-1} < 1$ from (44).

The SDDG and the distortion exponent of Prop-DF, Prop-AF, and Prop-MF are provided in Theorems 1–4 below.

Theorem 1: For Prop-DF, Prop-AF, and Prop-MF, the SDDG of layer l with the exponential power allocation is

$$d_l(\boldsymbol{r}) = \begin{cases} 2 - 4\sum_{m=1}^{l} r_m, & \text{if } \varsigma_{\text{sr}} \to \varsigma_{\text{sd}}, \\ 2 - 2\max_{1 \le i \le l} r_i - 2\sum_{m=1}^{l} r_m, & \text{if } \varsigma_{\text{sr}} \to 0. \end{cases} \tag{45}$$

Proof: See Appendix III. □

Theorem 1 implies that the SDDG of the proposed scheme is larger when the relay is closer to the source: Intuitively, when the relay is closer to the source, the relay can decode more layers successfully, and consequently, transmit the layers required at the destination more effectively. Since $P_{\text{out},l} \approx \Omega^{-d_l(\boldsymbol{r})}$ from (43) and $d_0(\boldsymbol{r}) = \infty > d_1(\boldsymbol{r}) > \cdots > d_{L+1}(\boldsymbol{r}) = 0$ in (45), the distortion exponent is obtained from (5) as

$$\Delta = \max_{\{\boldsymbol{r}:\boldsymbol{r}>0\}} \min_{l \in \mathbb{N}_1^{L+1}} \Delta_l, \tag{46}$$

where $\Delta_l = d_l(\boldsymbol{r}) + b\sum_{m=1}^{l-1} r_m$ with b the source-channel mismatch factor defined in (1).

Theorem 2: When $\varsigma_{\text{sr}} \to \varsigma_{\text{sd}}$, the distortion exponent of Prop-DF, Prop-AF, and Prop-MF with the exponential power allocation reaches

$$\Delta = 2 - 2\left\{\sum_{m=0}^{L} \left(\frac{b}{4}\right)^m\right\}^{-1} \tag{47}$$

when $r_1 = \left\{2\sum_{m=0}^{L} \left(\frac{b}{4}\right)^m\right\}^{-1}$ and $r_l = \left(\frac{b}{4}\right)^{l-1} r_1$ for $l \in \mathbb{N}_2^L$.

Proof: See Appendix IV. □

Theorem 3: When $\varsigma_{\text{sr}} \to 0$, the distortion exponent of Prop-DF, Prop-AF, and Prop-MF with the exponential power allocation reaches

$$\Delta = \begin{cases} 2 - \frac{2+b+(2-b)s^*}{1+s^*+\sum_{m=1}^{L-s^*+1}\left(\frac{b}{2}\right)^m}, & \text{if } 0 < b \le 2, \\ 2 - \frac{2}{1+\frac{b}{4}\sum_{m=1}^{L}\left(\frac{b+2}{4}\right)^{m-1}} & \text{if } b \ge 2, \end{cases} \tag{48}$$

where

$$s^* = \begin{cases} L, & \text{if } L \le 1 + \frac{2}{b}, \\ \min_{\{s\in\mathbb{N}_1^{L-1}:A(s)<0\}} s, & \text{if } L > 1 + \frac{2}{b} \end{cases} \tag{49}$$

with $A(s) = b - \{2 + (2-b)s\}\left(\frac{b}{2}\right)^{L-s+1}$. The multiplexing gain achieving (48) is given by

$$r_l = \begin{cases} \left\{1 + s^* + \sum_{m=1}^{L-s^*+1}\left(\frac{b}{2}\right)^m\right\}^{-1}, & \text{for } l \in \mathbb{N}_1^{s^*}, \\ \left(\frac{b}{2}\right)^{l-s^*} r_{s^*}, & \text{for } l \in \mathbb{N}_{s^*+1}^{L} \end{cases} \tag{50}$$

when $0 < b < 2$, and by

$$r_l = \left(\frac{b+2}{4}\right)^{l-1} r_1 \tag{51}$$

for $l \in \mathbb{N}_2^L$ with $r_1 = 2\left\{4 + b\sum_{m=1}^{L}\left(\frac{b+2}{4}\right)^{m-1}\right\}^{-1}$ when $b \ge 2$.

Proof: See Appendix V. □

When the relay is near the destination, it is immediate from Theorem 2 that the distortion exponent of the proposed scheme is identical to that of the conventional SC-AF [11]. When the relay is close to the source, we can in addition deduce, after some thought based on Theorems 1 and 2, that the distortion exponent (48) of the proposed scheme will be larger than that of SC-AF. Next, the behaviors of the distortion exponent for finite values of L can be predicted indirectly from the following theorem for $L \to \infty$.

Theorem 4: When $L \to \infty$, the distortion exponent of Prop-DF, Prop-AF, and Prop-MF converges to

$$\Delta^\infty = \begin{cases} \min\left(\frac{b}{2}, 2\right), & \text{if } \varsigma_{\text{sr}} \to \varsigma_{\text{sd}}, \\ \min(b, 2), & \text{if } \varsigma_{\text{sr}} \to 0. \end{cases} \tag{52}$$

Proof: From (47), $\Delta = 2 - 2\left(1 - \frac{b}{4}\right) = \frac{b}{2}$ if $0 < b < 4$ and $\Delta = 2$ if $b \ge 4$ when $\varsigma_{\text{sr}} \to \varsigma_{\text{sd}}$. When $\varsigma_{\text{sr}} \to 0$, $\Delta = 2$ if $b \ge 2$ from (48). Let us now consider the case $0 < b < 2$

when $\varsigma_{sr} \to 0$ in (48). Since $L > 1 + \frac{2}{b}$ when $L \to \infty$, we need to find $s^* = \min\limits_{s \in \mathbb{N}_1^{L-1}} s$ subject to $A(s) < 0$ in (49). Clearly, when $0 < b < 2$, $A(s)$ is a decreasing function of s with $A(1) = b - \{2 + (2-b)\}\left(\frac{b}{2}\right)^L$ and $A(L-1) = b - \{2 + (2-b)(L-1)\}\left(\frac{b}{2}\right)^2$ with $A(1) \to b$ and $A(L-1) \to -\infty$ as $L \to \infty$. Thus, there exists an $s^* \in \mathbb{N}_2^{L-1}$ such that $A(s^*-1) \geq 0$ and $A(s^*) < 0$. Now, when $L \to \infty$, the equation $A(s) = 0$, or equivalently $\left(\frac{b}{2}\right)^{L-s+1} = \frac{1}{2+(2-b)s}$, cannot be satisfied with a finite value of s: Thus, $s^* \to \infty$ as $L \to \infty$. This implies $\Delta = 2 - \frac{2+b+(2-b)s^*}{1+s^*+\sum\limits_{m=1}^{L-s^*+1}\left(\frac{b}{2}\right)^m} \to 2 - (2-b)$ as $L \to \infty$. $\square$

To confirm Theorems 2–4, the distortion exponent of the proposed scheme is shown in Fig. 2 as a function of the source-channel mismatch factor b. It is observed that the closed-form solutions (47), (48), and (52) (denoted by lines) agree well with those obtained from the optimization program [25] (denoted by marks) and that the distortion exponent increases with L as anticipated. The distortion exponent when $\varsigma_{sr} \to 0$ is larger than that when $\varsigma_{sr} \to \varsigma_{sd}$, which gets more noticeable as L increases. In other words, the proposed scheme would perform better when the relay is closer to the source and the number of layers L increases.

Fig. 3 compares the distortion exponent of the proposed and conventional schemes for four values of L when $\varsigma_{sr} \to 0$. In this figure, 'Prop' denotes the proposed scheme, 'Conv [10,11]' denotes the conventional SC-DF [10] and SC-AF [11] schemes, and 'DT [12]' denotes the direct transmission (DT) of SC [12]. As shown in Appendix VI, the distortion exponent of SC-DF and SC-AF does not depend on the relay location and is identical to that of the proposed scheme for $\varsigma_{sr} \to \varsigma_{sd}$. Clearly, at the cost of feedback overhead of $\lceil \log_2(L+1) \rceil$ bits, the proposed relaying scheme provides a higher distortion exponent than the conventional ones when the relay is close to the source, with the gain getting larger as L increases. When b is small, the high SNR performance of the relaying schemes is known to be inferior to that of DT since relaying using two slots can incur a larger distortion in source coding due to reduced source code rate: Over the interval (b_T, ∞) of b, on the contrary, it is clearly observed that the proposed scheme can enjoy a larger distortion exponent than DT, where b_T is approximately 1.3 when $L = 2$ and converges to 1 when $L \to \infty$. In addition, the proposed scheme provides the same performance as DT when $L \to \infty$ for $0 < b \leq 1$.

VI. PERFORMANCE EVALUATION

Let us now evaluate the performance of the proposed scheme in the finite SNR region. We assume the three nodes form a straight line ($\varsigma_{rd} = \varsigma_{sd} - \varsigma_{sr}$) with the same transmit power $\mathcal{P}_s = \mathcal{P}_r$ and path loss exponent $\nu = 3$. We numerically search for the minimum ED in the feasible region of $(\boldsymbol{R}, \boldsymbol{\alpha})$ in two steps as described in Subsection IV.E and show the result in dB relative to the worst ED.

