Statistical Channel Knowledge-Based Optimum Power Allocation for Relaying Protocols in the High SNR Regime

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***Abstract*— We are concerned with transmit power optimization in a wireless relay network with various cooperation protocols. With statistical channel knowledge (in the form of knowledge of the fading distribution and the path loss information across all the nodes) at the transmitters and perfect channel state information at the receivers, we derive the optimal power allocation that minimizes high signal-to-noise ratio (SNR) approximations of the outage probability of the mutual information (MI) with amplify-and-forward (AF), decode-and-forward (DF) and distributed space-time coded (DSTC) relaying protocols operating over Rayleigh fading channels. We demonstrate that the high SNR approximation-based outage probability expressions are convex functions of the transmit power vector, and the nature of the optimal power allocation depends on whether or not a direct link between the source and the destination exists. Interestingly, for AF and DF protocols, this allocation depends only on the ratio of mean channel power gains (i.e., the ratio of the source-relay gain to the relay-destination gain), whereas with a DSTC protocol this allocation also depends on the transmission rate when a direct link exists. In addition to the immediate benefits of improved outage behavior, our results show that optimal power allocation brings impressive coding gains over equal power allocation. Furthermore, our analysis reveals that the coding gain gap between the AF and DF protocols can also be reduced by the optimal power allocation.**

***Index Terms*— Distributed diversity, optimum power allocation, user cooperation, outage mutual information, relaying protocols.**

I. Introduction

MORE THAN two decades after the seminal works of van der Meulen [1], and Cover and El Gamal [2] on the capacity limits of relay channels, there is a renewed interest in the area of relay-assisted cooperative communication for mobile ad hoc wireless networks. By sharing the transmission resources efficiently in a collaborative manner, mobile nodes with single-antenna transceivers can increase their data rate, range and reliability by forming virtual antenna arrays [3]. Sendonaris *et al.* in [4] showed that with perfect channel state information (CSI) at the transmitters (CSIT), the sum capacity of a wireless network can be improved with user cooperation. With receiver CSI alone (or CSIR), Laneman and Wornell in [5] presented decode-and-forward (DF) and distributed space-time coded (DSTC) protocols for increasing the network reliability, whereas in [3], [6] the authors studied the performance of an amplify-and-forward (AF) protocol. The lifetime [7] of a wireless ad hoc network crucially depends on how efficiently the transmission power is utilized [8]. Conservation of transmit power not only increases the network lifetime, but also reduces undesirable interference to the other nodes in the network, thereby improving the communication reliability as well.

Numerous works have shown that with perfect CSI at both the transmitters and the receivers (denoted by CSI-TR), the relay channel performance can be improved significantly through optimal transmit power allocation. In [9], subject to a short term power constraint, the authors presented an information-theoretic study of the channel capacity as well as the outage probability of wireless relay channels with perfect CSI-TR. Outage behavior of various relaying protocols with optimal power control and CSI-TR is investigated in [10]. In [11], subject to individual long term power constraints on the source and the relay, Liang and Veeravalli consider transmitter power allocation on a Gaussian relay channel with perfect CSI-TR. With an average total energy constraint, Larsson and Cao in [12] consider the possibility of adapting not only the transmission power but also the time slot and bandwidth allocation for the source and the relay. Complementing [11] and [12], in [13] the authors address the outage minimization problem under a long-term total power constraint. In [14], the authors report the impact of relay gain allocation on the performance of an AF protocol. Adaptive transmit power allocation schemes are proposed in [15] and [16] for maximizing the instantaneous capacity of a two-hop Rayleigh fading relay channel. In [15], a regenerative (i.e., the relay decodes and then re-encodes the source bits) system is analyzed, whereas the performance of a non-regenerative system is considered in [16]. With an Alamouti space-time block code (STBC) [17], adaptive transmit power allocation based on perfect CSIT is investigated in [18].

While the above works assume that perfect CSI is available at the transmitters for optimum power allocation, power allocation can still be performed even when perfect CSIT is not available (which, for example, is true on a rapid varying fading channel), provided some statistical knowledge of the channel gains is available to the transmitting nodes. With the knowledge of the mean channel power gains (or simply, mean channel gains) alone, this idea is explored in [19] in the context

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of a multihop diversity system, whereas the authors in [20] investigate the optimal power allocation problem for a transmit diversity system. Recently, [21] presented both signal-to-noise ratio (SNR) maximizing and outage probability minimizing optimal power allocation schemes with the knowledge of the mean channel gains. However, the main limitations of [21] are that the results are valid for only AF protocol with a single relay node. The coding gain of AF and DF protocols, with equal power allocation, is computed in [22], wherein it is shown that when the average channel gain between the source and the relay is smaller than the average channel gain between the relay and the destination, the DF protocol is inferior to the AF protocol, in terms of the asymptotic coding gain (ACG) [23].

In this paper, building upon the equal power allocation-based information-theoretic results presented in [5] and [6], we study the optimal transmit power allocation problem for AF, DF and DSTC protocols with multiple relay nodes. Similar to [19], [20], and [21], we assume that only knowledge of the mean channel gains is known to the nodes in the network, which can easily be realized with a low-rate feedback for a slowly varying network topology, and obtain the optimum transmit power vector that minimizes the outage probability of the mutual information (or simply, outage probability) at the destination. We show that, at high SNR, the outage probability expressions for various protocols are convex functions of the transmit power vector, and the optimal power allocation depends on whether or not a direct link exists between the source and the destination. Additionally, for AF and DF protocols, this allocation depends only on the ratio of the mean channel gains (i.e., the ratio of the source-to-relay channel gain to the relay-to-destination channel gain), whereas with a DSTC protocol with a direct link this allocation also depends on the transmission rate. Interestingly, our results without a direct link show that both the DF and DSTC protocols have identical optimal power vectors and identical asymptotic coding gain ratios (CGR, i.e., the ratio of the ACG with optimal power allocation to the ACG with equal power allocation). Our analysis reveals that, in addition to the outage probability improvements, optimal power allocation also brings impressive coding gains over equal power allocation. Furthermore, with a single relay, our results show that optimal power allocation can also reduce the ACG gap between the DF and AF protocols. While our optimization is performed with a sum power constraint, our results can be modified to account for a per-node maximum power constraint by simply clipping the excess power of a given node, and reallocating the remaining power to the nodes satisfying the constraints in an optimal manner [19].

The rest of this paper is organized as follows. In Section II, we describe the system and the channel model. High SNR approximations for the outage probabilities expressions with AF, DF and DSTC protocols are developed, and validated through simulations, in Section III. We formulate the optimum power allocation problem in Section IV, and derive the optimum power allocation vector for AF, DF and DSTC protocols. The coding gain improvements with power allocation are presented in Section V. We provide numerical results and discussions in Section VI, and conclude our work in Section VII.

II. System Model

We assume a single source communicating with a single destination with the help of M relay nodes. The channels between all the nodes are assumed to be random, independent, frequency-flat, and constant over the signaling duration. We employ low-pass equivalent complex-valued representation for the transmit and receive signals, for the channel gains and for background additive noise. Specifically, the channel gain between the source and the destination is denoted by g_1, which is assumed to be a zero-mean, circularly symmetric, complex Gaussian random variable (r.v) with variance $E[|g_1|^2] = \Omega_1$. In a similar fashion, for the jth relay, the gain from the source to the relay is denoted by g_2^j, and the gain from the relay to the destination by g_3^j, with variances $E[|g_2^j|^2] = \Omega_2^j$ and $E[|g_3^j|^2] = \Omega_3^j$. The noise r.v on each link is assumed to be zero-mean, independent, additive, and Gaussian distributed. In this paper, we consider three relaying protocols, namely a) amplify-and-forward, b) decode-and-forward, and c) distributed space-time coded protocols. The description of these protocols can be found in [5] and [6]. While [5] and [6], in their mutual information (MI) analysis, always assume the existence of a direct link between the source and destination, in this paper we separately analyze the two systems with/without a direct link. When there is no direct link (NDL) between the source and the destination, which is true, for example, when there is an obstruction on the source-destination path, mathematically, we set $\Omega_1 = 0$. On the other hand, when there exists a direct link (DL) between the source and the destination Ω_1 is non-zero. The total bandwidth available for the source transmission without cooperation is denoted by W. Similar to [5] and [6], half-duplex constraints are imposed on the relay nodes (i.e., the relays cannot transmit and receive simultaneously). With repetition-based AF and DF protocols, we assume that the total bandwidth is divided into $M+1$ equi-width, disjoint channels, so that the bandwidth available for the source and for each one of the M relay nodes is $W/(M+1)$. Throughout this paper, the transmission rate of the source, $\tilde{R}$, is normalized by the bandwidth, W. That is, $R = \tilde{R}/W$. In a similar way, the MI is also normalized by W. Let us denote by $\tilde{P}_s$ the average transmit power of the source, and by $\tilde{P}_{r,j}$ the average transmit power of the jth relay. The single-sided power spectral density of the additive Gaussian noise is denoted by N_0, so that noise power in a bandwidth W is $\sigma_N^2 = N_0 W$.

We assume that the transmitted baseband samples of the nodes are independent Gaussian-distributed r.vs with zero-mean and variance equal to the respective average transmit power. When the source transmits at a power level of $\tilde{P}_s$, the instantaneous SNR at the destination is $\tilde{P}_s|g_1|^2/(N_0W/(M+1)) = (M+1)\tilde{P}_s|g_1|^2/\sigma_N^2$, which is denoted by γ_1. In a similar manner, the instantaneous received SNR at the jth relay is denoted by γ_2^j, which is given by $\gamma_2^j = (M+1)\tilde{P}_s|g_2^j|^2/\sigma_N^2$. When the relays transmit their respective signals to the destination, the SNR at the destination from the jth relay is $\gamma_3^j = (M+1)\tilde{P}_{r,j}|g_3^j|^2/\sigma_N^2$. Let us define $P_s = (M+1)\tilde{P}_s$, and, for $j = 1, \ldots, M$, $P_{r,j} = (M+1)\tilde{P}_{r,j}$; also, we denote by $\underline{P} = [P_s, P_{r,1}, \ldots, P_{r,M}]$ the transmit power vector. Finally, we define the following variables: $\overline{\gamma}_1 \triangleq E[\gamma_1] = P_s\Omega_1/\sigma_N^2$, and for $j = 1, \ldots, M$, $\overline{\gamma}_2^j \triangleq E[\gamma_2^j] = P_s \Omega_2^j / \sigma_N^2$ and $\overline{\gamma}_3^j \triangleq E[\gamma_3^j] = P_{r,j} \Omega_3^j / \sigma_N^2$.

III. High SNR Outage Analysis

In this section, we develop high SNR approximations for the outage probability of the MI with AF, DF and DSTC protocols, which is defined as the probability that the instantaneous MI at the destination falls below a target rate of R [24].