Fig. 4 compares the expected distortion performance of various relaying schemes as a function of the average SNR Ω when $b = 1$ and $L = 2$. The normalized relay location

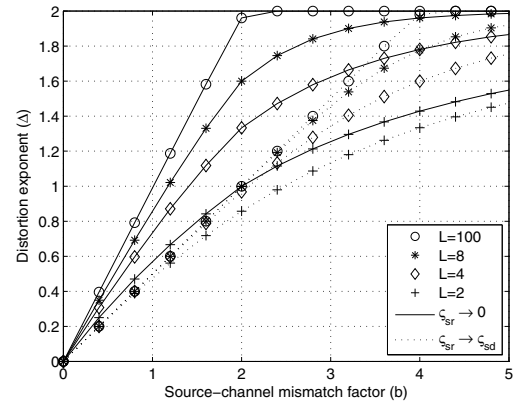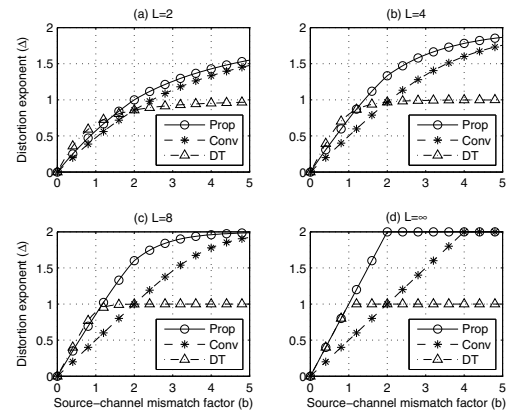Fig. 2. Distortion exponent of the proposed scheme as a function of the source-channel mismatch factor b.

Fig. 3. Comparison of the distortion exponents of the proposed and conventional schemes when $\varsigma_{sr} \to 0$.

$\frac{\varsigma_{sr}}{\varsigma_{sd}}$ is set to 0.2 and 0.8 in Figs. 4(a) and (b), respectively. As benchmarks, we have shown the performance of the optimal relaying scheme (denoted as 'CSIT') with optimal time allocation between the two transmission phases and full CSI at the source [21], [26], that of the basic AF and DF schemes without SC [3] (denoted as 'AF [3]' and 'DF [3]', respectively), and that of SC-DF [10] and SC-AF [11] schemes (denoted as 'SC-DF [10]' and 'SC-AF [11]', respectively): It should be noted that the optimal relaying scheme can be implemented only when the instantaneous CSI $\boldsymbol{\gamma}$ is fully available at the source for each data transmission whereas the other SC-based schemes can be implemented with the average CSI $\boldsymbol{\Omega}$ at the source. The solid and dash-dot lines represent the minimum ED evaluated with $P_{out,l}$ ('Exact') and the lower bound evaluated with $\tilde{P}_{out,l}$ ('LB'), respectively. It is observed that the lower bounds, obtained with significantly reduced computation time, are almost indistinguishable from the exact values. Prop-MF, Prop-AF, and Prop-DF perform similarly, all with some gain over AF, DF, SC-AF and SC-DF, when $\frac{\varsigma_{sr}}{\varsigma_{sd}} = 0.2$. When $\frac{\varsigma_{sr}}{\varsigma_{sd}} = 0.8$, Prop-MF, Prop-AF, and SC-AF perform similarly with some gain over Prop-DF, SC-DF, AF, and DF. The figure also reveals that the distortion exponent, i.e., the slope of the ED in the high SNR region, varies as the relay location. The slopes for the proposed scheme are steeper than those for the conventional schemes when $\frac{\varsigma_{sr}}{\varsigma_{sd}} = 0.2$, while the slopes are almost the same when $\frac{\varsigma_{sr}}{\varsigma_{sd}} = 0.8$, which agrees
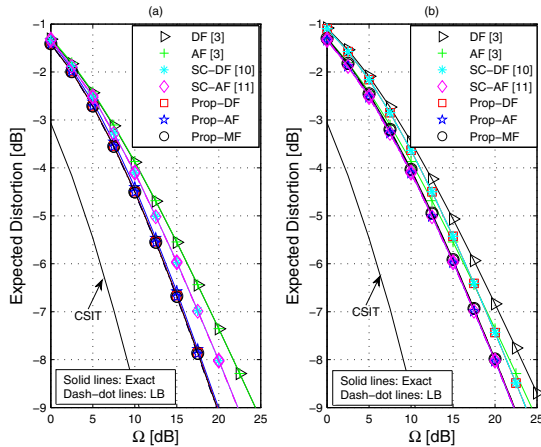
Fig. 4. Minimum ED as a function of the average SNR with exact and approximate evaluations: (a) $\frac{\varsigma_{sr}}{\varsigma_{sd}} = 0.2$ (b) $\frac{\varsigma_{sr}}{\varsigma_{sd}} = 0.8$.

Fig. 5. Minimum ED as a function of the normalized relay location $\frac{\varsigma_{sr}}{\varsigma_{sd}}$ with exact and approximate evaluations when $\Omega = 15$ dB.

Fig. 6. Minimum ED when $\frac{\varsigma_{sr}}{\varsigma_{sd}} = 0.2$: (a) When $b = 1$ (b) When $L = 2$.

with the analysis on the distortion exponent in Section V.

In Fig. 5, we show the expected distortion performance as a function of the normalized relay location $\frac{\varsigma_{sr}}{\varsigma_{sd}}$ when $b = 1$, $L = 2$, and $\Omega = 15$ dB. We can make observations similar to those in Fig. 4, except that lower bounds now tend to deviate from the exact values for Prop-AF and SC-AF around $\frac{\varsigma_{sr}}{\varsigma_{sd}} = 0.5$, since the inequality (66) becomes loose when $\gamma_{sr} \approx \gamma_{rd}$. It is observed that (i) Prop-DF (Prop-AF) outperforms, or performs similarly to, SC-DF (SC-AF), (ii) Prop-DF, not incurring any noise amplification, outperforms Prop-AF when the relay is near the source, and (iii) the AF-based schemes perform better than the DF-based schemes when the relay is near the destination as is well-known. We would like to add that Prop-MF provides uniformly the best performance at most relay locations by taking the advantages of both DF and AF signals. Prop-AF performs slightly better than Prop-MF when $\frac{\varsigma_{sr}}{\varsigma_{sd}} \gtrsim 0.75$, where the AF signals used in Prop-AF are more suitable than the DF signals used in Prop-MF for $q_d < q_r$: As the relay gets closer to the destination node, the number q_r tends to decrease, resulting in a smaller number of layers transmitted by the relay in the DF signal, which implies a better performance for the AF signal than the DF signal. Nonetheless, the gain is not significant, since the probability of the event $\{q_d < q_r\}$ is not that high. Let us mention that other assignments of the AF and DF signals can, of course, be devised, which would produce performance characteristics different from those of the proposed assignments. Specifically, if we modify Prop-MF so that AF signals, instead of DF signals, are sent in some cases where $q_d < q_r$, we could improve the performance when the relay is close to the destination at the expense of performance loss at other relay locations. In any case, we believe there does not exist a uniformly optimal scheme.

In Fig. 6, we compare the performance of some representative schemes for various values of L and b when $\frac{\varsigma_{sr}}{\varsigma_{sd}} = 0.2$. Again, the slopes of the ED curves in the high SNR region agree with the results in Fig. 3: That is, (i) the slope for Prop-MF is steeper than that for SC-AF as $\frac{\varsigma_{sr}}{\varsigma_{sd}} \to 0$, (ii) all the slopes become steeper when L increases with that for Prop-MF getting even steeper, and (iii) all the slopes become steeper
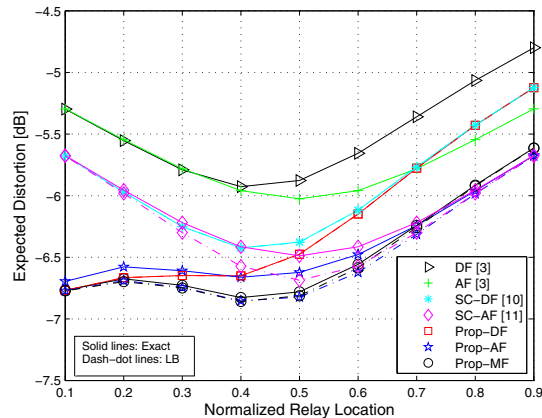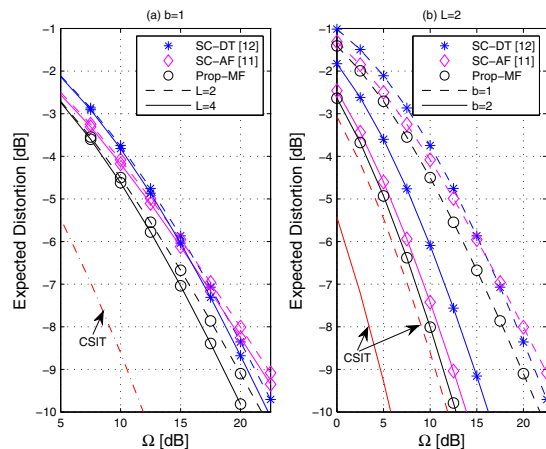
when b increases due to the decreased distortion from source coding. Unlike in the infinite SNR region, the relaying systems are shown to provide better outage performance than DT in the finite SNR region. In addition, Prop-MF again exhibits the best performance in most cases: As observed in Fig. 6(a), Prop-MF provides a gain of about 2.5 dB in the average SNR Ω over SC-AF and DT when $b = 1$, $L = 2$, $\frac{\varsigma_{sr}}{\varsigma_{sd}} = 0.2$, and $\mathbb{E}_{D_o} = -8$ dB, at the cost of 2-bit feedback overhead.

In passing, we would like to note that, as in the conventional relaying schemes [10], [11], [17]–[19], the rate $\boldsymbol{R}$ and power $\boldsymbol{\alpha}$ in the proposed scheme are determined once the average SNR levels $\boldsymbol{\Omega} = (\Omega_{sd}, \Omega_{sr}, \Omega_{rd})$ (in which the effect of relay position is incorporated inherently) of the channels are fixed. When the average SNR levels change, $\boldsymbol{R}$ and $\boldsymbol{\alpha}$ can be updated with a look-up table installed, or with a numerical search based on $\boldsymbol{\Omega}$ fed back, where the feedback and updating rate is much lower than that in the optimal scheme [21], [26] performing resource allocation based on the instantaneous SNRs.

VII. CONCLUSION

When superposition coding is adopted for the transmission of successive refinement layers in a relay network, we have proposed three types of relay signals by exploiting the decoding results at both the relay and destination nodes in the first slot. We have analyzed the outage probability of the proposed

$$\boldsymbol{C}_{\tilde{\boldsymbol{w}}_l} = \begin{bmatrix} |h_{\mathrm{sd}}|^2 \mathcal{P}_{\mathrm{s}} \bar{\alpha}_{l+1,L} + \sigma^2 & h_{\mathrm{sd}} h_{\mathrm{rd}}^* G_{q_{\mathrm{d}},q_{\mathrm{r}}}^D \sqrt{\mathcal{P}_{\mathrm{s}}\mathcal{P}_{\mathrm{r}}} \bar{\alpha}_{l+1,q_{\mathrm{r}}} \\ h_{\mathrm{sd}}^* h_{\mathrm{rd}} G_{q_{\mathrm{d}},q_{\mathrm{r}}}^D \sqrt{\mathcal{P}_{\mathrm{s}}\mathcal{P}_{\mathrm{r}}} \bar{\alpha}_{l+1,q_{\mathrm{r}}} & |h_{\mathrm{rd}}|^2 \left(G_{q_{\mathrm{d}},q_{\mathrm{r}}}^D\right)^2 \mathcal{P}_{\mathrm{r}} \bar{\alpha}_{l+1,q_{\mathrm{r}}} + \sigma^2 \end{bmatrix} \tag{57}$$

$$\eta_{\mathrm{f},l} = \frac{\alpha_l \left\{ \frac{\bar{\alpha}_{q_{\mathrm{r}}+1,L}}{\bar{\alpha}_{q_{\mathrm{d}}+1,q_{\mathrm{r}}}} \gamma_{\mathrm{sd}} \gamma_{\mathrm{rd}} + \frac{\gamma_{\mathrm{rd}}}{\bar{\alpha}_{q_{\mathrm{d}}+1,q_{\mathrm{r}}}} + \gamma_{\mathrm{sd}} \right\}}{\bar{\alpha}_{l+1,q_{\mathrm{r}}} \left\{ \frac{\bar{\alpha}_{q_{\mathrm{r}}+1,L}}{\bar{\alpha}_{q_{\mathrm{d}}+1,q_{\mathrm{r}}}} \gamma_{\mathrm{sd}} \gamma_{\mathrm{rd}} + \frac{\gamma_{\mathrm{rd}}}{\bar{\alpha}_{q_{\mathrm{d}}+1,q_{\mathrm{r}}}} + \gamma_{\mathrm{sd}} \right\} + \bar{\alpha}_{q_{\mathrm{r}}+1,L} \gamma_{\mathrm{sd}} + 1} \tag{58}$$

$$\boldsymbol{C}_{\tilde{\boldsymbol{w}}_l} = \begin{bmatrix} |h_{\mathrm{sd}}|^2 \mathcal{P}_{\mathrm{s}} \bar{\alpha}_{l+1,L} + \sigma^2 & h_{\mathrm{sd}} (h_{\mathrm{eq}}^A)^* \mathcal{P}_{\mathrm{s}} \bar{\alpha}_{l+1,L} \\ h_{\mathrm{sd}}^* h_{\mathrm{eq}}^A \mathcal{P}_{\mathrm{s}} \bar{\alpha}_{l+1,L} & |h_{\mathrm{eq}}^A|^2 \mathcal{P}_{\mathrm{s}} \bar{\alpha}_{l+1,L} + |h_{\mathrm{rd}}|^2 \left(G_{\min(q_{\mathrm{d}},q_{\mathrm{r}})}^A\right)^2 \sigma^2 + \sigma^2 \end{bmatrix} \tag{61}$$

$$\eta_{\mathrm{f},l} = \frac{\alpha_l \left\{ \gamma_{\mathrm{rd}} \gamma_{\mathrm{sd}} + \gamma_{\mathrm{rd}} \gamma_{\mathrm{sr}} + \bar{\alpha}_{\min(q_{\mathrm{d}},q_{\mathrm{r}})+1,L} \gamma_{\mathrm{sr}} \gamma_{\mathrm{sd}} + \gamma_{\mathrm{sd}} \right\}}{\bar{\alpha}_{l+1,L} \left\{ \gamma_{\mathrm{rd}} \gamma_{\mathrm{sd}} + \gamma_{\mathrm{rd}} \gamma_{\mathrm{sr}} + \bar{\alpha}_{\min(q_{\mathrm{d}},q_{\mathrm{r}})+1,L} \gamma_{\mathrm{sr}} \gamma_{\mathrm{sd}} + \gamma_{\mathrm{sd}} \right\} + \gamma_{\mathrm{rd}} + \bar{\alpha}_{\min(q_{\mathrm{d}},q_{\mathrm{r}})+1,L} \gamma_{\mathrm{sr}} + 1} \tag{62}$$

scheme, based upon which lower and upper bounds are derived for the fast evaluation of the minimum expected distortion in the finite SNR region and for the assessment of the asymptotic behavior of the expected distortion in terms of the distortion exponent, respectively. In investigating the distortion exponent by deriving the successive decoding diversity gain of the outage probability, we have also taken the relay location into consideration.

The results from the asymptotic analysis have exhibited that the difference in distortion exponents of the proposed and conventional relaying schemes becomes larger as the number L of layers increases, at the negligible cost of feedback overhead of $\lceil \log_2(L+1) \rceil$ bits for the proposed scheme, when the relay is close to the source. In particular, the proposed scheme can provide a distortion exponent of up to $\min(b, 2)$ as L tends to infinity, while the corresponding values for the conventional relaying and direct transmission are $\min\left(\frac{b}{2}, 2\right)$ and $\min(b, 1)$, respectively, where b is the source-channel mismatch factor.

In the finite SNR region, the proposed scheme can provide additional SNR gain over the conventional schemes by sending the relay signal more efficiently when the relay is close to the source. Among the three types of the proposed scheme, the type of selecting appropriately DF and AF relay signals is observed to perform the best over a wide range of relay locations.

APPENDIX I. DERIVATION OF FINAL EFFECTIVE SNRS

A. Prop-DF

For MRC at the destination, using $z(n)$ for $q_{\mathrm{d}} < q_{\mathrm{r}}$ shown in (11), we can rewrite (4) as

$$y_{\mathrm{d},2}(n) = h_{\mathrm{rd}} G_{q_{\mathrm{d}},q_{\mathrm{r}}}^D \sum_{l=q_{\mathrm{d}}+1}^{q_{\mathrm{r}}} \sqrt{\mathcal{P}_{\mathrm{r}} \alpha_l} x_l(n) + w_{\mathrm{d},2}(n). \tag{53}$$

Assuming that layers up to $l-1$ are decoded successfully at the destination and cancelled in the received signal (3) for $v = \mathrm{d}$ and in the received signal (53), the signals to be combined for decoding layer l can be expressed in a vector form as

$$\tilde{\boldsymbol{y}}_l(n) = \tilde{\boldsymbol{h}}_l x_l(n) + \tilde{\boldsymbol{w}}_l(n), \tag{54}$$

where $\tilde{\boldsymbol{h}}_l = \begin{bmatrix} h_{\mathrm{sd}} \sqrt{\mathcal{P}_{\mathrm{s}} \alpha_l} & h_{\mathrm{rd}} G_{q_{\mathrm{d}},q_{\mathrm{r}}}^D \sqrt{\mathcal{P}_{\mathrm{r}} \alpha_l} \end{bmatrix}^T$ and

$$\tilde{\boldsymbol{w}}_l(n) = \begin{bmatrix} h_{\mathrm{sd}} \sum_{m=l+1}^{L} \sqrt{\mathcal{P}_{\mathrm{s}} \alpha_m} x_m(n) + w_{\mathrm{d},1}(n) \\ h_{\mathrm{rd}} G_{q_{\mathrm{d}},q_{\mathrm{r}}}^D \sum_{m=l+1}^{q_{\mathrm{r}}} \sqrt{\mathcal{P}_{\mathrm{r}} \alpha_m} x_m(n) + w_{\mathrm{d},2}(n) \end{bmatrix}. \tag{55}$$

The final effective SNR is then obtained as [27]

$$\eta_{\mathrm{f},l} = \tilde{\boldsymbol{h}}_l^H \boldsymbol{C}_{\tilde{\boldsymbol{w}}_l}^{-1} \tilde{\boldsymbol{h}}_l, \tag{56}$$

where $\boldsymbol{C}_{\tilde{\boldsymbol{w}}_l} = E\{\tilde{\boldsymbol{w}}_l(n) \tilde{\boldsymbol{w}}_l^H(n)\}$. From (57) and (58) for $l \in \mathbb{N}_{q_{\mathrm{d}}+1}^{q_{\mathrm{r}}}$, we have (12).