A. AF Protocol

With the AF protocol, assuming a direct link between the source and the destination, the output SNR at the destination, with maximal ratio combining (MRC), is [25], [26]

$$\gamma_{AF,DL} = \gamma_1 + \sum_{j=1}^{M} \frac{\gamma_2^j \gamma_3^j}{1 + \gamma_2^j + \gamma_3^j}.$$

The instantaneous MI at the destination can be written as (1).

$$\begin{aligned} I_{AF,DL} &= \frac{1}{M+1} \log_2 \left(1 + \gamma_{AF,DL}\right) \\ &= \frac{1}{M+1} \log_2 \left(1 + \gamma_1 + \sum_{j=1}^{M} \frac{\gamma_2^j \gamma_3^j}{1 + \gamma_2^j + \gamma_3^j}\right). \quad (1) \end{aligned}$$

The fraction $1/(M+1)$ in (1) is due to the fact that the source uses only $1/(M+1)$ of the total bandwidth W. This outage probability with the AF protocol, $P_{Out,AF,DL}(\underline{P})$, is given by (2).

$$\begin{aligned} P_{Out,AF,DL}(\underline{P}) &= \text{Prob}(I_{AF,DL} < R) \\ &= \text{Prob}\left(\gamma_1 + \sum_{j=1}^{M} \frac{\gamma_2^j \gamma_3^j}{1 + \gamma_2^j + \gamma_3^j} < 2^{(M+1)R} - 1\right). \quad (2) \end{aligned}$$

At high SNR, following the approach of [26], we can approximate (2) as (3) on the following page.[1] Upon defining $\alpha_j = \Omega_2^j / \Omega_3^j$ and a constant $\mathsf{C}_{AF,DL} = \left[(2^{(M+1)R} - 1)\sigma_N^2\right]^{M+1} / ((M+1)! \Omega_1 \prod_{j=1}^{M} \Omega_2^j)$, (3) simplifies to the following compact form:

$$P_{Out,AF,DL}(\underline{P}) \approx \mathsf{C}_{AF,DL} \frac{1}{P_s} \prod_{j=1}^{M} \left(\frac{1}{P_s} + \frac{\alpha_j}{P_{r,j}}\right). \quad (4)$$

In the absence of a direct link, since $\Omega_1 = 0$, the source to the destination SNR r.v γ_1 does not contribute to the MI expression of (1). As a result, from (3) and (4), we can write the outage probability as

$$P_{Out,AF,NDL}(\underline{P}) \approx \mathsf{C}_{AF,NDL} \prod_{j=1}^{M} \left(\frac{1}{P_s} + \frac{\alpha_j}{P_{r,j}}\right), \quad (5)$$

where $\mathsf{C}_{AF,NDL} = \left[(2^{(M+1)R} - 1)\sigma_N^2\right]^M / (M! \prod_{j=1}^{M} \Omega_2^j)$.

[1] Note that the steps required to arrive at (3) are very similar to those in [26] and hence are skipped for brevity.

B. DF Protocol

With a DF protocol, a relay is assumed to correctly decode the source transmission if the instantaneous MI is above the attempted transmission rate R. A relay, after successfully decoding the source transmission, uses the same code book as that of the source for retransmission. Assuming a direct link, the instantaneous SNR at the destination, conditioned on a set $\mathcal{D}$ of correctly decoded relays, is given by $\gamma_{DF,DL} = \gamma_1 + \sum_{j \in \mathcal{D}} \gamma_3^j$, where we assumed that the destination performs MRC of the received signals. The instantaneous MI at the destination, conditioned on $\mathcal{D}$, can be written as (6).

$$\begin{aligned} I_{DF,DL}(\mathcal{D}) &= \frac{1}{M+1} \log_2 \left(1 + \gamma_{DF,DL}\right) \\ &= \frac{1}{M+1} \log_2 \left(1 + \gamma_1 + \sum_{j \in \mathcal{D}} \gamma_3^j\right). \quad (6) \end{aligned}$$

The outage probability at the destination can then be written as (7).

$$\begin{aligned} P_{Out,DF,DL}(\underline{P}) &= \text{Prob}(I_{DF,DL} < R) \\ &= \sum_{\mathcal{D}} \text{Prob}(\mathcal{D}) \text{Prob}\left(I_{DF,DL} < R | \mathcal{D}\right) \\ &= \sum_{\mathcal{D}} \text{Prob}(\mathcal{D}) \text{Prob}\left(\gamma_1 + \sum_{j \in \mathcal{D}} \gamma_3^j < 2^{(M+1)R} - 1\right). \quad (7) \end{aligned}$$

The probability of the decoding set, $\text{Prob}(\mathcal{D})$, is simply the probability that a subset $\mathcal{D}$ of relays correctly decodes the source signals. This event happens when all the relays in $\mathcal{D}$ have their conditional MI above the target rate R, and the relays outside $\mathcal{D}$ have their conditional MI below the rate R. That is,

$$\begin{aligned} \text{Prob}(\mathcal{D}) &= \left[\prod_{j \in \mathcal{D}} \text{Prob}\left(\frac{1}{M+1} \log_2(1 + \gamma_2^j) > R\right)\right] \\ &\quad \times \prod_{k \notin \mathcal{D}} \text{Prob}\left(\frac{1}{M+1} \log_2(1 + \gamma_2^k) < R\right) \\ &= \left[\prod_{j \in \mathcal{D}} e^{-\frac{2^{(M+1)R} - 1}{\overline{\gamma}_2^j}}\right] \times \prod_{k \notin \mathcal{D}} \left(1 - e^{-\frac{2^{(M+1)R} - 1}{\overline{\gamma}_2^k}}\right) \\ &\approx \prod_{k \notin \mathcal{D}} \frac{2^{(M+1)R} - 1}{\overline{\gamma}_2^k}, \end{aligned} \quad (8)$$

where the approximation in the last step of (8) is valid for high SNR, and is due to the fact that, for small x, $\exp(-x) \approx 1$ and $1 - \exp(-x) \approx x$ [5], [6], [20], [23]. The second term of (7) can be approximated as [27, Appendix G]

$$\begin{aligned} &\text{Prob}\left(\gamma_1 + \sum_{j \in \mathcal{D}} \gamma_3^j < 2^{(M+1)R} - 1\right) \\ &\approx \frac{\left[2^{(M+1)R} - 1\right]^{|\mathcal{D}|+1}}{(|\mathcal{D}| + 1)!} \frac{1}{\overline{\gamma}_1} \prod_{j \in \mathcal{D}} \frac{1}{\overline{\gamma}_3^j}. \end{aligned} \quad (9)$$

$$\begin{aligned} P_{Out,AF,DL}(\underline{P}) &\approx \frac{\left[2^{(M+1)R}-1\right]^{M+1}}{(M+1)!}\frac{1}{\overline{\gamma}_1}\prod_{j=1}^{M}\left(\frac{1}{\overline{\gamma}_2^j}+\frac{1}{\overline{\gamma}_3^j}\right) \\ &= \frac{\left[(2^{(M+1)R}-1)\sigma_N^2\right]^{M+1}}{(M+1)!}\frac{1}{\Omega_1 P_s}\prod_{j=1}^{M}\left(\frac{1}{P_s\Omega_2^j}+\frac{1}{P_{r,j}\Omega_3^j}\right). \end{aligned} \tag{3}$$

Using (8) and (9) in (7), we arrive at the high SNR approximation for $P_{Out,DF,DL}$:

$$\begin{aligned} P_{Out,DF,DL}(\underline{P}) &\approx \sum_{\mathcal{D}}\left[\prod_{k\notin\mathcal{D}}\frac{2^{(M+1)R}-1}{\overline{\gamma}_2^k}\right] \\ &\times\frac{\left[2^{(M+1)R}-1\right]^{|\mathcal{D}|+1}}{(|\mathcal{D}|+1)!}\frac{1}{\overline{\gamma}_1}\prod_{j\in\mathcal{D}}\frac{1}{\overline{\gamma}_3^j} \\ &= \mathsf{C}_{DF,DL}\sum_{\mathcal{D}}\frac{1}{(|\mathcal{D}|+1)!}\left(\frac{1}{P_s}\right)^{M+1-|\mathcal{D}|}\prod_{j\in\mathcal{D}}\frac{\alpha_j}{P_{r,j}}, \end{aligned} \tag{10}$$

where $\mathsf{C}_{DF,DL} = \left[(2^{(M+1)R}-1)\sigma_N^2\right]^{M+1}/(\Omega_1\prod_{j=1}^{M}\Omega_2^j)$. In the absence of a direct link, (10) can be modified as follows: First, the probability Prob($\mathcal{D}$) in (8) is not related to the existence of a direct link, and hence it remains unchanged. However, since there is no direct link, the r.v γ_1 does not contribute to the outage expression of (9), and the modified expression is [27, Appendix G]

$$\text{Prob}\left(\sum_{j\in\mathcal{D}}\gamma_3^j < 2^{(M+1)R}-1\right) \approx \frac{\left[2^{(M+1)R}-1\right]^{|\mathcal{D}|}}{(|\mathcal{D}|)!}\prod_{j\in\mathcal{D}}\frac{1}{\overline{\gamma}_3^j}. \tag{11}$$

Using (8) and (11) in (7), and following the steps in (10) for the simplification, we arrive at (12) on the following page, where $\mathsf{C}_{DF,NDL} = \left[(2^{(M+1)R}-1)\sigma_N^2\right]^{M}/(\prod_{j=1}^{M}\Omega_2^j)$.

C. Distributed STC Protocol

Let us now turn our attention to a DSTC protocol. With a DSTC protocol, the bandwidth is divided into two disjoint bands of width $W/2$ each. In the first phase of the protocol, the source transmits over a bandwidth of $W/2$. Each relay node independently attempts to decode the source transmission. In the event that multiple relay nodes are able to successfully decode the source information, they collaborate their transmissions by forming a virtual orthogonal STBC[2] and simultaneously transmit over the remaining bandwidth of $W/2$. Practical issues such as construction of distributed STBCs, channel feedback requirements, and communication-theoretic performances can be found, for example, in [28] and [29]. Compared with the repetition based AF/DF protocols, a DSTC protocol is bandwidth efficient by a factor of $(M+1)/2$. When a direct link exists, Laneman and Wornell [5] showed that a DSTC protocol achieves a full spatial diversity order equal to the total number of nodes (in our case, it is $M+1$). In the presence of a direct link, conditioned on the set of decoding nodes $\mathcal{D}$, the conditional MI at the destination with DSTC is

$$\begin{aligned} I_{DTSC,DL}(\mathcal{D}) &= \frac{1}{2}\log_2\left(1+\frac{\tilde{P}_s}{N_0W/2}|g_1|^2\right) \\ &+ \frac{1}{2}\log_2\left(1+\sum_{j\in\mathcal{D}}\frac{\tilde{P}_{r,j}}{N_0W/2}|g_3^j|^2\right) \\ &= \frac{1}{2}\log_2\left(1+\frac{2}{M+1}\gamma_1\right) \\ &+ \frac{1}{2}\log_2\left(1+\frac{2}{M+1}\sum_{j\in\mathcal{D}}\gamma_3^j\right), \end{aligned} \tag{13}$$

which is the sum of the MIs of two independent parallel channels, the first one from the source to the destination, and the second one from the successfully decoded relays to the destination. Eqn. (13) is achievable when relays re-encode the decoded source information using independent code books, and when all the code books are available to the destination [2]. The factor $1/2$ in front of the logarithm in (13) is due to the fact that the nodes transmit in half of the available bandwidth.

In the absence of a direct link, (13) reduces to

$$I_{DTSC,NDL}(\mathcal{D}) = \frac{1}{2}\log_2\left(1+\frac{2}{M+1}\sum_{j\in\mathcal{D}}\gamma_3^j\right). \tag{14}$$

Similar to (7), the outage probability with DSTC is shown in (15) on the following page, with a direct link, and

$$\begin{aligned} &P_{Out,DSTC,NDL}(\underline{P}) \\ &= \sum_{\mathcal{D}}\text{Prob}(\mathcal{D})\text{Prob}\left(\frac{2}{M+1}\sum_{j\in\mathcal{D}}\gamma_3^j < 2^{2R}-1\right) \end{aligned} \tag{16}$$

without a direct link. Analogous to (8), the probability Prob($\mathcal{D}$) is

$$\begin{aligned} \text{Prob}(\mathcal{D}) &= \left[\prod_{j\in\mathcal{D}}\text{Prob}\left(\frac{1}{2}\log_2(1+\frac{2}{M+1}\gamma_2^j) > R\right)\right] \\ &\times \prod_{k\notin\mathcal{D}}\text{Prob}\left(\frac{1}{2}\log_2(1+\frac{2}{M+1}\gamma_2^k) < R\right) \\ &\approx \prod_{k\notin\mathcal{D}}\frac{(2^{2R}-1)(M+1)}{2\overline{\gamma}_2^k}, \end{aligned} \tag{17}$$

[2]Note that very low rate but highly reliable side channels are assumed to exist between the relays to communicate which nodes have successfully decoded, and to convey the choice of the space-time block code to be used.

$$
\begin{aligned}
P_{Out,DF,NDL}(\underline{P}) &\approx \frac{\left[(2^{(M+1)R}-1)\sigma_N^2\right]^M}{\prod_{j=1}^{M}\Omega_2^j}\sum_{\mathcal{D}}\frac{1}{(|\mathcal{D}|)!}\left(\frac{1}{P_s}\right)^{M-|\mathcal{D}|}\prod_{j\in\mathcal{D}}\frac{\Omega_2^j}{\Omega_3^j P_{r,j}} \\
&= \mathsf{C}_{DF,NDL}\sum_{\mathcal{D}}\frac{1}{(|\mathcal{D}|)!}\left(\frac{1}{P_s}\right)^{M-|\mathcal{D}|}\prod_{j\in\mathcal{D}}\frac{\alpha_j}{P_{r,j}}, \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
P_{Out,DSTC,DL}(\underline{P}) &= \mathrm{Prob}(I_{DSTC,DL}<R)=\sum_{\mathcal{D}}\mathrm{Prob}(\mathcal{D})\mathrm{Prob}\left(I_{DSTC,DL}<R|\mathcal{D}\right) \\
&= \sum_{\mathcal{D}}\mathrm{Prob}(\mathcal{D})\mathrm{Prob}\left((1+\frac{2}{M+1}\gamma_1)\times(1+\frac{2}{M+1}\sum_{j\in\mathcal{D}}\gamma_3^j)<2^{2R}\right) \qquad (15)
\end{aligned}
$$

whereas the second term of (15) can be approximated as[3].

$$
\begin{aligned}
&\mathrm{Prob}\left((1+\frac{2}{M+1}\gamma_1)\times(1+\frac{2}{M+1}\sum_{j\in\mathcal{D}}\gamma_3^j)<2^{2R}\right) \\
&\approx\left(\frac{(2^{2R}-1)(M+1)}{2}\right)^{|\mathcal{D}|+1} \\
&\times\mathcal{A}_{|\mathcal{D}|}(2^{2R}-1)\frac{1}{\overline{\gamma}_1}\prod_{j\in\mathcal{D}}\frac{1}{\overline{\gamma}_3^j}, \qquad (18)
\end{aligned}
$$

where

$$
\mathcal{A}_n(t)=\frac{1}{(n-1)!}\int_{u=0}^{1}\frac{u^{n-1}(1-u)}{1+tu}du \qquad n>0, \qquad (19)
$$

and $\mathcal{A}_0(t)=1$. Without a direct link, the second term of (16) approximately equals

$$
\begin{aligned}
&\mathrm{Prob}\left(\frac{2}{M+1}\sum_{j\in\mathcal{D}}\gamma_3^j<2^{2R}-1\right) \\
&\approx\left(\frac{(2^{2R}-1)(M+1)}{2}\right)^{|\mathcal{D}|}\frac{1}{(|\mathcal{D}|)!}\prod_{j\in\mathcal{D}}\frac{1}{\overline{\gamma}_3^j}. \qquad (20)
\end{aligned}
$$

Plugging (17) and (18) in (15), we have

$$
\begin{aligned}
&P_{Out,DSTC,DL}(\underline{P})\approx\left(\frac{(2^{2R}-1)(M+1)}{2}\right)^{M+1} \\
&\sum_{\mathcal{D}}\mathcal{A}_{|\mathcal{D}|}(2^{2R}-1)\frac{1}{\overline{\gamma}_1}\left[\prod_{k\notin\mathcal{D}}\frac{1}{\overline{\gamma}_2^k}\right]\prod_{j\in\mathcal{D}}\frac{1}{\overline{\gamma}_3^j} \\
&=\mathsf{C}_{DSTC,DL}\times\sum_{\mathcal{D}}\mathcal{A}_{|\mathcal{D}|}(2^{2R}-1)\left(\frac{1}{P_s}\right)^{M+1-|\mathcal{D}|}\prod_{j\in\mathcal{D}}\frac{\alpha_j}{P_{r,j}}, \qquad (21)
\end{aligned}
$$

where

$$
\mathsf{C}_{DSTC,DL}=\left((2^{2R}-1)(M+1)\sigma_N^2/2\right)^{M+1}/(\Omega_1\prod_{j=1}^{M}\Omega_2^j).
$$

[3]Eqn. (18) can be obtained from [5] by setting the variables $\mathsf{SNR}=1$, $\lambda_{s,d(s)}=1/\overline{\gamma}_1$, and $\lambda_{r,d(s)}=1/\overline{\gamma}_3^r$ in [5, Eqn. (19)].

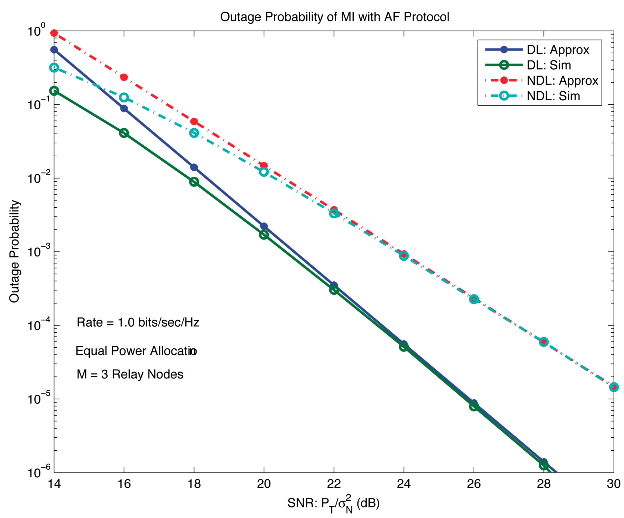

Fig. 1. Comparison of exact outage probability against the high SNR approximation for an amplify-and-forward protocol. $M=3$ relay nodes are considered both without and with a direct link between the source and the destination. Equal power allocation is assumed with $R=1.0$ bits/sec/Hz.

Using (17) and (20) in (16), the approximate outage probability without a direct link is

$$
\begin{aligned}
&P_{Out,DSTC,NDL}(\underline{P}) \\
&\approx\mathsf{C}_{DSTC,NDL}\sum_{\mathcal{D}}\frac{1}{(|\mathcal{D}|)!}\left(\frac{1}{P_s}\right)^{M-|\mathcal{D}|}\prod_{j\in\mathcal{D}}\frac{\alpha_j}{P_{r,j}}, \qquad (22)
\end{aligned}
$$

where

$$
\mathsf{C}_{DSTC,NDL}=\left((2^{2R}-1)(M+1)\sigma_N^2/2\right)^{M}/(\prod_{j=1}^{M}\Omega_2^j).
$$

It is worth noticing the similarity between (22) and (12), which can be explained by the fact that the same number, $|\mathcal{D}|$, of relay nodes are employed in both DF and DSTC protocols[4].

D. Accuracy of High SNR Outage Probability Approximations

We now compare the high SNR approximations of (4), (5), (10), (12), (21), and (22) against their respective exact outage expressions. One hundred million (10^8) channel realizations,

[4]Eqns. (12) and (22) differ only in the scale factor, which captures the bandwidth efficiency of a given protocol.

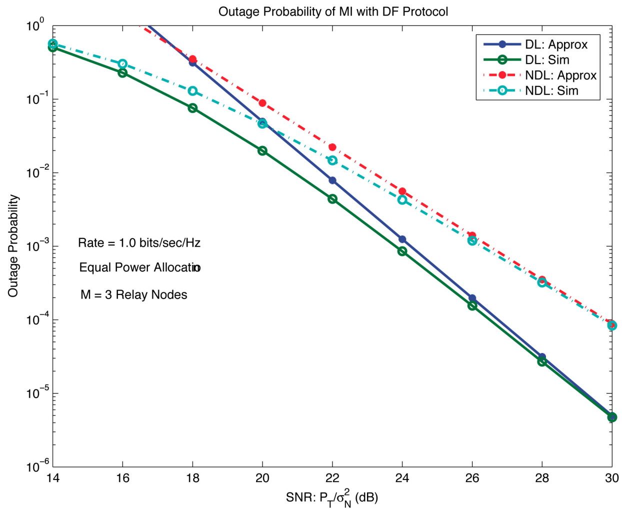

Fig. 2. Comparison of exact outage probability against the high SNR approximation for a decode-and-forward protocol. $M = 3$ relay nodes are considered both without and with a direct link between the source and the destination. Equal power allocation is assumed with $R = 1.0$ bits/sec/Hz.