B. Prop-AF

With $z(n)$ given in (14), we have

$$y_{\mathrm{d},2}(n) = h_{\mathrm{eq}}^A \sum_{l=\min(q_{\mathrm{d}},q_{\mathrm{r}})+1}^{L} \sqrt{\mathcal{P}_{\mathrm{s}} \alpha_l} x_l(n) + \tilde{w}_{\mathrm{d},2}(n), \tag{59}$$

where $h_{\mathrm{eq}}^A = h_{\mathrm{sr}} h_{\mathrm{rd}} G_{\min(q_{\mathrm{d}},q_{\mathrm{r}})}^A$ and $\tilde{w}_{\mathrm{d},2}(n) = h_{\mathrm{rd}} G_{\min(q_{\mathrm{d}},q_{\mathrm{r}})}^A w_{\mathrm{d},1}(n) + w_{\mathrm{d},2}(n)$. Hence, for Prop-AF, we again have (54), now with $\tilde{\boldsymbol{h}}_l = \begin{bmatrix} h_{\mathrm{sd}} \sqrt{\mathcal{P}_{\mathrm{s}} \alpha_l} & h_{\mathrm{eq}}^A \sqrt{\mathcal{P}_{\mathrm{s}} \alpha_l} \end{bmatrix}^T$ and

$$\tilde{\boldsymbol{w}}_l(n) = \begin{bmatrix} h_{\mathrm{sd}} \sum_{m=l+1}^{L} \sqrt{\mathcal{P}_{\mathrm{s}} \alpha_m} x_m(n) + w_{\mathrm{d},1}(n) \\ h_{\mathrm{eq}}^A \sum_{m=l+1}^{L} \sqrt{\mathcal{P}_{\mathrm{s}} \alpha_m} x_m(n) + \tilde{w}_{\mathrm{d},2}(n) \end{bmatrix}. \tag{60}$$

We thus have (61) and (62) for $l \in \mathbb{N}_{q_{\mathrm{d}}+1}^L$, leading to (15).

APPENDIX II. UPPER BOUNDS ON $P_{l,i}^D(j)$, $P_{l,i*}^A$, AND $P_{l,*j}^A$

Since $\Phi_{j,L}(x)$ is a monotonically increasing concave function for $x > 0$, we have

$$\Phi_{j,L}(x) \leq \min \left\{ \varphi_{j,L}(x; \Gamma_{i,L}), \varphi_{j,L}(x; \Gamma_{i+1,L}) \right\} \tag{63}$$

for $\Gamma_{i,L} < x \leq \Gamma_{i+1,L}$, where $\varphi_{j,L}(x; a) = \Phi_{j,L}'(a)(x-a) + \Phi_{j,L}(a)$ is the tangent line to $\Phi_{j,L}(x)$ at $x = a$. From (28), we then have an upper bound

$$\breve{P}_{l,i}^D(j) = \mathbb{P}\left[\Gamma_{i,L} < \gamma_{\mathrm{sd}} \leq \Gamma_j^+(i), \frac{\gamma_{\mathrm{rd}}}{\bar{\alpha}_{i+1,j}} > \Gamma_{l,j} - \frac{\varphi_{j,L}(\gamma_{\mathrm{sd}}, \Gamma_{i,L})}{\alpha_j} \right]$$
$$+ \mathbb{P}\left[\Gamma_j^+(i) < \gamma_{\mathrm{sd}} \leq \Gamma_{i+1,L}, \right.$$
$$\left. \frac{\gamma_{\mathrm{rd}}}{\bar{\alpha}_{i+1,j}} > \Gamma_{l,j} - \frac{\varphi_{j,L}(\gamma_{\mathrm{sd}}, \Gamma_{i+1,L})}{\alpha_j} \right], \tag{64}$$

$$\breve{P}_{l,i*}^A = \mathbb{P}\left[\Gamma_{i,L} < \gamma_{\rm sd} \le \Gamma_{i+1,L}, \ \gamma_{\rm sr} > \Gamma_{i+1,L}, \ \gamma_{\rm sd} + \gamma_{\rm sr} > \Gamma_{l,L}, \ \gamma_{\rm sd} + \frac{\gamma_{\rm rd}}{\bar{\alpha}_{i+1,L}} > \Gamma_{l,L}\right]$$

$$= \int_{\Gamma_{i,L}}^{\tau_{li}}\int_{\Gamma_{l,L}-x}^{\infty}\int_{(\Gamma_{l,L}-x)\bar{\alpha}_{i+1,L}}^{\infty}\frac{1}{\Omega_{\rm sd}\Omega_{\rm sr}\Omega_{\rm rd}}e^{-\left(\frac{x}{\Omega_{\rm sd}}+\frac{y}{\Omega_{\rm sr}}+\frac{z}{\Omega_{\rm rd}}\right)}dzdydx$$

$$+ \int_{\tau_{li}}^{\Gamma_{i+1,L}}\int_{\Gamma_{i+1,L}}^{\infty}\int_{(\Gamma_{l,L}-x)\bar{\alpha}_{i+1,L}}^{\infty}\frac{1}{\Omega_{\rm sd}\Omega_{\rm sr}\Omega_{\rm rd}}e^{-\left(\frac{x}{\Omega_{\rm sd}}+\frac{y}{\Omega_{\rm sr}}+\frac{z}{\Omega_{\rm rd}}\right)}dzdydx$$

$$= \frac{1}{\zeta_{i2}}g\left(\frac{\zeta_{i2}\Gamma_{i,L}}{\Omega_{\rm sd}}, \frac{\zeta_{i2}\tau_{li}}{\Omega_{\rm sd}}\right)g\left(\frac{\Gamma_{l,L}}{\Omega_{\rm sr}}+\frac{\bar{\alpha}_{i+1,L}\Gamma_{l,L}}{\Omega_{\rm rd}}, \infty\right)$$

$$+ \frac{1}{\zeta_{i1}}g\left(\frac{\zeta_{i1}\tau_{li}}{\Omega_{\rm sd}}, \frac{\zeta_{i1}\Gamma_{i,L}}{\Omega_{\rm sd}}\right)g\left(\frac{\Gamma_{i+1,L}}{\Omega_{\rm sr}}+\frac{\bar{\alpha}_{i+1,L}\Gamma_{l,L}}{\Omega_{\rm rd}}, \infty\right), \tag{67}$$

$$\breve{P}_{l,*j}^A = \mathbb{P}\left[\Gamma_{j,L} < \gamma_{\rm sd} \le \Gamma_{l,L}, \ \Gamma_{j,L} < \gamma_{\rm sr} \le \Gamma_{j+1,L}, \gamma_{\rm sd}+\gamma_{\rm sr} > \Gamma_{l,L}, \ \gamma_{\rm sd}+\frac{\gamma_{\rm rd}}{\bar{\alpha}_{j+1,L}} > \Gamma_{l,L}\right]$$

$$= \frac{1}{\zeta_{j2}}g\left(\frac{\zeta_{j2}\tau_{lj1}}{\Omega_{\rm sd}}, \frac{\zeta_{j2}\tau_{lj2}}{\Omega_{\rm sd}}\right)g\left(\frac{\Gamma_{l,L}}{\Omega_{\rm sr}}+\frac{\bar{\alpha}_{j+1,L}\Gamma_{l,L}}{\Omega_{\rm rd}}, \infty\right)$$

$$- \frac{1}{\zeta_{j1}}g\left(\frac{\zeta_{j1}\tau_{lj1}}{\Omega_{\rm sd}}, \frac{\zeta_{j1}\tau_{lj2}}{\Omega_{\rm sd}}\right)g\left(\frac{\Gamma_{j+1,L}}{\Omega_{\rm sr}}+\frac{\bar{\alpha}_{j+1,L}\Gamma_{l,L}}{\Omega_{\rm rd}}, \infty\right)$$

$$+ \frac{1}{\zeta_{j1}}g\left(\frac{\zeta_{j1}\tau_{lj2}}{\Omega_{\rm sd}}, \frac{\zeta_{j1}\Gamma_{l,L}}{\Omega_{\rm sd}}\right)g\left(\frac{\Gamma_{j,L}}{\Omega_{\rm sr}}+\frac{\bar{\alpha}_{j+1,L}\Gamma_{l,L}}{\Omega_{\rm rd}}, \frac{\Gamma_{j+1,L}}{\Omega_{\rm sr}}+\frac{\bar{\alpha}_{j+1,L}\Gamma_{l,L}}{\Omega_{\rm rd}}\right), \tag{68}$$

on $P_{l,i}^D(j)$, where $\Gamma_j^+(i)$ is the solution x to $\varphi_{j,L}(x; \Gamma_{i,L}) = \varphi_{j,L}(x; \Gamma_{i+1,L})$. When $l > i$, since $\Phi_{j,L}(\Gamma_{l,L}) = \frac{\alpha_l\kappa_l}{\alpha_l - \bar{\alpha}_{i+1,L}\kappa_l + \bar{\alpha}_{j+1,L}\kappa_l} = \alpha_j\Gamma_{l,j}$ and $\Phi_{j,L}(\Gamma_{l,L}) \ge \min\{\varphi_{j,L}(x; \Gamma_{i,L}), \varphi_{j,L}(x; \Gamma_{i+1,L})\}$ for $\Gamma_{i,L} < x \le \Gamma_{i+1,L}$, we have $\Gamma_{l,j} - \frac{\varphi_{j,L}(x,\Gamma_{i,L})}{\alpha_j} \ge 0$ for $\Gamma_{i,L} < x \le \Gamma_j^+(i)$ and $\Gamma_{l,j} - \frac{\varphi_{j,L}(x,\Gamma_{i+1,L})}{\alpha_j} \ge 0$ for $\Gamma_j^+(i) < x \le \Gamma_{i+1,L}$. Therefore, (64) can be rewritten

$$\breve{P}_{l,i}^D(j) = \int_{\Gamma_{i,L}}^{\Gamma_j^+(i)}\frac{1}{\Omega_{\rm sd}}e^{-\frac{x}{\Omega_{\rm sd}}-\frac{\bar{\alpha}_{i+1,j}\left\{\Phi_{j,L}(\Gamma_{l,L})-\varphi_{j,L}(x;\Gamma_{i,L})\right\}}{\alpha_j\Omega_{\rm rd}}}dx$$

$$+ \int_{\Gamma_j^+(i)}^{\Gamma_{i+1,L}}\frac{1}{\Omega_{\rm sd}}e^{-\frac{x}{\Omega_{\rm sd}}-\frac{\bar{\alpha}_{i+1,j}\left\{\Phi_{j,L}(\Gamma_{l,L})-\varphi_{j,L}(x;\Gamma_{i,L})\right\}}{\alpha_j\Omega_{\rm rd}}}dx$$