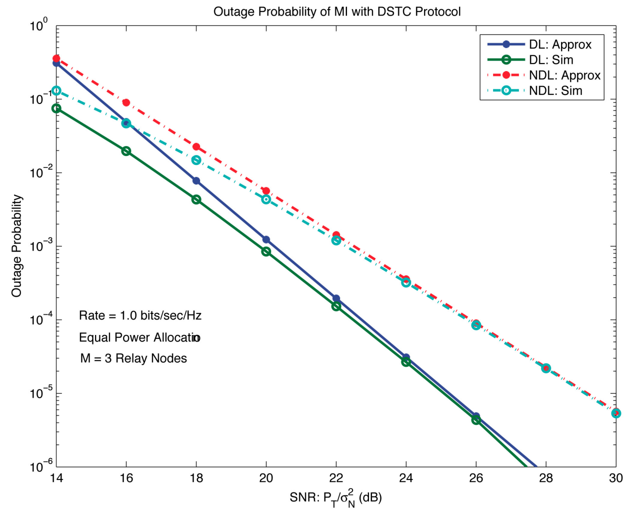

Fig. 3. Comparison of exact outage probability against the high SNR approximation for a distributed space-time code protocol. $M = 3$ relay nodes are considered both without and with a direct link between the source and the destination. Equal power allocation is assumed with $R = 1.0$ bits/sec/Hz.

for each SNR value, were simulated to evaluate the exact outage expressions. We used $M = 3$ relay nodes, both with and without a direct link between the source and the destination. For simplicity we assumed that the source, the destination, and the relay nodes were placed on a circle with radius $1/2$. The source was located at $(0, 0)$, the destination at $(1, 0)$, and the jth relay at $((1 + \cos\theta_j)/2, (\sin\theta_j)/2)$, $j = 1, 2, 3$, with $\theta_1 = \pi/3$, $\theta_2 = \pi/4$ and $\theta_3 = \pi/6$. For a given path loss exponent η, we have $\Omega_1 = 1$, $\Omega_2^j = [\cos(\theta_j/2)]^{-\eta}$, and $\Omega_3^j = [\sin(\theta_j/2)]^{-\eta}$, $j = 1, 2, 3$. Throughout this paper, we use $\eta = 4$. For simplicity, we assumed equal power allocation among the source and the 3 relays. Then, $P_s = P_T/4$, and $P_{r,j} = P_T/4$, $j = 1, \ldots, 4$, where P_T is the average total power. The target information rate was set to $R = 1.0$ bits/sec/Hz. The outage probability results are shown, as a function of P_T/σ_N^2, in Figs. 1, 2 and 3 for AF, DF, and DSTC protocols, respectively. From Figs. 1, 2 and 3, we conclude that the high SNR approximations are quite accurate[5].

E. Convexity of High SNR Outage Probability Expressions

Upon examining (4), (5), (10), (12), (21), and (22), we notice that the outage probability of each protocol can be expressed as a linear combination (with positive weights) of the function

$$\Psi(\underline{P}) = \frac{1}{P_s^{n_0} \prod_{j=1}^{M} P_{r,j}^{n_j}}, \tag{23}$$

where $n_j \geq 0$ for $j = 0, 1, \ldots, M$. The determinant of the Hessian matrix of the objective function of (23) can be shown to be

$$\det\left(\nabla^2 \Psi(\underline{P})\right) = \Psi(\underline{P})^{M+1} \times \frac{(1 + \sum_{j=0}^{M} n_j) \prod_{j=0}^{M} n_j}{P_s^2 \prod_{j=1}^{M} P_{r,j}^2} \tag{24}$$

[5]At an outage of 10^{-6}, there is a very small discrepancy between the exact and high SNR-based curves in Figs. 1 and 3, and this appears to be due to numerical inaccuracy.

which is strictly positive. It can also be shown that all the principal sub-matrices of $\nabla^2\Psi(\underline{P})$ have positive determinants, which can be expressed in a form similar to (24). This implies that (23) is a strictly convex function of $\underline{P}$. Since a linear combination (with positive weights) of convex functions is also convex, we conclude that (4), (5), (10), (12), (21), and (22) are also convex functions of $\underline{P}$. Since (23) is in the form of a *monomial* [30] in $\underline{P}$, upon changing the variables $P_s = e^{x_s}$, $x_s \in \Re$, and $P_{r,j} = e^{x_j}$, $x_j \in \Re$, $j = 1, \ldots, M$, we can express each one of the objective functions, together with the modified constraint $e^{x_s} + \sum_{j=1}^{M} e^{x_j} \leq P_T$, as a standard *geometric programming* optimization problem [30, Chap. 4.5]. For a large number of relay nodes, this optimization can be performed efficiently using any commercial solver such as MOSEK [31].

IV. Optimal Power Allocation

In this section, for the AF, DF and DSTC protocols, we derive the optimal transmit power vector, $\underline{P}$, that minimizes the outage probability, subject to a sum power constraint. That is, our optimization problem is

$$\text{minimize} \quad P_{Out}(\underline{P}) \quad \text{subject to} \quad P_s + \sum_{j=1}^{M} P_{r,j} \leq P_T, \tag{25}$$

where P_T is the total transmit power. With equal power allocation, we have $P_s = P_{r,j} = P_T/(M+1)$, $j = 1, \ldots, M$. In our optimization, we devote equal attention to the cases without and with a direct link between the source and the destination. As will be clear at the end of this section, the presence or absence of a direct link significantly affects the optimum power vector, and the resulting performance gains. For all the protocols, we simply ignore the constants $\mathsf{C}_{AF,DL}, \mathsf{C}_{AF,NDL}, \mathsf{C}_{DF,DL}, \mathsf{C}_{DF,NDL}, \mathsf{C}_{DSTC,DL}$, and $\mathsf{C}_{DSTC,NDL}$, as they appear as multiplicative factors to the objective functions, and hence do not affect the resulting optimal power vector.

A. Amplify-and-Forward Protocol

When there is a direct link, using (4), the optimization problem of (25) is

$$\begin{aligned} &\text{minimize} \quad \frac{1}{P_s}\prod_{j=1}^{M}\left(\frac{1}{P_s}+\frac{\alpha_j}{P_{r,j}}\right) \\ &\text{subject to} \quad P_s+\sum_{j=1}^{M}P_{r,j}\le P_T. \end{aligned} \tag{26}$$

The Lagrange cost function can be written as

$$\begin{aligned} \mathcal{J}(\underline{P},\lambda) &= \frac{1}{P_s}\prod_{j=1}^{M}\left(\frac{1}{P_s}+\frac{\alpha_j}{P_{r,j}}\right) \\ &+ \lambda\left(P_s+\sum_{j=1}^{M}P_{r,j}-P_T\right), \end{aligned} \tag{27}$$

where λ is the Lagrange parameter.

Upon setting the derivatives of $\mathcal{J}(\underline{P},\lambda)$ with respect to (w.r.t) P_s, $P_{r,j}$, $j=1,\ldots,M$, and λ, to zero, we have (28) and (29) on the following page, and

$$P_s+\sum_{j=1}^{M}P_{r,j}=P_T. \tag{30}$$

Using (29) and (30) in (28), we obtain

$$\frac{\lambda}{P_{Out,AF,NDL}(\underline{P})}=\frac{M+1}{P_T}. \tag{31}$$

Substituting (31) in (29), we arrive at the following quadratic equation over $P_{r,j}$:

$$P_{r,j}^2+P_{r,j}P_s\alpha_j-\alpha_jP_sP_T/(M+1)=0, \tag{32}$$

whose solution, in terms of P_s, is

$$P_{r,j}=\frac{-P_s\alpha_j+\sqrt{P_s^2\alpha_j^2+4P_s\alpha_jP_T/(M+1)}}{2}, \quad j=1,\ldots,M. \tag{33}$$

Let $P_s=\delta_0P_T$, and for $j=1,\ldots,M$, $P_{r,j}=\delta_jP_T$, where $\delta_0>0$, $0\le\delta_j\le 1$, $j=1,\ldots,M$, and $\sum_{j=0}^{M}\delta_j=1$. Then, by substituting (33) in (30), δ_0 can be expressed as the following transcendental equation:

$$\sum_{j=1}^{M}\sqrt{\delta_0^2\alpha_j^2+4\alpha_j\delta_0/(M+1)}=2(1-\delta_0)+\delta_0\sum_{j=1}^{M}\alpha_j. \tag{34}$$

Once δ_0 is found, P_s can be obtained as $P_s=\delta_0P_T$ and (33) yields $P_{r,j}$. As a special case, let us assume $\alpha_j=\alpha$, $\forall j=1,\ldots,M$, which might be thought of as a result of a symmetric relay placement. In this case, $\delta_j=(1-\delta_0)/M$, $j=1,\ldots,M$, and we obtain the following closed-form expression for δ_0:

$$\delta_0=\frac{1}{1-M\alpha}\left[1-\frac{\alpha M}{2(M+1)}\left(1+\sqrt{1+4\frac{M+1}{\alpha}}\right)\right]. \tag{35}$$

As $\alpha\to 0$, $\delta_0\to 1$ indicating that all the power should be allocated to the source. Intuitively, this makes sense, since, when the relay is arbitrarily close to the destination, we expect the source to use as much of the available power as possible to reach the destination, and only a small amount of power is needed for the relay to reach the destination. On the other hand, when $\alpha\to\infty$ (i.e., the relay is arbitrarily close to the source) $\delta_0\to\frac{1}{M+1}$. That is, equal power allocation is optimal only for large values of α. As $\alpha\to 1/M$, there is a discontinuity in the function, but using the L'Hospital rule in (35), we have $\delta_0\to(M+1)/(1+2M)$.

In the absence of a direct link, with (5), the optimization problem is

$$\begin{aligned} &\text{minimize} \quad \prod_{j=1}^{M}\left(\frac{1}{P_s}+\frac{\alpha_j}{P_{r,j}}\right) \\ &\text{subject to} \quad P_s+\sum_{j=1}^{M}P_{r,j}\le P_T. \end{aligned} \tag{36}$$

The Lagrange cost function can be written as

$$\mathcal{J}(\underline{P},\lambda)=\prod_{j=1}^{M}\left(\frac{1}{P_s}+\frac{\alpha_j}{P_{r,j}}\right)+\lambda\left(P_s+\sum_{j=1}^{M}P_{r,j}-P_T\right). \tag{37}$$

Upon setting the derivatives of $\mathcal{J}(\underline{P},\lambda)$ w.r.t P_s, $P_{r,j}$, $j=1,\ldots,M$, and λ, to zero, we have (38) and (39) on the following page, and (30). Using (39) and (30) in (38), we obtain

$$\frac{\lambda}{P_{Out,AF,NDL}(\underline{P})}=\frac{M}{P_T}. \tag{40}$$

Substituting (40) in (39), we arrive at the quadratic

$$P_{r,j}^2+P_{r,j}P_s\alpha_j-\alpha_jP_sP_T/M=0, \tag{41}$$

whose solution, as a function of P_s, is

$$P_{r,j}=\frac{-P_s\alpha_j+\sqrt{P_s^2\alpha_j^2+4P_s\alpha_jP_T/M}}{2}, \quad j=1,\ldots,M. \tag{42}$$

Let $P_s=\zeta_0P_T$, and for $j=1,\ldots,M$, $P_{r,j}=\zeta_jP_T$, where $\zeta_0>0$, $0\le\zeta_j\le 1$, $j=1,\ldots,M$, and $\sum_{j=0}^{M}\zeta_j=1$. Then, by substituting (42) in (30), ζ_0 can be expressed in the following implicit equation:

$$\sum_{j=1}^{M}\sqrt{\zeta_0^2\alpha_j^2+4\alpha_j\zeta_0/M}=2(1-\zeta_0)+\zeta_0\sum_{j=1}^{M}\alpha_j. \tag{43}$$

Once ζ_0 is found, we get $P_s=\zeta_0P_T$, and (42) yields $P_{r,j}$. As a special case, let us assume $\alpha_j=\alpha$, $\forall j=1,\ldots,M$, so that $\zeta_j=(1-\zeta_0)/M$, $j=1,\ldots,M$, where ζ_0 is given by

$$\zeta_0=\frac{1}{1+\sqrt{M\alpha}}. \tag{44}$$

As $\alpha\to 0$, $\zeta_0\to 1$ indicating that a large fraction of the available power should be allocated to the source, consistent with the case with a direct link. On the other hand, unlike the case with a direct link, $\alpha\to\infty$ gives us $\zeta_0\to 0$. That is, since the relay is arbitrarily close to the source, very little transmit power is needed by the source, and the rest of the available power has to be shared by the relays equally. Only when $\alpha=M$ does the equal power allocation become optimal.

$$-\frac{1}{P_s^2}\prod_{j=1}^{M}\left(\frac{1}{P_s}+\frac{\alpha_j}{P_{r,j}}\right)+\sum_{k=1}^{M}\frac{1}{P_s}\left(\prod_{j=1,j\neq k}^{M}\left(\frac{1}{P_s}+\frac{\alpha_j}{P_{r,j}}\right)\right)\left(-\frac{1}{P_s^2}\right)+\lambda=0$$

$$\Longrightarrow \quad \lambda=\frac{P_{Out,AF,NDL}(\underline{P})}{P_s}\left(1+\sum_{k=1}^{M}\frac{P_{r,k}}{P_s\alpha_k+P_{r,k}}\right), \tag{28}$$

$$\frac{1}{P_s}\left[\prod_{k=1,k\neq j}^{M}\left(\frac{1}{P_s}+\frac{\alpha_k}{P_{r,k}}\right)\right]\left(-\frac{\alpha_j}{P_{r,j}}\right)+\lambda=0 \Longrightarrow \lambda=\frac{P_{Out,AF,NDL}(\underline{P})\alpha_j P_s}{P_{r,j}(P_s\alpha_j+P_{r,j})},\ j=1,\ldots,M, \tag{29}$$

$$\sum_{k=1}^{M}\left(-\frac{1}{P_s^2}\right)\prod_{i=1,i\neq k}^{M}\left(\frac{1}{P_s}+\frac{\alpha_i}{P_{r,i}}\right)+\lambda=0 \Longrightarrow \lambda=\frac{P_{Out,AF,NDL}(\underline{P})}{P_s}\sum_{k=1}^{M}\frac{P_{r,k}}{P_s(P_s\alpha_k+P_{r,k})} \tag{38}$$

$$\left[\prod_{i=1,i\neq j}^{M}\left(\frac{1}{P_s}+\frac{\alpha_i}{P_{r,i}}\right)\right]\left(-\frac{\alpha_j}{P_{r,j}^2}\right)+\lambda=0 \Longrightarrow \lambda=\frac{P_{Out,AF,NDL}(\underline{P})\alpha_j P_s}{P_{r,j}(P_s\alpha_j+P_{r,j})}\ j=1,\ldots,M, \tag{39}$$

B. Decode-and-Forward Protocol

Unlike the case with the AF protocol, due to the nature of (10) and (12), arriving at an optimal power vector for the DF protocol is rather cumbersome, even for a symmetric relay network with $\alpha_j=\alpha$, $\forall\, j=1,\ldots,M$. In what follows, we restrict our attention to $M=1$ and 2 relay nodes.

1) $M=1$ Relay Node: Let us consider $M=1$ first. The possible decoding sets are $\mathcal{D}=\phi$ (i.e., the relay is unable to decode) and $\mathcal{D}=\{1\}$ (the relay successfully decodes). Then, with a direct link, using (10), and ignoring the constant, the optimization problem of (25) reduces to

$$\begin{aligned}&\text{minimize} && \frac{1}{P_s^2}+\frac{1}{2}\frac{1}{P_s}\frac{\alpha_1}{P_{r,1}}=\frac{1}{P_s}\left(\frac{1}{P_s}+\frac{\widehat{\alpha}_1}{P_{r,1}}\right)\\ &\text{subject to} && P_s+P_{r,1}\le P_T,\end{aligned} \tag{45}$$

where $\widehat{\alpha}_1=\alpha_1/2$. Comparing (45) with (26) with $M=1$, we notice that the DF protocol outage probability expression is very similar to that of the AF protocol. It then follows that, upon defining $P_s=\tau_0 P_T$ and $P_{r,1}=(1-\tau_0)P_T$, $0<\tau_0\le 1$, τ_0 can be obtained directly from (35) with $M=1$ and α in (35) replaced by $\widehat{\alpha}_1=\alpha_1/2$. That is,

$$\tau_0=\frac{2}{2-\alpha_1}\left[1-\frac{\alpha_1}{8}\left(1+\sqrt{1+\frac{16}{\alpha_1}}\right)\right]. \tag{46}$$

As $\alpha_1\to 2$, using the L'Hospital rule in (46), we have $\tau_0\to 2/3$.

In the absence of a direct link, from (12) with $\mathcal{D}=\phi$ and $\{1\}$, the optimization problem is

$$\begin{aligned}&\text{minimize} && \frac{1}{P_s}+\frac{\alpha_1}{P_{r,1}}\\ &\text{subject to} && P_s+P_{r,1}\le P_T.\end{aligned} \tag{47}$$

Comparing (47) with (36) with $M=1$, we notice that the DF protocol outage probability expression is exactly the same as that of the AF protocol. It then follows that, upon defining $P_s=\mu_0 P_T$ and $P_{r,1}=(1-\mu_0)P_T$, $0<\mu_0\le 1$, μ_0 can be obtained directly from (44) with $M=1$ as

$$\mu_0=\frac{1}{1+\sqrt{\alpha_1}}, \tag{48}$$

which also coincides with [19, Eqn. (8)].

2) $M=2$ Relay Nodes: With $M=2$ relays, we have $\mathcal{D}=\phi,\{1\},\{2\}$, and $\{1,2\}$, and the optimization problem with a direct link is

$$\begin{aligned}&\text{minimize} && \frac{1}{P_s^3}+\frac{1}{2}\frac{1}{P_s^2}\left(\frac{\alpha_1}{P_{r,1}}+\frac{\alpha_2}{P_{r,2}}\right)+\frac{1}{6}\frac{1}{P_s}\frac{\alpha_1\alpha_2}{P_{r,1}P_{r,2}}\\ &\text{subject to} && P_s+P_{r,1}+P_{r,2}\le P_T.\end{aligned} \tag{49}$$

Upon setting the derivatives of the Lagrange cost function w.r.t P_s, $P_{r,1}$, $P_{r,2}$, and λ to zero, we have (50), (51), and (52) on the following page, and

$$P_s+P_{r,1}+P_{r,2}=P_T. \tag{53}$$

Equating (51) with (50), and (51) with (52), we have

$$\begin{aligned}&P_s^3(\alpha_1\alpha_2/3)-P_s^2(\alpha_1\alpha_2 P_{r,1}/3-\alpha_1 P_{r,2})\\ &-P_s(2\alpha_1 P_{r,1}P_{r,2}+2\alpha_2 P_{r,1}^2)-6P_{r,1}^2P_{r,2}=0\end{aligned} \tag{54}$$

and

$$P_s(P_{r,2}-P_{r,1})\alpha_1\alpha_2/3-(\alpha_2 P_{r,1}^2-\alpha_1 P_{r,2}^2)=0. \tag{55}$$

Eqns. (53), (54) and (55) constitute three equations in three unknowns, P_s, $P_{r,1}$, and $P_{r,2}$, and can be solved numerically to arrive at the optimal power vector. For the case of a symmetric relay network, we have $\alpha_1=\alpha_2$. With this, (55) gives us $P_{r,1}=P_{r,2}$. Further, let $P_s=\epsilon P_{r,1}=\epsilon P_T/(2+\epsilon)$, $0<\epsilon<1$. Then substituting in (55) results in the following cubic equation[6] in ϵ:

$$\epsilon^3-\epsilon^2(1-3/\alpha)-\epsilon(12/\alpha)-18/\alpha^2=0 \tag{56}$$

which has at least one real root. Since the objective function is strictly convex in $\underline{P}$, it then follows that there exists only

[6]In general, for a symmetric relay network with M relay nodes, one has to find the unique positive root of a polynomial of degree $M+1$.

$$-\frac{3}{P_s^4}-\frac{2}{P_s^3}\left(\frac{\alpha_1}{P_{r,1}}+\frac{\alpha_2}{P_{r,2}}\right)-\frac{1}{P_s^2}\frac{1}{6}\frac{\alpha_1\alpha_2}{P_{r,1}P_{r,2}}+\lambda=0\Longrightarrow\lambda=\frac{3}{P_s^4}+\frac{2}{P_s^3}\left(\frac{\alpha_1}{P_{r,1}}+\frac{\alpha_2}{P_{r,2}}\right)+\frac{1}{P_s^2}\frac{1}{6}\frac{\alpha_1\alpha_2}{P_{r,1}P_{r,2}}, \tag{50}$$

$$-\frac{1}{2P_s^2}\frac{\alpha_1}{P_{r,1}^2}-\frac{1}{6P_s}\frac{\alpha_1\alpha_2}{P_{r,1}^2P_{r,2}}+\lambda=0 \quad\Longrightarrow\lambda=\frac{1}{2P_s^2}\frac{\alpha_1}{P_{r,1}^2}+\frac{1}{6P_s}\frac{\alpha_1\alpha_2}{P_{r,1}^2P_{r,2}}, \tag{51}$$

$$-\frac{1}{2P_s^2}\frac{\alpha_2}{P_{r,2}^2}-\frac{1}{6P_s}\frac{\alpha_1\alpha_2}{P_{r,1}P_{r,2}^2}+\lambda=0 \quad\Longrightarrow\lambda=\frac{1}{2P_s^2}\frac{\alpha_2}{P_{r,2}^2}+\frac{1}{6P_s}\frac{\alpha_1\alpha_2}{P_{r,1}P_{r,2}^2}, \tag{52}$$

one positive root of (56). When $\alpha\to\infty$, (56) yields $\epsilon=1$. That is, $P_s=P_{r,1}=P_{r,2}=P_T/3$, implying the optimality of equal power allocation as $\alpha\to\infty$.