$$= \frac{1}{\vartheta_{ij0}}g\left(\frac{\vartheta_{ij0}\Gamma_{i,L}}{\Omega_{\rm sd}}, \frac{\vartheta_{ij0}\Gamma_j^+(i)}{\Omega_{\rm sd}}\right)g\left(\delta_{lij0}, \infty\right)$$

$$+ \frac{1}{\vartheta_{ij1}}g\left(\frac{\vartheta_{ij1}\Gamma_j^+(i)}{\Omega_{\rm sd}}, \frac{\vartheta_{ij1}\Gamma_{i+1,L}}{\Omega_{\rm sd}}\right)g\left(\delta_{lij1}, \infty\right), \tag{65}$$

with $\Gamma_{l,j} = \frac{\Phi_{j,L}(\Gamma_{l,L})}{\alpha_j}$, where $g(x,y) = e^{-x} - e^{-y}$, $\vartheta_{ijn} = 1 - \Phi_{j,L}'(\Gamma_{i+n,L})\frac{\bar{\alpha}_{i+1,j}}{\alpha_j}\frac{\Omega_{\rm sd}}{\Omega_{\rm rd}}$, and $\delta_{lijn} = \frac{\bar{\alpha}_{i+1,j}}{\alpha_j\Omega_{\rm rd}}\{\Phi_{j,L}(\Gamma_{l,L}) + \Phi_{j,L}(\Gamma_{i+n,L}) - \Gamma_{i+n,L}\Phi_{j,L}'(\Gamma_{i+n,L})\}$ for $n = 0, 1$.

Next, by applying the inequality

$$\frac{\gamma_{\rm sr}\gamma_{\rm rd}}{\gamma_{\rm sr}c + \gamma_{\rm rd} + 1} \le \min\left(\gamma_{\rm sr}, \frac{\gamma_{\rm rd}}{c}\right) \tag{66}$$

for $c > 0$ to (31), we can obtain an upper bound on $P_{l,i*}^A$ as (67), where $\zeta_{i1} = 1 - \bar{\alpha}_{i+1,L}\frac{\Omega_{\rm sd}}{\Omega_{\rm rd}}$, $\zeta_{i2} = \zeta_{i1} - \frac{\Omega_{\rm sd}}{\Omega_{\rm sr}}$, and $\tau_{li} = \min\{\max(\Gamma_{l,L} - \Gamma_{i+1,L}, \Gamma_{i,L}), \Gamma_{i+1,L}\}$. Note that $\Gamma_{i,L} \le \tau_{li} \le \Gamma_{i+1,L}$, and therefore, $\{\gamma_{\rm sr} > \Gamma_{i+1,L}\} \cap \{\gamma_{\rm sd} + \gamma_{\rm sr} > \Gamma_{l,L}\} = \{\gamma_{\rm sd} + \gamma_{\rm sr} > \Gamma_{l,L}\}$ when $\gamma_{\rm sd} \in [\Gamma_{i,L}, \tau_{li}]$, and $\{\gamma_{\rm sr} > \Gamma_{i+1,L}\} \cap \{\gamma_{\rm sd} + \gamma_{\rm sr} > \Gamma_{l,L}\} = \{\gamma_{\rm sr} > \Gamma_{i+1,L}\}$ when $\gamma_{\rm sd} \in [\tau_{li}, \Gamma_{i+1,L}]$. An upper bound on $P_{l,*j}^A$ can next be obtained in a similar way as (68), where $\tau_{lj1} = \max(\Gamma_{l,L} - \Gamma_{j+1,L}, \Gamma_{j,L})$ and $\tau_{lj2} = \max(\Gamma_{l,L} - \Gamma_{j,L}, \Gamma_{j,L})$.

APPENDIX III. PROOF OF THEOREM 1

Let us first derive upper bounds on the outage probability $P_{\rm out,l}$ using

$$\mathbb{P}[X + Y > c] \ge \mathbb{P}[X \le c, Y > c] + \mathbb{P}[X > c] \tag{69}$$

for nonnegative random variables X and Y and a positive constant c. For DF signals, let us obtain a lower bound $\tilde{P}_{l,i}^D(j)$ on $P_{l,i}^D(j)$ shown in (28), in order to obtain an upper bound on $P_{\rm out,l}$ shown in (29). Applying (69) with $X = \frac{\Phi_{j,L}(\gamma_{\rm sd})}{\alpha_j}$ and $Y = \frac{\gamma_{\rm rd}}{\bar{\alpha}_{i+1,j}}$ in (28), we have

$$\tilde{P}_{l,i}^D(j) = \mathbb{P}\left[\mathcal{E}_{d,i} \cap \{\Phi_{j,L}(\gamma_{\rm sd}) \le \alpha_j\Gamma_{l,j}, \gamma_{\rm rd} > \bar{\alpha}_{i+1,j}\Gamma_{l,j}\}\right]$$
$$+ \mathbb{P}\left[\mathcal{E}_{d,i} \cap \{\Phi_{j,L}(\gamma_{\rm sd}) > \alpha_j\Gamma_{l,j}\}\right]. \tag{70}$$

Since $\{\Phi_{j,L}(\gamma_{\rm sd}) \le \alpha_j\Gamma_{l,j}\} = \{\gamma_{\rm sd} \le \Gamma_{l,L}\}$ and $\{\Phi_{j,L}(\gamma_{\rm sd}) > \alpha_j\Gamma_{l,j}\} = \{\gamma_{\rm sd} > \Gamma_{l,L}\}$ from $\alpha_j\Gamma_{l,j} = \Phi_{j,L}(\Gamma_{l,L})$, $\bar{\alpha}_{i+1,j} \approx \bar{\alpha}_{i+1,L}$, and $\Gamma_{l,j} \approx \Gamma_{l,L}$ in the high SNR region as observed in (44), we can derive (70) as

$$\tilde{P}_{l,i}^D(j) = \mathbb{P}\left[\mathcal{E}_{d,i} \cap \{\gamma_{\rm sd} \le \Gamma_l, \ \gamma_{\rm rd} > \bar{\alpha}_{i+1,j}\Gamma_{l,j}\}\right]$$
$$\approx g\left(\frac{\Gamma_{i,L}}{\Omega_{\rm sd}}, \frac{\Gamma_{i+1,L}}{\Omega_{\rm sd}}\right)g\left(\frac{\bar{\alpha}_{i+1,L}\Gamma_{l,L}}{\Omega_{\rm rd}}, \infty\right). \tag{71}$$

For AF signals, lower bounds $\tilde{P}_{l,i*}^A$ and $\tilde{P}_{l,*j}^A$ on (31) and (33), respectively, can be obtained by setting $X = \gamma_{\rm sd}$ and $Y = \frac{\gamma_{\rm sr}\gamma_{\rm rd}}{\bar{\alpha}_{i+1,L}\gamma_{\rm sr}+\gamma_{\rm rd}+1}$ in (69) and then applying

$$\frac{\gamma_{\rm sr}\gamma_{\rm rd}}{\gamma_{\rm sr}c+\gamma_{\rm rd}+1} \ge \frac{1}{2}\min\left(\gamma_{\rm sr}, \frac{\gamma_{\rm rd}-1}{c}\right) \tag{72}$$

for $c > 0$. Specifically, we get

$$\tilde{P}_{l,i*}^A = \mathbb{P}\Big[\Gamma_{i,L} < \gamma_{\rm sd} \le \Gamma_{i+1,L}, \ \gamma_{\rm sr} > \Gamma_{i+1,L},$$
$$\gamma_{\rm sd} \le \Gamma_{l,L}, \ \tfrac{1}{2}\min\left(\gamma_{\rm sr}, \tfrac{\gamma_{\rm rd}-1}{\bar{\alpha}_{i+1,L}}\right) > \Gamma_{l,L}\Big]$$
$$= g\left(\frac{\Gamma_{i,L}}{\Omega_{\rm sd}}, \frac{\Gamma_{i+1,L}}{\Omega_{\rm sd}}\right)g\left(\frac{2\Gamma_{l,L}}{\Omega_{\rm sr}}+\frac{2\bar{\alpha}_{i+1,L}\Gamma_{l,L}+1}{\Omega_{\rm rd}}, \infty\right) \tag{73}$$

and

$$\tilde{P}_{l,*j}^A = \mathbb{P}\Big[\Gamma_{j,L} < \gamma_{\rm sd} \le \Gamma_{l,L}, \ \Gamma_{j,L} < \gamma_{\rm sr} \le \Gamma_{j+1,L},$$
$$\gamma_{\rm sd} \le \Gamma_{l,L}, \ \tfrac{1}{2}\min\left(\gamma_{\rm sr}, \tfrac{\gamma_{\rm rd}-1}{\bar{\alpha}_{j+1,L}}\right) > \Gamma_{l,L}\Big] = 0. \tag{74}$$

With the lower bounds (71), (73), and (74), the outage probabilities (29), (34), and (35) of Prop-DF, Prop-AF, and Prop-MF can be approximated in the high SNR region as

$$P_{\text{out},l} \approx 1 - e^{-\frac{\Gamma_{l,L}}{\Omega_{\text{sd}}}}$$
$$- \sum_{i=0}^{l-1} g\left(\frac{\Gamma_{i,L}}{\Omega_{\text{sd}}}, \frac{\Gamma_{i+1,L}}{\Omega_{\text{sd}}}\right) g\left(\frac{\mu\Gamma_{l,L}}{\Omega_{\text{sr}}} + \frac{\mu\bar{\alpha}_{i+1,L}\Gamma_{l,L}}{\Omega_{\text{rd}}}, \infty\right), (75)$$

where $\mu = 1$ or 2. When $\Omega_{\text{sd}} = \Omega$, $\Omega_{\text{sr}} = \rho_1\Omega$, and $\Omega_{\text{rd}} = \rho_2\Omega$, if we consider only the dominant terms in the Taylor series expansion of (75) in the high SNR region, we get

$$P_{\text{out},l} \approx \frac{\mu\Gamma_{l,L}^2}{\rho_1\Omega^2} + \sum_{i=0}^{l-1} \frac{\mu\bar{\alpha}_{i+1,L}\Gamma_{i+1,L}\Gamma_{l,L}}{\rho_2\Omega^2}$$
$$\approx \frac{\mu}{\rho_1}\Omega^{2\epsilon_{l-1}-2+4\sum_{m=1}^{l}r_m} + \frac{\mu}{\rho_2}\sum_{i=1}^{l}\Omega^{\epsilon_{l-1}-2+2r_i+2\sum_{m=1}^{l}r_m}, (76)$$

where all possible candidates for the lowest power of Ω^{-1} are preserved, since the dominant term varies depending on the multiplexing gain vector $\boldsymbol{r}$ and (ρ_1, ρ_2).