In the absence of a direct link, with $M=2$, the Lagrangian cost function is

$$\begin{aligned}\mathcal{J}(\underline{P},\lambda) &= \frac{1}{P_s^2}+\frac{1}{P_s}\frac{\alpha_1}{P_{r,1}}+\frac{1}{P_s}\frac{\alpha_2}{P_{r,2}}\\ &+ \frac{1}{2}\frac{\alpha_1\alpha_2}{P_{r,1}P_{r,2}}+\lambda\left(P_s+\sum_{j=1}^{M}P_{r,j}-P_T\right).\end{aligned} \tag{57}$$

Upon setting the derivatives of (57) w.r.t P_s, $P_{r,1}$, $P_{r,2}$, and λ to zero, we have (58), (59), and (60) on the following page, and (53). Upon equating (59) with (60), and (59) with (58), we have

$$(\alpha_1P_{r,2}^2-\alpha_2P_{r,1}^2)=\alpha_1\alpha_2P_s(P_{r,1}-P_{r,2})/2 \tag{61}$$

and

$$\begin{aligned}P_s^3(\alpha_1\alpha_2/2) &+ P_s^2(\alpha_1P_{r,2})\\ &- P_s(\alpha_1P_{r,1}P_{r,2}+\alpha_2P_{r,1}^2)\\ &- 2P_{r,1}^2P_{r,2}=0.\end{aligned} \tag{62}$$

For a general (α_1,α_2), a numerical approach is needed to solve the above nonlinear equations. On the other hand, with $\alpha_1=\alpha_2=\alpha$, we once again have $P_{r,1}=P_{r,2}$ from (61). Upon letting $P_s=\kappa P_{r,1}=\kappa P_T/(2+\kappa)$, (62) results in

$$\kappa^3+\kappa^2(2/\alpha)-\kappa(4/\alpha)-4/\alpha^2=0. \tag{63}$$

Eqn. (63) is significantly different from (56) in the following way: While (56) shows that $\epsilon\to1$ as $\alpha\to\infty$, (63) gives us $\kappa\to0$ as $\alpha\to\infty$. That is, arbitrarily small power is needed for the source to transmit, instead of one-third of the total power allocation, when $\alpha\to\infty$.

C. Distributed Space-Time Coded Protocol

Upon comparing the outage probability expression for DSTC of (21) with that of (10) for the DF protocol, we notice that, in addition to differing in multiplicative constants, the factor $1/(|\mathcal{D}|+1)!$ in (10) is replaced by $\mathcal{A}_{|\mathcal{D}|}(2^{2R}-1)$ in (21). In fact,

$$\mathcal{A}_n(0)=\frac{1}{(n-1)!}\int_{u=0}^{1}u^{n-1}(1-u)du=\frac{1}{(n+1)!}, \tag{64}$$

which implies that the optimal power vector of the DF protocol is indeed a special case of that of the DSTC protocol. The important difference is that the optimal power allocation vector of the DSTC protocol *depends on the transmission rate* R. A simple, but interesting, case is that of $M=2$ relay nodes, which forms a basis for implementing distributed Alamouti STBC.

With $M=2$ relays, the analysis is very similar to that in Section IV-B, and hence we skip it for brevity. For a general (α_1,α_2), we obtain sets of equations very similar to (54) and (55), but with a dependence on $\mathcal{A}_1(2^{2R}-1)$ and $\mathcal{A}_2(2^{2R}-1)$. That is, we have (65) and (66) on the following page. On the other hand, when $\alpha_1=\alpha_2=\alpha$, similar to (56) we have the cubic equation

$$\begin{aligned}&\epsilon^3-\epsilon^2\left(1-\mathcal{A}_1(2^{2R}-1)/(\mathcal{A}_2(2^{2R}-1)\alpha)\right)\\ &\quad-\epsilon\left(4\mathcal{A}_1(2^{2R}-1)/(\mathcal{A}_2(2^{2R}-1)\alpha)\right)\\ &\quad-\left(3/(\mathcal{A}_2(2^{2R}-1)\alpha^2)\right)=0.\end{aligned} \tag{67}$$

As $\alpha\to\infty$, (67) gives us $\epsilon\to1$, which is not a function of R, thus showing the optimality of equal power allocation. As a quick check, by setting $\mathcal{A}_1(2^{2R}-1)=1/2$ and $\mathcal{A}_2(2^{2R}-1)=1/6$, (67) reduces to (56) of the DF protocol.

Without a direct link, ignoring the constant, the outage probability expression in (22) of the DSTC protocol has a form very similar to that of the DF protocol of (12). Since the optimum power vector is not a function of the multiplicative constant of the objective function, it then follows that the optimal power vector of the DSTC protocol is exactly the same as that of the DF protocol.

V. Coding Gain Considerations

For simplicity, let us define $\Gamma=P_T/\sigma_N^2$, as the average SNR. For sufficiently large Γ, the outage probability can be written as $P_{Out}\approx(\mathsf{G}_c\Gamma)^{-\mathsf{G}_d}$ [23], where G_d is the so-called diversity gain, and G_c can be viewed as the asymptotic coding gain. Note that the use of the term "coding gain" is seemingly a misnomer, since there is no explicit forward error correction in the systems being analyzed. However, the term has been used in the literature, and so we adopt it here. With equal power allocation, [22] studied the coding gain performance of both AF and DF protocols. We now present the coding gain expressions for the AF, DF and DSTC protocols with optimal power allocation. First, upon setting $\underline{P}=[\delta_0,\delta_1,\ldots,\delta_M]\times P_T$ in (4), the ACG of the AF protocol with a direct link is given in (68) below. Upon letting $\underline{P}=[\zeta_0,\zeta_1,\ldots,\zeta_M]\times P_T$ in (5), the ACG without a direct link is

$$\mathsf{G}_{c,AF,NDL}=\frac{\zeta_0}{2^{(M+1)R}-1}\left[M!\prod_{j=1}^{M}\left(\frac{\Omega_2^j\zeta_j}{\zeta_j+\zeta_0\alpha_j}\right)\right]^{\frac{1}{M}}. \tag{69}$$

$$-\frac{2}{P_s^3}-\frac{1}{P_s^2}\frac{\alpha_1}{P_{r,1}}-\frac{1}{P_s^2}\frac{\alpha_2}{P_{r,2}}+\lambda=0 \implies \lambda=\frac{2}{P_s^3}+\frac{1}{P_s^2}\left(\frac{\alpha_1}{P_{r,1}}+\frac{\alpha_2}{P_{r,2}}\right), \tag{58}$$

$$-\frac{1}{P_s}\frac{\alpha_1}{P_{r,1}^2}-\frac{1}{2}\frac{\alpha_1\alpha_2}{P_{r,1}^2P_{r,2}}+\lambda=0 \implies \lambda=\frac{1}{P_{r,1}}\left(\frac{1}{P_s}\frac{\alpha_1}{P_{r,1}}+\frac{1}{2}\frac{\alpha_1\alpha_2}{P_{r,1}P_{r,2}}\right), \tag{59}$$

$$-\frac{1}{P_s}\frac{\alpha_2}{P_{r,2}^2}-\frac{1}{2}\frac{\alpha_1\alpha_2}{P_{r,1}P_{r,2}^2}+\lambda=0 \implies \lambda=\frac{1}{P_{r,2}}\left(\frac{1}{P_s}\frac{\alpha_2}{P_{r,2}}+\frac{1}{2}\frac{\alpha_1\alpha_2}{P_{r,1}P_{r,2}}\right), \tag{60}$$

$$P_s^3(\mathcal{A}_2(2^{2R}-1)\alpha_1\alpha_2)-P_s^2(\mathcal{A}_2(2^{2R}-1)\alpha_1\alpha_2P_{r,1}-\mathcal{A}_1(2^{2R}-1)\alpha_1P_{r,2})$$
$$-P_s(2\mathcal{A}_1(2^{2R}-1)\alpha_1P_{r,1}P_{r,2}+2\mathcal{A}_1(2^{2R}-1)\alpha_2P_{r,1}^2)-3P_{r,1}^2P_{r,2}=0, \tag{65}$$
$$\text{and}\quad \mathcal{A}_1(2^{2R}-1)\left(\alpha_1P_{r,2}^2-\alpha_2P_{r,1}^2\right)-\mathcal{A}_2(2^{2R}-1)\alpha_1\alpha_2P_s\left(P_{r,1}-P_{r,2}\right)=0. \tag{66}$$

$$\mathsf{G}_{c,AF,DL}=\frac{\delta_0}{2^{(M+1)R}-1}\left[(M+1)!\Omega_1\prod_{j=1}^{M}\left(\frac{\Omega_2^j\delta_j}{\delta_j+\delta_0\alpha_j}\right)\right]^{\frac{1}{M+1}}. \tag{68}$$

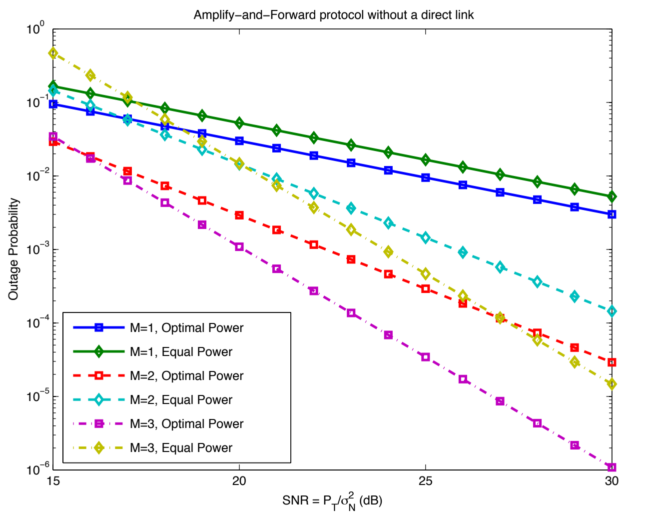

Fig. 4. Outage probability of the AF protocol with both equal and optimal power allocation. One, two and three relays are considered without a direct link between the source and the destination.