When $\varsigma_{\text{sr}} \to \varsigma_{\text{sd}}$ (i.e., $\rho_1 \to 1$ and $\rho_2 \gg \rho_1$), the first term in (76) dominates, leading to the successive decoding diversity gain $d_l(\boldsymbol{r}) = 2 - 4\sum_{m=1}^{l} r_m - 2\epsilon_{l-1}$. When $\varsigma_{\text{sr}} \to 0$ (i.e., $\rho_1 \gg \rho_2$ and $\rho_2 \to 1$), on the other hand, the second term in (76) dominates, leading to $d_l(\boldsymbol{r}) = 2 - 2\max_{1\leq i \leq l} r_i - 2\sum_{m=1}^{l} r_m - \epsilon_{l-1}$. Letting $\epsilon_{l-1} \to 0$, we get (45).

APPENDIX IV. PROOF OF THEOREM 2

With the SDDG for $\varsigma_{\text{sr}} \to \varsigma_{\text{sd}}$ given in (45), we have $\Delta_l = 2 - (4-b)\sum_{m=1}^{l-1} r_m - 4r_l$ for $l \in \mathbb{N}_1^L$ and $\Delta_{L+1} = b\sum_{m=1}^{L} r_m$. It is easy to see that $\{\Delta_l\}_{l=1}^{L}$ are active[1] in the feasible set $\mathcal{F} = \{\boldsymbol{r} : \boldsymbol{r} \geq 0\}$ since $\Delta_{l+1} - \Delta_l = br_l - 4r_{l+1}$ for $l \in \mathbb{N}_1^{L-1}$ and $\Delta_{L+1} - \Delta_L = br_L + 4\sum_{m=1}^{L} r_m - 2$. Now, when $l \in \mathbb{N}_1^L$, Δ_l is a decreasing function of r_l while Δ_{L+1} is an increasing function of r_l. Hence, to maximize $\min\left(\{\Delta_l\}_{l=1}^{L+1}\right)$, r_l should satisfy $\Delta_l = \Delta_{L+1}$ for $l \in \mathbb{N}_1^L$. The solution of such L equations is given by $r_l = \left(\frac{b}{4}\right)^{l-1} r_1$ for $l \in \mathbb{N}_2^L$ and $r_1 = \left\{2\sum_{m=0}^{L}\left(\frac{b}{4}\right)^m\right\}^{-1}$, leading to (47).

APPENDIX V. PROOF OF THEOREM 3

With the SDDG for $\varsigma_{\text{sr}} \to 0$ given in (45), we have $\Delta_l = 2 - 2\max_{i \in \mathbb{N}_1^l} r_i - (2-b)\sum_{m=1}^{l-1} r_m - 2r_l$ for $l \in \mathbb{N}_1^L$ and $\Delta_{L+1} = b\sum_{m=1}^{L} r_m$. To solve the optimization problem (46), we first subdivide the set $\mathcal{F} = \{\boldsymbol{r} : \boldsymbol{r} \geq 0\}$ as $\mathcal{F} = \bigcup_{s=1}^{L} \mathcal{F}(s)$, where $\mathcal{F}(s) = \{\boldsymbol{r} : \boldsymbol{r} \geq 0, r_s \geq r_l, l \in \mathbb{N}_1^L\}$. Let us next find the solutions $\Delta(s) = \max_{\boldsymbol{r} \in \mathcal{F}(s)} \min_{l \in \mathbb{N}_1^{L+1}} \Delta_l$ and

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[1]If there is any $\Delta_j, j \neq l$ such that $\Delta_j \leq \Delta_l$ for all possible values of $\boldsymbol{r} \in \mathcal{S}$, Δ_l is called inactive in the set $\mathcal{S}$. Otherwise, Δ_l is called active in the set $\mathcal{S}$.

$$\boldsymbol{r}(s) = [r_1(s)\ r_2(s)\ \cdots\ r_L(s)] = \arg\max_{\boldsymbol{r} \in \mathcal{F}(s)} \min_{l \in \mathbb{N}_1^{L+1}} \Delta_l \text{ for}$$

each subset $\mathcal{F}(s)$.

When $\boldsymbol{r} \in \mathcal{F}(s)$, we can rewrite Δ_l as

$$\Delta_l = \begin{cases} 2 - 2r_{i(l)} - (2-b)\sum_{m=1}^{l-1} r_m - 2r_l, & l \in \mathbb{N}_1^{s-1}, \\ 2 - 2r_s - (2-b)\sum_{m=1}^{l-1} r_m - 2r_l, & l \in \mathbb{N}_s^L, \end{cases} (77)$$

where $r_s = \max_{l \in \mathbb{N}_1^L} r_l$ and $i(l) = \arg\max_{m \in \mathbb{N}_1^l} r_m$. Note that, since $\max_{m \in \mathbb{N}_1^l} r_m = \max(r_{i(l-1)}, r_l)$, we have $i(1) = 1$, $i(s) = s$, and $i(l) = l$ or $i(l-1)$ for $l \in \mathbb{N}_2^{s-1}$. Observe from (77) that Δ_l can be inactive for some b in $\mathcal{F}(s)$: We will obtain $\Delta(s)$ and $\boldsymbol{r}(s)$, and then, $\Delta = \max_{s \in \mathbb{N}_1^L} \Delta(s)$ in three distinct cases as follows:

Case *1:* $0 < b < 2$

When $l \in \mathbb{N}_1^{s-1}$, we have $\Delta_l - \Delta_s = 4r_s - 2r_l - 2r_{i(l)} + (2-b)\sum_{m=l}^{s-1} r_m \geq 0$ since $r_s \geq r_l$ and $0 < b < 2$. Therefore, $\{\Delta_l\}_{l=1}^{s-1}$ are inactive in $\mathcal{F}(s)$. When $l \in \mathbb{N}_{s+1}^L$, the difference $\Delta_l - \Delta_s = -2r_l + 2r_s + (b-2)\sum_{m=s}^{l-1} r_m$ can be either nonnegative or negative in $\mathcal{F}(s)$: For example, $\Delta_l - \Delta_s = br_s \geq 0$ if $r_m = 0$ for $m \in \mathbb{N}_{s+1}^l$, and $\Delta_l - \Delta_s = (b-2)(l-s)r_s \leq 0$ if $r_m = r_s$ for $m \in \mathbb{N}_{s+1}^l$. Similarly, for $l_1, l_2 \in \mathbb{N}_{s+1}^L$ and $l_1 < l_2$, the difference $\Delta_{l_2} - \Delta_{l_1} = -2(r_{l_2} - r_{l_1}) - (2-b)\sum_{m=l_1}^{l_2-1} r_m$ can be either nonnegative or negative in $\mathcal{F}(s)$. In addition, when $l \in \mathbb{N}_s^L$, we have $\Delta_{L+1} - \Delta_l = b\sum_{m=l}^{L} r_m + 2\sum_{m=1}^{l} r_m + 2r_s - 2$, which can also be either nonnegative or negative. Therefore, the $\{\Delta_l\}_{l=s}^{L+1}$ are active in $\mathcal{F}(s)$ for $0 < b < 2$, with which the optimization problem (46) can be restated as

$$\Delta(s) = \max_{\{\boldsymbol{r}:r_s \geq r_l \geq 0\}} \min(\Delta_s, \Delta_{s+1}, \cdots, \Delta_L, \Delta_{L+1}). \quad (78)$$

Here, the optimal r_l should satisfy $\Delta_l = \Delta_{L+1}$ for $l \in \mathbb{N}_s^L$, since Δ_l is a decreasing function of r_l for $l \in \mathbb{N}_s^L$, while Δ_{L+1} is an increasing function of r_l. Now, $\Delta_{L+1} = b\sum_{l=1}^{L} r_l$ can be maximized in $\mathcal{F}(s)$ when $r_l = r_s$ for $l \in \mathbb{N}_1^{s-1}$, since $r_s \geq r_l$. Thus, from (77), the solution of (78) is given by

$$\begin{aligned} \Delta(s) &= 2 - \{2 + b + (2-b)s\}r_s(s) \\ &= 2 - \frac{2 + b + (2-b)s}{1 + s + \sum_{m=1}^{L-s+1}\left(\frac{b}{2}\right)^m} \end{aligned} \quad (79)$$

with

$$r_l(s) = \begin{cases} \left\{1 + s + \sum_{m=1}^{L-s+1}\left(\frac{b}{2}\right)^m\right\}^{-1}, & l \in \mathbb{N}_1^s, \\ \left(\frac{b}{2}\right)^{l-s} r_1(s), & l \in \mathbb{N}_{s+1}^L. \end{cases} \quad (80)$$