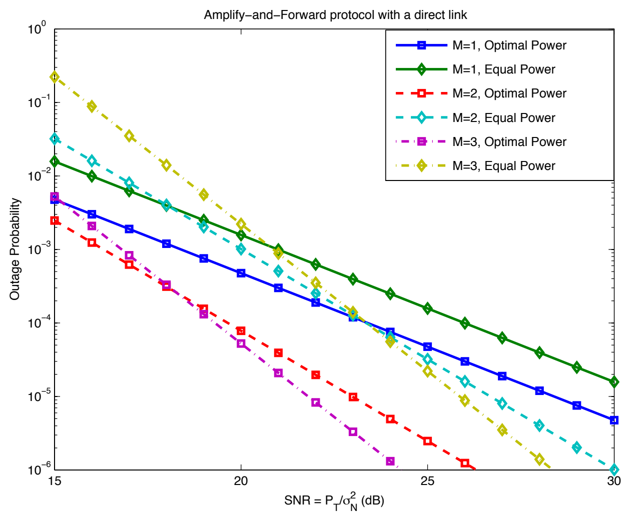

Fig. 5. Outage probability of the AF protocol with both equal and optimal power allocation. One, two and three relays are considered with a direct link between the source and the destination.

By setting $\delta_j=P_T/(M+1)$ in (68) and $\zeta_j=P_T/(M+1)$ in (69), $j=0,1,\ldots,M$, we obtain the ACG expressions with an equal power allocation [22]. Similar expressions can easily be found for the DF and DSTC protocols. The coding gain ratio of a protocol with optimal power allocation is defined as $\mathsf{CGR}=\mathsf{G}_c(\text{Optimal Alloc})/\mathsf{G}_c(\text{Equal Alloc})$. With the AF protocol, these are

$$\mathsf{CGR}_{AF,DL}=\delta_0\times(M+1)\times\left(\prod_{j=1}^{M}\frac{\delta_j(1+\alpha_j)}{\delta_j+\delta_0\alpha_j}\right)^{\frac{1}{M+1}} \tag{70}$$

and

$$\mathsf{CGR}_{AF,NDL}=\zeta_0\times(M+1)\times\left(\prod_{j=1}^{M}\frac{\zeta_j(1+\alpha_j)}{\zeta_j+\zeta_0\alpha_j}\right)^{\frac{1}{M}}. \tag{71}$$

With a single relay, the CGRs of the DF protocol are

$$\mathsf{CGR}_{DF,DL}=2\tau_0\sqrt{\frac{(1-\tau_0)(2+\alpha_1)}{2(1-\tau_0)+\tau_0\alpha_1}} \tag{72}$$

and

$$\text{and}\quad \mathsf{CGR}_{DF,NDL}=\frac{2(1+\alpha_1)}{(1+\sqrt{\alpha_1})^2}. \tag{73}$$

With either $\alpha_1\to 0$ or $\alpha_1\to\infty$, (73) predicts a 3 dB improvement in ACG with a single relay node and optimal power allocation.

VI. Results and Discussion

We now present some numerical results illustrating the performance gains of various relaying protocols with optimal transmit power allocation. The relay network topology is the same as that described in Section III-D. The outage performance of the AF protocol is shown in Figs. 4 and

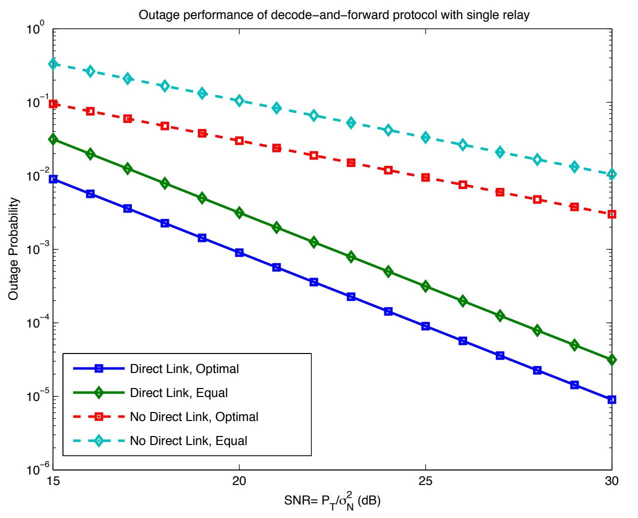

Fig. 6. Outage probability of the DF protocol with both equal and optimal power allocation. A single relay is assumed without and with a direct link between the source and the destination.

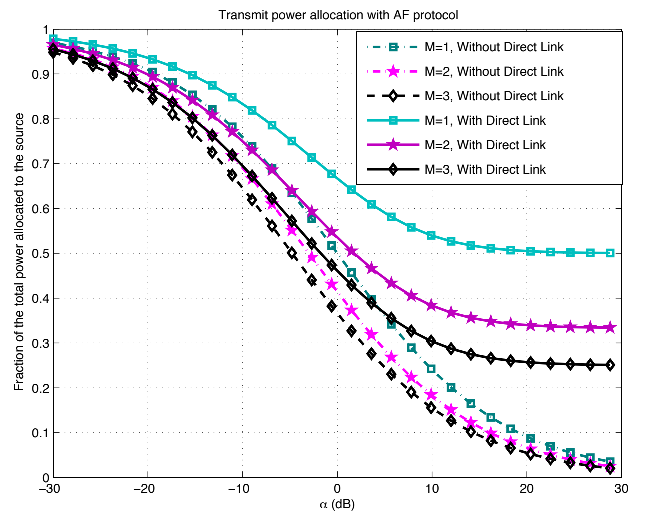

Fig. 7. The fraction of the total power allocated to the source with M-relay AF protocol, for a symmetric relay network, as a function of $\alpha = \Omega_2/\Omega_3$. Both the cases without and with a direct link between the source and the destination are considered.

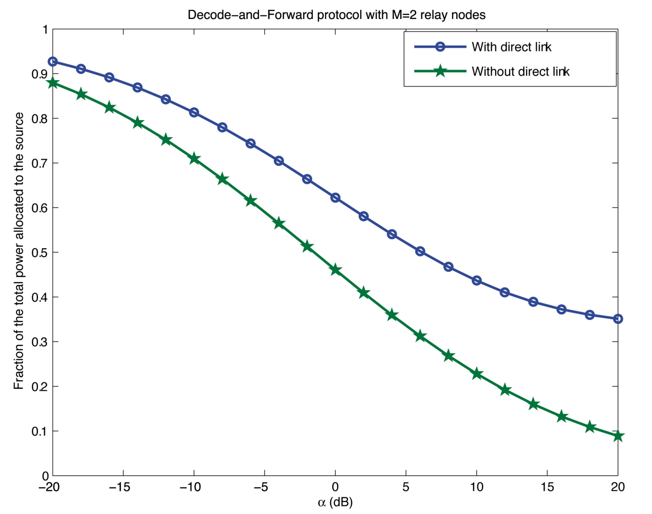

Fig. 8. Fraction of the total power expended by the source as a function of $\alpha = \Omega_2/\Omega_3$. A DF protocol is assumed with a direct link from the source to the destination with $M = 2$ relay nodes in a symmetric relay network (i.e., $\alpha_1 = \alpha_2 = \alpha = \Omega_2/\Omega_3$).

5 without and with a direct link, respectively. $M = 1, 2,$ and 3 relays are considered. The information rate is $R = 1$ bit/sec/Hz. Without a direct link, Fig. 4 shows that a 2 dB improvement can be obtained with optimal power allocation at an outage level of 10^{-2} with a single relay, whereas these gains, at an outage level of 10^{-3}, increase to 3.5 dB with two relays, and 4 dB with three relays. As shown in Fig. 5, existence of a direct link boosts the outage performance by providing an additional diversity path.

Fig. 5 shows that, at an outage of 10^{-3}, optimal allocation with a single relay provides a significant gain of 3 dB. The gains increase to 3.8 and 4 dB with two and three relay nodes, respectively, at an outage level of 10^{-5}. With the same topology as that of the AF protocol, the outage performance of a single relay (location at θ_1) based DF protocol is presented in Fig. 6 without and with a direct link. Fig. 6 shows that, at an outage probability of 10^{-2}, the SNR gain without a direct link is 5 dB, whereas, at 10^{-3} outage probability, the existence of a direct link provides a gain around 2.0 dB. These gains, by judicious allocation of transmit power across various nodes, and based on the knowledge of the mean channel power gains alone, directly improve the energy efficiency, and thus lead to a longer network lifetime.

Fig. 7 shows the optimal transmit power allocation for the AF protocol under the symmetric relay network assumption. That is, $\alpha_j = \alpha, \forall j = 1, \ldots, M$. Due to this symmetry, as shown before, the source transmits at a power level of P_s, and all other relays transmit at an identical power level of P_r such that $P_s + MP_r = P_T$. Fig. 7 shows P_s as a function of α, parameterized by the number of relays, M. Both the cases of direct and no direct link between the source and the destination are considered. Two observations are made regarding the behavior of P_s as α is varied: First, when there is no direct link between the source and the destination, a larger value of α implies that the relay is closer to the source than the destination, and it enjoys less path loss from the source. This allows the source to reduce its transmit power, and helps the relay to transmit at a higher power level in order to compensate for the path loss between the relay and the destination. This also explains why the source transmit power decreases gradually with α as the number of relays is increased. However, the situation is quite different when there exists a direct link between the source and the destination. In this case, irrespective of the value of α, the source has to expend a non-negligible amount of transmit power in order to reach the destination via the direct link. As $\alpha \to \infty$, the existence of a direct link leads to $P_s \to 1/(M+1)$ (i.e., equal power allocation is asymptotically optimum as $\alpha \to \infty$), whereas without the direct link we have $P_s \to 0$ (which is to be interpreted as the negligible transmit power required by the source to reach the relays).

Next, we present the optimal transmit power allocation for the DF and DSTC protocols for a symmetric network topology. With $M = 2$ relays, the fraction of the power utilized by the source is plotted in Fig. 8 with $M = 2$ relay nodes, without and with a direct link. Similar to the case of the

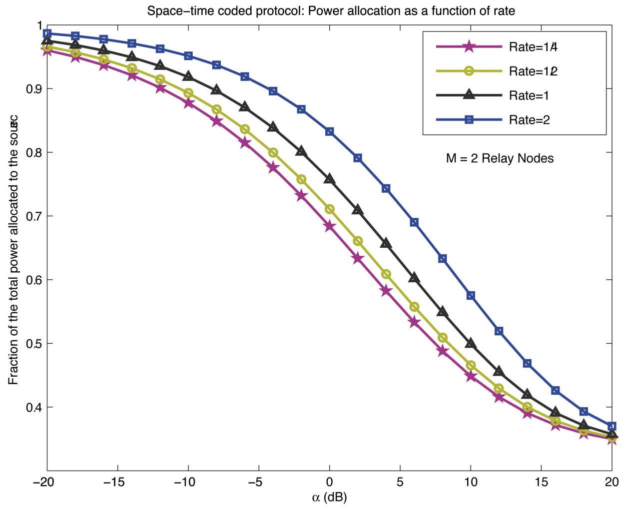

Fig. 9. Fraction of the total power expended by the source as a function of α. A DSTC protocol is assumed with a direct link from the source to the destination with $M = 2$ relay nodes in a symmetric relay network (i.e., $\alpha_1 = \alpha_2 = \alpha = \Omega_2/\Omega_3$).