We now obtain the solution $\Delta = \max_{s \in \mathbb{N}_1^L} \Delta(s)$ in $\mathcal{F}$. Let

$$\Delta(s+1) - \Delta(s) = \frac{A(s)}{B(s)B(s+1)}, \text{ where } A(s) = b -$$

$\{2 + (2-b)s\} \left(\frac{b}{2}\right)^{L-s+1}$ and $B(s) = 1 + s + \sum_{l=1}^{L-s+1} \left(\frac{b}{2}\right)^{l}$.
Observe that $\Delta(s+1) - \Delta(s)$ is a monotonically decreasing function of s for $s \in \mathbb{N}_1^{L-1}$ when $0 < b < 2$, since $A(s)$ is a decreasing function of s and $B(s)$ is an increasing function of s, since $A(s+1) - A(s) = \left(\frac{b}{2} - 1\right)\left(\frac{b}{2}\right)^{L-s}\{4 + (2-b)s\} < 0$ and $B(s+1) - B(s) = 1 - \left(\frac{b}{2}\right)^{L-s+1} > 0$ for $s \in \mathbb{N}_1^{L-1}$. Thus, if $A(s_o) > 0$ for $s_0 \in \mathbb{N}_1^{L-1}$, we have $\Delta(s) < \Delta(s_o+1)$ for $s \in \mathbb{N}_1^{s_o}$, and we have $\Delta(s) < \Delta(s_o)$ for $s \in \mathbb{N}_{s_o+1}^{L}$ if $A(s_o) < 0$ for $s_0 \in \mathbb{N}_1^{L-1}$. The optimal value $\Delta = \Delta(s^*)$ is then obtained when $s^* = L$ if $A(L-1) > 0$ and $s^* = \min_{\{s \in \mathbb{N}_1^{L-1}: A(s) < 0\}} s$ if $A(L-1) \le 0$, which is equivalent to (49).

Case *2:* $b = 2$

If $b = 2$, (77) becomes $\Delta_l = 2 - 2r_{i(l)} - 2r_l$ for $l \in \mathbb{N}_1^{s-1}$ and $\Delta_l = 2 - 2r_s - 2r_l$ for $l \in \mathbb{N}_s^{L}$. Therefore, $\Delta_s - \Delta_l = 2r_{i(l)} + 2r_l - 4r_s$ for $l \in \mathbb{N}_1^{s-1}$ and $\Delta_s - \Delta_l = 2r_l - 2r_s$ for $l \in \mathbb{N}_{s+1}^{L}$, resulting in $\Delta_s \le \Delta_l$ for all $l \in \mathbb{N}_1^{L}$, since $r_s \ge r_l$ for all $l \in \mathbb{N}_1^{L}$. The optimization problem then becomes $\Delta(s) = \max_{\{r:r_s \ge r_l \ge 0\}} \min(\Delta_s, \Delta_{L+1})$, for which the solution occurs when $\Delta_s = \Delta_{L+1}$ and $r_1 = r_2 = \cdots = r_L$ from the same reasoning as that for $0 < b < 2$: In essence, the solution for $b = 2$ is given by

$$\Delta(s) = 2 - 4r_s(s) = 2 - \frac{4}{2+L} \tag{81}$$

with $r_l(s) = \frac{1}{2+L}$ for $l \in \mathbb{N}_1^{L}$. Thus, the solution $\Delta = \max_{s \in \mathbb{N}_1^{L}} \Delta(s)$ in $\mathcal{F}$ is $\Delta = 2 - \frac{4}{2+L}$.

Case *3:* $b > 2$

We first observe that $\{\Delta_l\}_{l=s+1}^{L}$ are inactive in $\mathcal{F}(s)$, since

$\Delta_l - \Delta_s = 2(r_s - r_l) + (b-2)\sum_{m=s}^{l-1} r_m > 0$ from $r_s \ge r_l > 0$ and $b > 2$. Then, the optimization problem when $b > 2$ is restated as

$$\Delta(s) = \max_{\{r:r_s \ge r_l \ge 0\}} \min(\Delta_1, \Delta_2, \cdots, \Delta_s, \Delta_{L+1}). \tag{82}$$

Next observe that Δ_{L+1} is active with $\{\Delta_l\}_{l=1}^{s}$ in $\mathcal{F}(s)$, since the difference $\Delta_{L+1} - \Delta_l = b\sum_{m=l}^{L} r_m + 2\sum_{m=1}^{l} r_m + 2r_{i(l)} - 2$ can be either nonnegative or negative regardless of the value of $i(l)$ for $l \in \mathbb{N}_1^{s}$ if $b > 2$. On the other hand, the activeness of $\{\Delta_l\}_{l=1}^{s}$ depends on the values of $\{i(l)\}_{l=2}^{s-1}$. We thus have to investigate the activeness of $\{\Delta_l\}_{l=1}^{s}$ for all possible cases of $\{i(l)\}_{l=2}^{s-1}$ and derive the solution in each case. To facilitate this, define $\mathcal{I}_s = \{l : i(l) = l, 1 \le l \le s\}$ and its complement $\bar{\mathcal{I}}_s = \mathbb{N}_1^{s} - \mathcal{I}_s$: Note that $\{1, s\} \subseteq \mathcal{I}_s \subseteq \mathbb{N}_1^{s}$ since $i(1) = \arg\max_{m \in \mathbb{N}_1^{1}} r_m = 1$ and $i(s) = \arg\max_{m \in \mathbb{N}_1^{s}} r_m = s$. For notational convenience, arrange the elements of $\mathcal{I}_s$ as $\mathcal{I}_s = \{k_1, k_2, \cdots, k_{|\mathcal{I}_s|}\}$, where $k_1 = 1$, $k_{|\mathcal{I}_s|} = s$, and $k_m < k_{m+1}$ with $|\mathcal{I}_s|$ the cardinality of $\mathcal{I}_s$. In fact, $\mathcal{F}(s)$ can be partitioned with all possible cases of $\mathcal{I}_s$. For example, $\mathcal{I}_s = \mathbb{N}_1^{s}$ corresponds to the case $r_1 < r_2 < \cdots < r_s$ while $\mathcal{I}_s = \{1, s\}$ corresponds to the case $r_s \ge r_1 \ge r_l$ for all $l \in \mathbb{N}_2^{s-1}$.

Let $\mathcal{F}_{\mathcal{I}_s}(s)$ be the set of r in $\mathcal{F}(s)$ leading to $\mathcal{I}_s$: Specifically, $\mathcal{F}_{\mathcal{I}_s}(s) = \mathcal{F}(s) \cap \{r : \max_{m \in \mathbb{N}_1^{l}} r_m =$

r_l if $l \in \mathcal{I}_s$, $\max_{m \in \mathbb{N}_1^{l}} r_m = \max_{m \in \mathbb{N}_1^{l-1}} r_m$ if $l \in \bar{\mathcal{I}}_s\}$. By partitioning $\mathcal{F}(s)$ into $\{\mathcal{F}_{\mathcal{I}_s}(s), \forall \mathcal{I}_s\}$, we can obtain the solution when $b > 2$ as follows:

Lemma *1:* When $b > 2$, Δ_l is active (inactive) in $\mathcal{F}_{\mathcal{I}_s}(s)$ if $l \in \mathcal{I}_s$ (if $l \in \bar{\mathcal{I}}_s$).

Proof: Consider $l \in \bar{\mathcal{I}}_s$ such that $k_i < l < k_{i+1}$ for $i \in \mathbb{N}_1^{|\mathcal{I}_s|-1}$. Then, $\Delta_l - \Delta_{k_i} = 2(r_{k_i} - r_l) + (b-2)\sum_{m=k_i}^{l-1} r_m > 0$ since $r_{k_i} \ge r_l$ and $b > 2$. Hence, Δ_l for $l \in \bar{\mathcal{I}}_s$ is inactive in $\mathcal{F}_{\mathcal{I}_s}(s)$. On the other hand, when $k_i < k_j$, the difference $\Delta_{k_j} - \Delta_{k_i} = 4(r_{k_i} - r_{k_j}) + (b-2)\sum_{m=k_i}^{k_j-1} r_m$ can be either negative or positive from the following observations (O1) and (O2): (O1) Since $\Delta_{k_j} - \Delta_{k_i} \le 4(r_{k_{j-1}} - r_{k_j}) + (b-2)(k_j - k_i)r_{k_{j-1}}$, we have $\Delta_{k_j} - \Delta_{k_i} < 0$ if $r_{k_j} > \left\{1 + \frac{(b-2)(k_j-k_i)}{4}\right\} r_{k_{j-1}}$. (O2) Since $\Delta_{k_j} - \Delta_{k_i} \ge 4(r_{k_i} - r_{k_j}) + (b-2)r_{k_i}$, we have $\Delta_{k_j} - \Delta_{k_i} > 0$ if $r_{k_j} < \left(1 + \frac{b-2}{4}\right)r_{k_i}$. Since we can always find an r in $\mathcal{F}_{\mathcal{I}_s}(s)$ with either $r_{k_j} > \left\{1 + \frac{(b-2)(k_j-k_i)}{4}\right\} r_{k_{j-1}}$ or $r_{k_j} < \left(1 + \frac{b-2}{4}\right)r_{k_i}$, Δ_l for $l \in \mathcal{I}_s$ is active in $\mathcal{F}_{\mathcal{I}_s}(s)$. $\square$

Lemma *2:* The optimization problem $\Delta_{\mathcal{I}_s}(s) = \max_{r \in \mathcal{F}_{\mathcal{I}_s}(s)} \min(\Delta_1, \Delta_2, \cdots, \Delta_s, \Delta_{L+1})$ in partition $\mathcal{F}_{\mathcal{I}_s}(s)$ for $b > 2$ has the solution

$$\Delta_{\mathcal{I}_s}(s) = 2 - 4r_{1,\mathcal{I}_s}(s) \tag{83}$$

with

$$r_{1,\mathcal{I}_s}(s) = 2\left\{4 + b\sum_{m=1}^{s} \beta_m + b(L-s)\beta_s\right\}^{-1}, \tag{84}$$

$r_{l,\mathcal{I}_s}(s) = \beta_l r_{1,\mathcal{I}_s}(s)$ for $l \in \mathbb{N}_2^{s-1}$, and $r_{l,\mathcal{I}_s}(s) = \beta_s r_{1,\mathcal{I}_s}(s)$ for $l \in \mathbb{N}_s^{L}$. Here, $\beta_1 = 1$ and

$$\beta_l = \beta_{k_m} \tag{85}$$

for $l \in \mathbb{N}_{k_m}^{k_{m+1}-1}$ with $\beta_{k_m} = \prod_{i=1}^{m-1}\left\{1 - \frac{(2-b)(k_{i+1}-k_i)}{4}\right\}$ for $m \in \mathbb{N}_2^{|\mathcal{I}_s|}$.