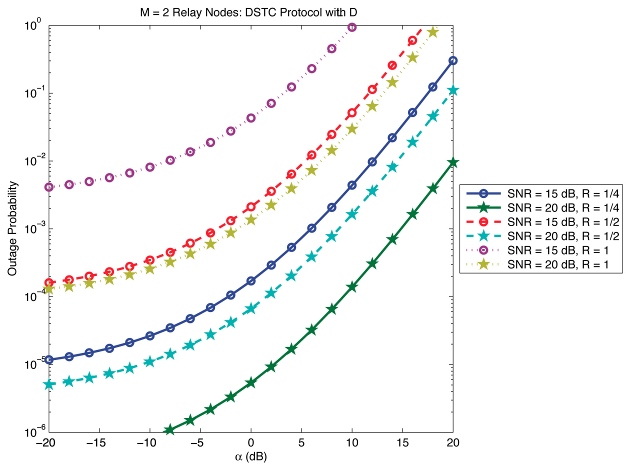

Fig. 10. Outage probability of DSTC protocol with $M = 2$ relay nodes and optimum transmission power allocation. A symmetric relay placement is assumed with $\alpha_1 = \alpha_2 = \alpha = \Omega_2/\Omega_3$ and a direct link between the source and the destination. The outage curves are parameterized by the transmission rate R, in bits/sec/Hz, and the SNR P_T/σ_N^2.

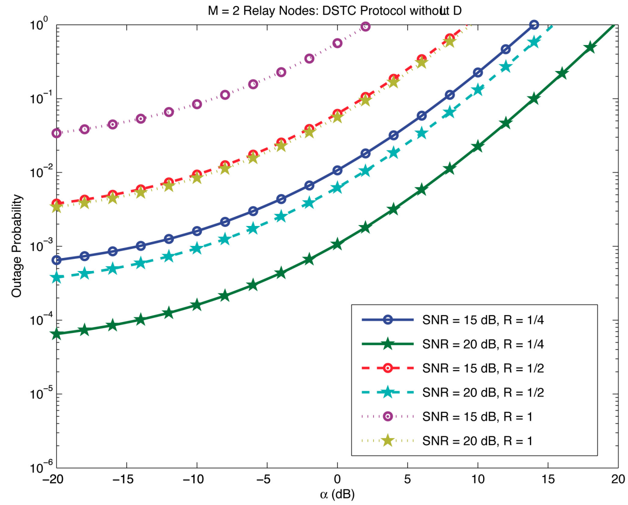

Fig. 11. Outage probability of DSTC protocol with $M = 2$ relay nodes and optimum transmission power allocation. A symmetric relay placement is assumed with $\alpha_1 = \alpha_2 = \alpha = \Omega_2/\Omega_3$ *without* a direct link between the source and the destination. The outage curves are parameterized by the transmission rate R, in bits/sec/Hz, and the SNR P_T/σ_N^2.

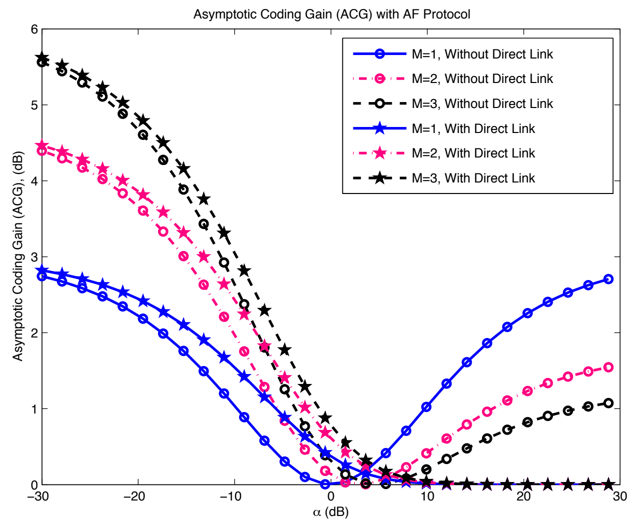

Fig. 12. Asymptotic coding gain (ACG) improvement with the AF protocol with optimum power allocation. Both the cases without and with a direct link between the source and the destination are considered. Here, $\alpha = \Omega_2/\Omega_3$.

AF protocol, existence of a direct link requires the source to draw significantly more power than without a direct link. For example, when $\alpha = 10$ dB, the source power with a direct link is twice the power without a direct link, and is approximately three times the power without a direct link when $\alpha = 17$ dB. The results of Fig. 8 are also applicable for a DSTC protocol without a direct link. However, with a direct link, the power allocation for a DSTC protocol is dependent on the transmission rate. Fig. 9 shows this dependency with the transmission rate $R \in \{1/4, 1/2, 1, 2\}$ bits/sec/Hz, and with two relays. From Fig. 9, we notice that more power should be invested in the source transmissions for increasing data rates.

Outage performance of the DSTC protocol with two relays is presented, as a function of α, for a symmetric network in Figs. 10 and 11 for scenarios with and without a direct link, respectively. The outage curves are parameterized by the rate $R \in \{1/4, 1/2, 1\}$ and the average SNR, $P_T/\sigma_N^2 \in \{15, 20\}$ dB. Without loss of generality, we set $\Omega_2^j = 1$ for $j = 1, 2$, and let $\Omega_1 = 1$ with a direct link. In Figs. 10 and 11, a large value of α implies that the relay nodes are far away from the destination, and their signals are received at the destination with severe attenuation. This leads to a degradation in the outage performance, and is more pronounced at low SNR and when there is no direct link between the source and the destination (i.e., no signal contribution from the source to the destination). Upon comparing Fig. 11 with Fig. 10 in terms of attempted transmission rate, we observe that an increase in the rate can significantly degrade the outage performance when there is no direct link.

The CGR improvement of the AF protocol with optimal power allocation is plotted as a function of α in Fig. 12 for both the cases without and with a direct link between the source and the destination. The following conclusions can be drawn from Fig. 12: i) In the absence of a direct link, the optimal power allocation provides more coding gain than the equal

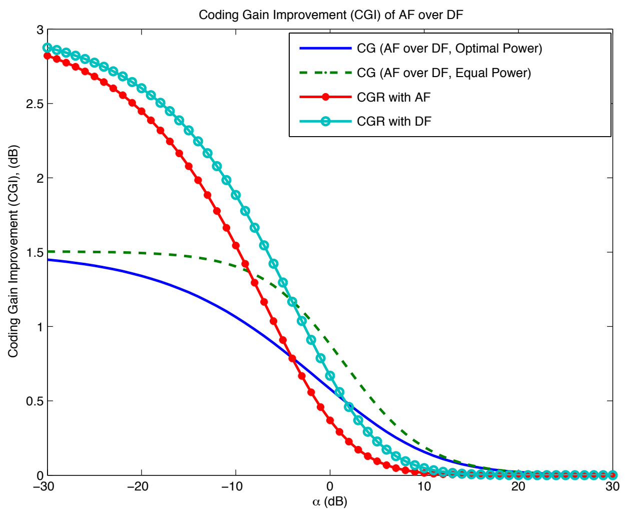

Fig. 13. Comparison of coding gain improvements (CGI) for both AF and DF protocols with optimal power allocation as a function of $\alpha = \Omega_2/\Omega_3$. A single relay node is considered. The coding gain gap between AF and DF protocols is also shown. As seen in this figure, this gap is reduced by the use of optimal power allocation.

power allocation for all values of α, except when $\alpha = M$. When $\alpha = M$, from (44), we have $P_s = P_r = P_T/(M+1)$, and the resulting ACG over equal power allocation is zero. $ii)$ When a direct link exists, the CGR improvement is significant for smaller values of α (i.e., when the relay is farther away from the source than from the destination); on the other hand, as $\alpha \to \infty$, we have $P_s = P_r = P_T/(M+1)$, leading to no further improvement in CGR. In this regime (i.e., for large values of α) the case without a direct link provides more coding gain than the case with a direct link. $iii)$ Interestingly, without a direct link, a large number of relays is beneficial for smaller values of α, in terms of the CGR benefits, whereas a single relay is good enough for large α.

Fig. 13 presents a comparative study of the CGR improvements of the AF and DF protocols with a direct link when a single relay is employed. The ACG of AF over DF, without and with optimal allocation, and the individual CGRs of AF and DF protocols with optimal power allocation over equal power allocation, are considered. Fig. 13 shows that the mean channel gain-based optimal power allocation is more beneficial to the DF protocol than to the AF protocol. We also notice from Fig. 13 that, for $-20 \leq \alpha\,(\text{dB}) \leq 0$, the coding gain gap between the AF and DF protocols is reduced by as much as 0.4 dB with optimal power allocation.

VII. Conclusion and Future Directions

We have presented the optimum transmit power allocation for wireless relaying protocols with mean channel gain information. The results were established, at high SNR, for AF, DF and DSTC protocols operating over a Rayleigh fading channel. At high SNR, the optimal power allocation was shown to depend not only on the ratio of mean power gains, but also on whether or not a direct link between the source and the destination exists. Our results showed that, in addition to the improvements in the outage probability, optimal power allocation yields impressive coding gains over equal power allocation. Furthermore, our analysis revealed that the coding gain gap between the AF and DF protocols can be reduced by the optimal power allocation.

Some future research directions are the following. The present work assumes equal bandwidth for the source and the M relays. However, it may be possible to improve the outage performance by jointly optimizing the transmission power and the channel time/bandwidth resources [12]. Note that our power allocation approach requires a centralized controller to optimally allocate the transmission powers. In practical systems, due to complexity/implementation concerns, it may be highly useful to have a distributed alternative to the approach presented here. In addition to the Lagrange formulation, ideas such as primal- or dual-decomposition techniques [32] may be starting points to perform power allocation in a distributed manner. Finally, we assume that the mean channel gains are perfectly known, which, in practice, have to be estimated from the received signal [33]. Assessing the effects of noisy estimates of mean channel gains on the outage performance is very important to gain a better understanding of the power allocation policies.

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