Proof: From Lemma 1, the optimal solution occurs when $\Delta_l = \Delta_{L+1}$ for $l \in \mathcal{I}_s$, $r_l = r_{k_m}$ for $l \in \mathbb{N}_{k_m}^{k_{m+1}-1}$, and $r_l = r_s$ for $l \in \mathbb{N}_{s+1}^{L}$. Now, from $\Delta_{k_{m+1}} = \Delta_{k_m}$, we have $r_{k_{m+1}} = \left\{1 - \frac{(2-b)(k_{m+1}-k_m)}{4}\right\} r_{k_m}$ so that $r_{k_m} = \beta_{k_m} r_1$ with $\beta_{k_m} = \prod_{i=1}^{m-1}\left\{1 - \frac{(2-b)(k_{i+1}-k_i)}{4}\right\}$. Since $r_l = r_{k_m}$ for $l \in \mathbb{N}_{k_m}^{k_{m+1}-1}$, we have $r_l = \beta_l r_1$ with $\beta_l = \beta_{k_m}$ for $l \in \mathbb{N}_{k_m}^{k_{m+1}-1}$. In addition, from $r_l = r_s$ for $l \in \mathbb{N}_{s+1}^{L}$, we have $r_l = \beta_s r_1$ for $l \in \mathbb{N}_{s+1}^{L}$. By solving $\Delta_1 = \Delta_{L+1}$, i.e., $2 - 4r_1 = b\sum_{m=1}^{L} r_m$, we will get (84), leading to (83) from $\Delta_1 = 2 - 4r_1$. $\square$

Lemma *3:* The solution of $\Delta(s) = \max_{\mathcal{I}_s} \Delta_{\mathcal{I}_s}(s)$ for $b > 2$ is obtained with $\mathcal{I}_s = \mathbb{N}_1^{s}$ as

$$\Delta(s) = 2 - 4r_1(s) \tag{86}$$

with

$$r_1(s) = 2\left\{4 + b\sum_{m=1}^{s} \left(\frac{b+2}{4}\right)^{m-1} + b(L-s)\left(\frac{b+2}{4}\right)^{s-1}\right\}^{-1} \tag{87}$$

$r_l(s) = \left(\frac{b+2}{4}\right)^{l-1} r_1(s)$ if $l \in \mathbb{N}_2^{s-1}$, and $r_l(s) = \left(\frac{b+2}{4}\right)^{s-1} r_1(s)$ if $l \in \mathbb{N}_s^L$.

Proof: If $\mathcal{I}_s = \mathbb{N}_1^s$, we have $k_l = l$ for $l \in \mathbb{N}_1^s$ in (85), and consequently, $\beta_l = \left(\frac{2+b}{4}\right)^{l-1} = \beta_l^\star$. With $\beta_l^\star$, (83) and (84) become (86) and (87), respectively. We now prove $\beta_l^\star \geq \beta_l$ for all possible β_l given in (85) via induction. First, it is clear $\beta_1^\star \geq \beta_1$ since $\beta_1^\star = \beta_1 = 1$. Next, assume that $\beta_l^\star \geq \beta_l$ for $l \in \mathbb{N}_2^{k_m}$. Then, we have $\beta_l^\star \geq \beta_l$ for $l \in \mathbb{N}_{k_m+1}^{k_{m+1}-1}$ since $\beta_l^\star = \left(\frac{2+b}{4}\right)^{l-k_m}\beta_{k_m}^\star$, $\beta_l = \beta_{k_m}$, and $\left(\frac{2+b}{4}\right)^{l-k_m} > 1$ for $b > 2$. In addition, $\beta_{k_{m+1}}^\star \geq \beta_{k_{m+1}}$ since $\beta_{k_{m+1}}^\star = \left(\frac{2+b}{4}\right)^{k_{m+1}-k_m}\beta_{k_m}^\star$, $\beta_{k_{m+1}} = \left\{1 - \frac{(2-b)(k_{m+1}-k_m)}{4}\right\}\beta_{k_m}$, and $\left(1-\frac{2-b}{4}\right)^{k_{m+1}-k_m} > \left\{1 - \frac{(2-b)(k_{m+1}-k_m)}{4}\right\}$, where we have used $(1+x)^n \geq (1+nx)$ for $x > 0$ and any positive integer n. Therefore, $\beta_l^\star \geq \beta_l$ for $l \in \mathbb{N}_1^s$. It is then straightforward that $r_1(s) = 2\left\{4 + b\sum_{m=1}^{s}\beta_m^\star + b(L-s)\beta_s^\star\right\}^{-1} \leq r_{1,\mathcal{I}_s}(s) = 2\left\{4 + b\sum_{m=1}^{s}\beta_m + b(L-s)\beta_s\right\}^{-1}$ and thus $\Delta(s) \geq \Delta_{\mathcal{I}_s}(s)$. $\square$

Now, $r_1(s)$ in (87) is a decreasing function of s since $\frac{1}{r_1(s+1)} - \frac{1}{r_1(s)} = (L-s)\frac{b(b-2)}{8}\left(\frac{b+2}{4}\right)^{s-1} > 0$ for $s < L$. Thus, $\Delta(s) = 2 - 4r_1(s)$ is maximum at $s = L$, resulting in $\Delta = \max_{s \in \mathbb{N}_1^L}\Delta(s) = 2 - 4r_1(L) = 2 - 2\left\{1 + \frac{b}{4}\sum_{m=1}^{L}\left(\frac{b+2}{4}\right)^{m-1}\right\}^{-1}$ from (86) and (87).

APPENDIX VI. ANALYSIS ON SC-DF [10] AND SC-AF [11]

SC-DF with relay signal (7) has $\eta_{f,l} = \Phi_{l,L}(\gamma_{sd})$ if $l \in \mathbb{N}_{q_r+1}^L$ and $\eta_{f,l} = \Phi_{l,q_r}(\lambda_{q_r}^C)$ if $l \in \mathbb{N}_1^{q_r}$, where $\lambda_j^C = \frac{\Phi_{j,L}(\gamma_{sd})}{\alpha_j} + \frac{\gamma_{rd}}{\bar{\alpha}_{1,j}}$. Following the approach used for Prop-DF, we have an upper bound

$$\breve{P}_{out,l}^{CD} = g\left(0, \frac{\Gamma_{l,L}}{\Omega_{sd}}\right)$$
$$\cdot\left\{1 - \sum_{j=l}^{L} g\left(\frac{\Gamma_{j,L}}{\Omega_{sr}}, \frac{\Gamma_{j+1,L}}{\Omega_{sr}}\right) g\left(\frac{\bar{\alpha}_{1,j}\Gamma_{l,j}}{\Omega_{rd}}, \infty\right)\right\} \quad (88)$$

on $P_{out,l}$ for SC-DF. In the high SNR region, where $\bar{\alpha}_{1,j} \approx 1$ and $\Gamma_{l,j} \approx \Omega^{2\sum_{m=1}^{l}r_m + \epsilon_{l-1}}$, we have

$$\breve{P}_{out,l}^{CD} \approx \frac{\Gamma_l^2}{\Omega_{sd}\Omega_{sr}} + \frac{\Gamma_l^2}{\Omega_{sd}\Omega_{rd}}$$
$$\approx \left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)\Omega^{2\epsilon_{l-1}-2+4\sum_{m=1}^{l}r_m}. \quad (89)$$

Similarly, SC-AF with relay signal (8) has $\eta_{f,l} = \Phi_{l,L}\left(\gamma_{sd} + \frac{\gamma_{sr}\gamma_{rd}}{\gamma_{sr}+\gamma_{rd}+1}\right)$, and an upper bound

$$\breve{P}_{out,l}^{CA} = g\left(0, \frac{\Gamma_{l,L}}{\Omega_{sd}}\right) g\left(\frac{2\Gamma_{l,L}}{\Omega_{sr}} + \frac{2\Gamma_{l,L}+1}{\Omega_{rd}}, \infty\right) \quad (90)$$

on $P_{out,l}$ for SC-AF can be obtained by following the approach used for Prop-AF: In the high SNR region, (90) becomes

$$\breve{P}_{out,l}^{CA} \approx \left(\frac{2}{\rho_1} + \frac{2}{\rho_2}\right)\Omega^{2\epsilon_{l-1}-2+4\sum_{m=1}^{l}r_m}. \quad (91)$$

From (89) and (91), it is clear that SC-DF and SC-AF have $d_l(\boldsymbol{r}) = 2 - 4\sum_{m=1}^{l} r_m - 2\epsilon_{l-1}$, identical to (45) for $\varsigma_{sr} \to \varsigma_{sd}$ with $\epsilon_{l-1} \to 0$, irrespective of relay location. Consequently, the distortion exponent of SC-DF and SC-AF is also given by (47).

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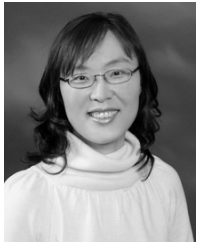
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